W. TH. HETTERSCHEID

DESIGNING TRANSISTOR

BANDPASS AMPLIFIERS

in active preparation

The design and construction of I.F. amplifiers with transistors for radio, television and radar receivers is the subject of this work. The properties of the transistors are assumed to be expressed in the admittance parameter system. These parameters are considered in detail as regards their dependence on the d.c. operating point as well as environmental conditions.

A survey of the theory of designing transistor I.F. amplifiers is presented, from which a practical design procedure is developed making use of special design charts. The book contains a large number of these normalized design charts which facilitate a rapid evaluation of the transducer gain, the amplitude response curve and the envelope delay curve of the complete amplifier when the number of transistors in the amplifier, their biasing points and the types of interstage networks have been chosen. The design charts moreover present the necessary information for constructing single or double-tuned interstage networks by a simple conversion of the normalized variables to real variables.

A separate chapter deals with automatic gain control in transistorized amplifiers in relation to both forward and reversed biased gain control methods.

The design procedures described are elucidated by means of six fully worked-out examples.

PHILIPS TECHNICAL LIBRARY

W. Th. HETTERSCHEID

TRANSISTOR BANDPASS AMPLIFIERS

W. Th. HETTERSCHEID

Transistor bandpass amplifiers

PHILIPS TECHNICAL LIBRARY

TRANSISTOR

BANDPASS AMPLIFIERS

The theory of analysis and design of selective amplifiers as used in the I.F. parts of radio, television and radar receivers, is here dealt with, especially in relation to the application of transistors.

Use is made of a four-terminal network representation of the transistors (or vacuum tubes) which facilitates a mathematical description of the performance of the complete amplifier by means of a single determinant. The properties of the transistor are assumed to be expressed in the small signal admittance — or hybrid — *h* parameters.

Single-stage amplifiers as well as multi-stage amplifiers, with arbitrary types of interstage or terminating networks are treated in detail as regards stability, power gain, amplitude response curve and envelope delay curve: also neutralization of the transistor internal feedback and problems associated with spreads in transistor parameters.

THEORY OF TRANSISTOR BANDPASS AMPLIFIERS

HETTERSCHEID

PHILIPS TECHNICAL LIBRARY

TRANSISTOR

BANDPASS AMPLIFIERS

Shortly, by the same author:

DESIGNING TRANSISTOR
BANDPASS AMPLIFIERS

dealing with the practical aspects of the design and construction of I.F. amplifiers with transistors for radio, television and radar receivers. Contains a large number of normalized design charts which facilitate a rapid evaluation of the specific data of the complete amplifier after the number of transistors in the amplifier, their biasing points and the types of interstage networks have been chosen.

Distributors for the United Kingdom and Eire:

CLEAVER-HUME PRESS LTD.

(Macmillan & Co. Ltd.)

10-15 St. Martin's Street, London, W.C. 2

TRANSISTOR BANDPASS AMPLIFIERS

W. Th. H. HETTERSCHEID

1964

PHILIPS TECHNICAL LIBRARY

This book contains xxiv + 314 pages and 189 illustrations
Other editions of this book in Philips Technical Library
in the German and French languages
U.D.C. nr. 621.375.121 : 621.382.3

Printed in The Netherlands

PREFACE

During recent years the transistor has achieved great importance for use as amplifying element in bandpass amplifiers. Although the design of bandpass amplifiers equipped with transistors follows the same lines as comparable amplifiers equipped with electron tubes there are a number of differences which justify the analysis of transistor bandpass amplifiers presented in this book.

If both transistors and electron tubes are considered as four-terminal networks with their inherent parameters the design of the bandpass amplifiers differs mainly in the magnitudes of the parameters of both devices. It are those differences which render the design of bandpass amplifiers equipped with transistors — or, more specifically, amplifiers the interstage coupling of which consists of single or double-tuned bandpass filters — more difficult than that of similar amplifiers equipped with electron tubes.

In the first place, the input- and output dampings of the transistor usually load the tuned circuits of which the bandpass filters are composed to such an extent that the resulting increase in bandwidth and loss in power gain are by no means negligible. Secondly, no matter which electrode is chosen as the common terminal, considerable feedback is present in the transistor, and this also influences the bandwidth and the power gain, and possibly even the tuning. Thirdly, like in all circuits with feedback, there is a risk of instability or considerable asymmetry in the response curve when the circuit is on the verge of becoming unstable. Special measures must therefore be taken to ensure stable operation of the amplifier circuit.

The above aspects, here summarized in only a general form, must be taken into account when designing bandpass amplifiers equipped with transistors. The main design parameter is obviously the stability, the other parameters being the adjacent channel selectivity (especially in I.F. amplifiers) and/or the 3 dB bandwidth and the powergain, whilst in some cases consideration must also be given to the envelope delay curve. Since all these points depend more or less on the method of aligning the amplifier, the tuning procedure must also be investigated.

The points mentioned above also apply to tube amplifiers to some extent. However, if modern penthodes are used and the signal frequency is not very high the influences of these effects on the performance of the amplifier are much less than in amplifiers equipped with transistors.

The stage gain obtainable with a transistor amplifier is limited by the properties of the transistor with which the stage is equipped and by the gain which is permissible in view of the stability requirements of the amplifier. To obtain the specified overall gain of the amplifier with the smallest number of stages, the amplifier must so be designed that each individual stage gives the maximum obtainable gain. Moreover, stability must be ensured and the requirements as to the 3 dB bandwidth and adjacent channel selectivity must be satisfied. As a rule, it will be necessary to seek a compromise between power gain and 3 dB bandwidth and/or adjacent channel selectivity requirements.

As already referred to, in the analyses presented in this book, the transistor will be considered as a four-terminal network specified by either the admittance or hybrid-h parameter matrices. Various aspects of this four-terminal network representation are considered in Chapter 1.

To obtain a clear picture of the various design aspects and of their consequences, in Chapter 2 a detailed discussion is given of a single-stage amplifier. Although such an amplifier is of little practical use except for some specialized cases, its analysis will be most helpful in defining a number of quantities and concepts, the understanding of which is essential for the investigation and design of more complex amplifier arrangements, to be considered in later chapters.

The specialized design aspects of neutralization or unilateralization is dealt with in Chapter 3.

Chapter 4 is devoted to a further analysis of the single-stage amplifier with two single-tuned bandpass filters of Chapter 2. In this chapter especially the problem of optimization of power gain is considered.

The considerations of Chapters 2 and 4 regarding an amplifier stage with two single-tuned bandpass filters are extended in Chapter 6 to an amplifier with two double-tuned bandpass filters.

Chapters 5, 7, 8 and 9 are devoted to the analysis of general n-stage amplifiers with single-tuned bandpass filters, double-tuned bandpass filters or combinations of both. The mathematical formulation of the complete amplifier design problem obtained facilitates the complete calculation of the stability, the gain and the amplitude response and envelope delay curves.

Spreads in transistor parameters and their influences upon the amplifier performance are considered in Chapter 11.

The last chapter deals with problems associated with the influences of practical taps on the tuned circuits of the performance of the amplifier. The taps on the tuned circuits, which are used as impedance transforming devices, are considered as transformers which depart more or less from an ideal transformer.

The theory of transistor bandpass amplifiers presented in this book forms the basis for a second book by the autor entitled "Designing Transistor I.F. Amplifiers" which comprises a complete description of the practical design procedure of I.F. amplifiers. The latter book (Book II) contains a large number of "design charts" with which an optimum design of an I.F. amplifier can be ascertained with great ease and accuracy. These design charts are calculated making use of the theory presented the underlying volume.

This book is based on research carried out during the last years in the Philips Semiconductor Application Laboratory at Nijmegen, Netherlands, under the leadership of Mr. H.H. van Abbe.

The subject-matter of this book forms an extension of early unpublished work carried out by Messrs. C. le Can and A. H. J. Nieveen van Dijkum of this laboratory and of research carried out by Mr. C. J. McCluskey of Philips Electrical Industries, Ltd, Ontario, Canada.

The author wishes to express his gratitude towards his colleagues for the many stimulating discussions and the helpful suggestions. In this respect, he especially wishes to mention Mr. A. H. J. Nieveen van Dijkum, Mr. J. J. Rongen, and Mr. R. J. Nienhuis.

June 1964

The Author

CONTENTS

IMPORTANT TABLES

LIST OF SYMBOLS

A_r	product of the dampings at the input and output terminals of the r^{th} transistor of an amplifier, including g_{11} and g_{22} of this transistor
α	shape factor of a double-tuned bandpass filter
B_{3dB}	bandwidth of an amplifier between the points on bandpass curve at which the responses is 3dB down
β	relative detuning
C	capacitance
γ_{11}, γ_{12} γ_{21}, γ_{22}	general notation for the four-terminal network parameters of an active network, in particular, a transistor or electron tube
$\Gamma_{11}, \Gamma_{12},$ Γ_{21}, Γ_{22}	general notation for the four-terminal network parameters of a passive network, in particular, a double-tuned bandpass filter
${}_n\Delta$	main determinant of a complete n-stage amplifier
$n\delta_0$	value of ${}_n\delta$ for a n-stage amplifier at the tuning frequency ($x = 0$)
$n\delta$	reduced determinant of a complete n-stage amplifier
Φ	power gain
Φ_{M}	maximum power gain
Φ_a	available power gain
$\Phi_{a\mathrm{M}}$	maximum available power gain
$n\Phi_f$	gain (or losses) due to the feedback of a complete n-stage amplifier
Φ_{fr}	gain (or losses) due to the feedback of the r^{th} stage of an n-stage amplifier
Φ_i	insertion losses of a complete bandpass filter (either single-tuned or double-tuned)
Φ_{mm}	mismatch losses
Φ_p	damping ratio of the primary of a double-tuned bandpass filter
Φ_q	losses due to a non-critical coupling of a double-tuned bandpass filter
Φ_s	damping ratio of the secondary of a double-tuned bandpass filter
${}_n\Phi_t$	transducer gain of an n-stage amplifier
$\Phi_{u\mathrm{M}}$	maximum unilateralized power gain of amplifying device
g_{inf}	extra input damping due to feedback
f_t	cut-off frequency

G_r	damping at resonance of the r^{th} tuned circuit of an amplifier, this circuit being loaded
$G_r{}^*$	damping at resonance of the r^{th} tuned circuit of an amplifier, this circuit being unloaded
h_{11}, h_{12}, h_{21}, h_{22}	hybrid-h four-terminal network parameters of a transistor
H_{11}, H_{12}, H_{12}, H_{22}	hybrid-h four-terminal network parameters of a passive network
${}_nH_t$	forward current transfer ratio of an n-stage amplifier
K_{11}, K_{12}, K_{21}, K_{22}	hybrid-k four-terminal network parameters of a passive network, in particular, a double-tuned bandpass filter
${}_nK_t$	forward voltage transfer ratio of an n-stage amplifier
M	$M = \lvert\gamma_{12}\gamma_{21}\rvert$
N	$N = \left\lvert\dfrac{\gamma_{21}}{\gamma_{12}}\right\rvert$
n	number of stages; tapping ratio
P_i	input power
P_o	output power
$P_{r\mathrm{M}}$	value of minor determinant of order r at the tuning frequency, tuning method B being applied
P_{Sav}	available power from the source
p_1, p_2	co-factors for the tuning correction terms
Q_r	loaded quality factor of the r^{th} tuned circuit of an amplifier
Q_o	unloaded quality factor of a tuned circuit
$Q_{r\mathrm{M}}$	value of minor determinant of order r at the tuning frequency, tuning method C being applied
q_r	coupling coefficient of the r^{th} double-tuned bandpass filter
r	suffix attached to symbols referring to items which occur several times in an amplifier. The suffix r denotes that the symbol refers to the r^{th} item, starting at the output side
r_r	factor indicating the ratio of the secondary to primary loaded quality factors of the r^{th} double-tuned bandpass filter
${}_nS_r$	cascaded stability factor of the r^{th} stage of a cascade of n-stages
s_r	isolated stability factor of the r^{th} stage of an amplifier
T_g	regeneration coefficient on the boundary of stability
T_r	regeneration coefficient of the r^{th} active fourpole of an amplifier
t	intrinsic regeneration coefficient
Θ_r	regeneration phase angle of the r^{th} active fourpole of an amplifier: $\Theta = \arg {}_r\gamma_{12} + \arg {}_r\gamma_{21}$

t_e	envelope delay
τ_e	reduced envelope delay
${}_nU_r$	loopgain of the r^{th} stage of a cascade of n amplifier stages
u_r	loopgain of an isolated amplifier stage
w_r	ratio of the loaded to the unloaded quality factor of the r^{th} tuned circuit of an amplifier
x_r	normalized frequency (or normalized detuning) of the r^{th} tuned circuit
x'_r	tuning correction term of the r^{th} tuned circuit of an amplifier for tuning method B
x_r''	tuning correction term of the r^{th} tuned circuit of an amplifier for tuning method C
y_{11}, y_{12}, y_{21}, y_{22}	four-terminal network admittance parameters of a transistor or electron tube
y_{ib}, y_{rb}, y_{fb}, y_{ob}	four-terminal network admittance parameters of a transistor in common base connection
y_{ie}, y_{re}, y_{fe}, y_{oe}	four-terminal network admittance parameters of a transistor in common emitter connection
y_{ic}, y_{rc}, y_{fc}, y_{oc}	four-terminal network admittance parameters of a transistor in common collector connection
Y_{11}, Y_{12}, Y_{21}, Y_{22}	four-terminal network admittance parameters of a passive network, in particular, a double-tuned bandpass filter
Y	general admittance symbol: since $Y = G+\mathrm{j}B$, both G and B are not defined separately when the corresponding Y is defined
Y_L	load admittance
Y_r	admittance of the r^{th} tuned circuit of an amplifier, this circuit being loaded.
Y_r*	admittance of the r^{th} tuned circuit of an amplifier, this circuit being unloaded
Y_S	source admittance
${}_nY_t$	forward transfer admittance of an n-stage amplifier
${}_nZ_t$	forward transfer impedance of an n-stage amplifier

CHAPTER 1

REPRESENTATION OF TRANSISTORS BY A FOUR-TERMINAL NETWORK

In order to facilitate the design of amplifiers equipped with transistors a method must be found of representing the transistor. For bandpass amplifiers which are to be considered in this book the method chosen must be suitable for solving problems of stability, gain, amplitude response and phase response of the amplifier. The amplifier characteristics mentioned depend, as far as the transistors are concerned, only on the external electrical properties of these active devices. To design or analyse such an amplifier only the current and voltages at the input and output terminals of the transistor need thus be investigated. The transistor may therefore be represented as a "black box" with a number of terminals.

1.1 The Transistor as a Four-Terminal Network

The transistor, which is basically a three-terminal device, may thus be considered as a "black box" provided with two pairs of terminals as represented in Fig. 1.1. One terminal of the transistor is used as a common terminal in forming input and output pairs of terminals. Depending on which terminal is taken as common, the transistor is said to be connected in either the common-base, the common-emitter or the common-collector configuration.

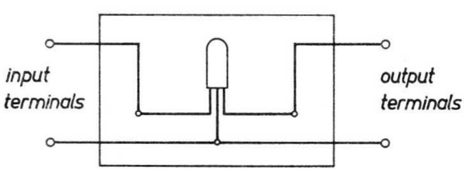


Fig. 1.1. The transistor as a four-terminal network

For our amplifier design or analysis we may therefore consider the transistor as a two-terminal pair network (or four-terminal network) to which the results and methods of four-pole theory may be applied. Fig. 1.2 representing such a "black box", gives the notation of instantaneous currents and voltages. The arrows for the currents indicate positive directions, whereas the arrows for the voltages point to terminals at which the voltage is posi-

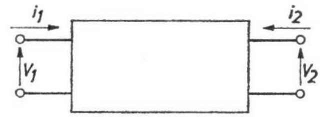


Fig. 1.2. "Black box" representation of a transistor showing the notation of the instantaneous currents and voltages. The arrows for the currents indicate positive direction and the arrows for the voltages point to terminals at which the voltage is positive.

tive. This method of indicating the signs of currents and voltages will be adopted throughout this book.

For this black box or fourpole, six different combinations of functions can be written down which relate the quantities I_1, V_1, I_2 and V_2 in various ways depending on which of the two quantities is taken as the independent variables and which as the dependent variables. Of these six combinations of functions four combinations are of interest for our amplifier considerations. These combinations are:

$$\left.\begin{array}{ll} V_1 = Z_1(I_1, I_2), & I_1 = Y_1(V_1, V_2), \\ V_2 = Z_2(I_1, I_2), & I_2 = Y_2(V_1, V_2), \\ & \\ V_1 = H_1(I_1, V_2), & I_1 = K_1(V_1, I_2), \\ I_2 = H_2(I_1, V_2), & V_2 = K_2(V_1, I_2). \end{array}\right\} \quad (1.1.1)$$

Fundamentally these four combinations of functions are all suited for representing the transistor. Depending on the type of application of the transistor, however, it may happen that a certain combination of functions characterizes the transistor four-terminal network better than other combinations. Furthermore there might be a preference for a certain set of combinations because of the circuitry around the transistor. Both cases will be dealt with in the following sections.

The relations between the voltages and currents at the input and output pairs of terminals of the transistor are generally non-linear functions. In the bandpass amplifiers under consideration, only small signal operation has to be dealt with. The signals can then be considered as incremental variations of the direct currents and voltages at the terminals and the increments can be expressed by means of a Taylor series.

1.1.1 ADMITTANCE PARAMETER REPRESENTATION

To investigate the functions describing the electrical behaviour of the transistor as given by Eq. (1.1.1) in more detail we first consider the combination:

$$\left.\begin{aligned} I_1 &= Y_1,(V_1,\ V_2) \\ I_2 &= Y_2(V_1,\ V_2). \end{aligned}\right\} \qquad (1.1.2)$$

Assuming that the increments δV_1 and δV_2 of V_1 and V_2 respectively are small, the increments δI_1 and δI_2 of I_1 and I_2 respectively can be expressed in a Taylor series for two variables as:

$$\left.\begin{aligned} \delta I_1 &= \frac{\partial I_1}{\partial V_1}\cdot\delta V_1 + \frac{\partial I_1}{\partial V_2}\cdot\delta V_2 + \frac{1}{2}\frac{\partial^2 I_1}{\partial V_1{}^2}\,\delta V_1{}^2 + \frac{1}{2}\frac{\partial^2 I_1}{\partial V_2{}^2}\,\delta V_2{}^2 + \\ &\qquad + \frac{1}{2}\frac{\partial I_1}{\partial V_1}\cdot\frac{\partial I_1}{\partial V_2}\,\delta V_1\,.\,\delta V_2 + \ldots, \\ \delta I_2 &= \frac{\partial I_2}{\partial V_1}\cdot\delta V_1 + \frac{\partial I_2}{\partial V_2}\,\delta V_2 + \frac{1}{2}\frac{\partial^2 I_2}{\partial V_1{}^2}\,\delta V_1{}^2 + \frac{1}{2}\frac{\partial^2 I_2}{\partial V_2{}^2}\,\delta V_2{}^2 + \\ &\qquad + \frac{1}{2}\frac{\partial I_2}{\partial V_1}\cdot\frac{\partial I_2}{\partial V_2}\cdot\delta V_1\,.\,\delta V_2 + \ldots. \end{aligned}\right\} \qquad (1.1.3)$$

Since it is sufficient for our purpose to consider small variations of the quantities I_1, I_2, V_1 and V_2, to represent the d.c. values at the chosen working point, the higher order terms in Eq. (1.1.3) may be disregarded:

$$\left.\begin{aligned} \delta I_1 &= \frac{\partial I_1}{\partial V_1}\cdot\delta V_1 + \frac{\partial I_1}{\partial V_2}\cdot\delta V_2\,, \\ \delta I_2 &= \frac{\partial I_2}{\partial V_1}\cdot\delta V_1 + \frac{\partial I_2}{\partial V_2}\cdot\delta V_2\,. \end{aligned}\right\} \qquad (1.1.4)$$

The partial derivatives are thus proportionality constants relating the increments of I_1 and I_2 to those of V_1 and V_2. The proportionality constants have the dimensions of admittances and are dependent on the values of the direct currents and voltages applied to the terminal pairs.

Furthermore, the currents in Eq. (1.1.4) are generally periodical functions of time and hence, the proportionality constants are dependent on the components constituting the current functions. Using a Fourier expansion these periodical functions may, however, be expressed as a sum of components of different frequencies. Considering the first current of the right hand side of Eq. (1.1.4) we may put:

$$\frac{\partial I_1}{\partial V_1} \cdot \delta V_1 \equiv \sum_{n=1}^{\infty} i_{1,n}(\omega) \cdot \exp(\mathrm{j}n\omega t) \,. \qquad (1.1.5)$$

The Fourier coefficients $i_{1,n}(\omega)$ are functions of frequency only. Also δV_1 may be expanded into a Fourier series as:

$$\delta V_1 \equiv \sum_{n=1}^{\infty} v_{1,n}(\omega) \cdot \exp(\mathrm{j}n\omega t) \,. \qquad (1.1.6)$$

Hence the proportionality constant, which has the dimension of an admittance becomes:

$$\frac{\partial I_1}{\partial V_1} = \sum_{n=1}^{\infty} \frac{i_{1,n}(\omega)}{v_{1,n}(\omega)} = \sum_{n=1}^{\infty} y_{1,n}(\omega) \,. \qquad (1.1.7)$$

The proportionality constant is thus a function of frequency and not of times. It comprises an admittance $y_{1,n}$ for a signal component of frequency $n\omega$. The other proportionality constants of Eq. (1.1.4) may be considered in an analogous way.

By putting:

$$\left.\begin{aligned} \frac{\partial I_1}{\partial V_1} &= y_{11}\,, & \frac{\partial I_1}{\partial V_2} &= y_{12}\,, \\ \frac{\partial I_2}{\partial V_1} &= y_{21}\,, & \frac{\partial I_2}{\partial V_2} &= y_{22}\,, \end{aligned}\right\} \qquad (1.1.8)$$

and considering δI_1, δI_2, δV_1 and δV_2 as small alternating currents and voltages superimposed on much larger direct currents and voltages, we may write for Eq. (1.1.4):

$$\left.\begin{aligned} i_1 &= y_{11}\,v_1 + y_{12}\,v_2\,, \\ i_2 &= y_{21}\,v_1 + y_{22}\,v_2\,. \end{aligned}\right\} \qquad (1.1.9)$$

Here i and v denote the alternating currents and voltages of frequency ω at which y_{11}, y_{12}, y_{21} and y_{22} are measured or specified.

The proportionality constants y_{11}, y_{12}, y_{21} and y_{22} are referred to as the *admittance parameters* or *y-parameters* of the four-terminal network (transistor) under consideration. These admittance parameters, which are small-

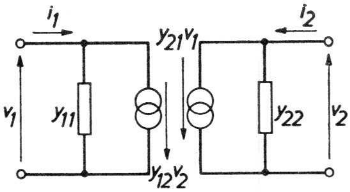


Fig. 1.3. Admittance parameter equivalent circuit.

signal quantities, are shown to be dependent on the biasing point of the transistor as well as on the frequency.

According to Eq. (1.1.9) the transistor can be represented by the equivalent four-terminal network as shown in Fig. 1.3. It follows from this figure and from Eq. (1.1.8) and (1.1.9) that:

y_{11} is the small-signal input admittance of the transistor four-terminal network with the output terminals short-circuited (v_2 being zero of V_2 being constant);

y_{22} is the small-signal output admittance of the transistor four-terminal network, with the input terminals short-circuited (v_1 being zero);

y_{12} is the small-signal reverse transfer admittance of the transistor four-terminal network, that is to say the ratio of the short-circuited input current to the output voltage (v_1 being zero);

y_{21} is the small-signal forward transfer admittance of the transistor four-terminal network, that is to say the ratio of the short-circuited output current to the input voltage (v_2 being zero).

We thus have:

$$\left.\begin{aligned} y_{11} &= \frac{i_1}{v_1}\bigg|_{v_2=0}, \qquad & y_{12} &= \frac{i_2}{v_2}\bigg|_{v_1=0}, \\ y_{21} &= \frac{i_2}{v_1}\bigg|_{v_2=0}, \qquad & y_{22} &= \frac{i_2}{v_2}\bigg|_{v_1=0}. \end{aligned}\right\} \tag{1.1.10}$$

The quantities y_{11}, y_{12}, y_{21} and y_{22} are generally complex in character, so that each should be split up into a real and an imaginary part:

$$y_{11} = g_{11} + jb_{11} = g_{11} + j\omega C_{11}, \tag{1.1.11}$$

$$y_{12} = g_{12} + jb_{12} = |y_{12}| \exp(j \arg y_{12}), \tag{1.1.12}$$

$$y_{21} = g_{21} + jb_{21} = |y_{21}| \exp(j \arg y_{21}), \tag{1.1.13}$$

$$y_{22} = g_{22} + jb_{22} = g_{22} + j\omega C_{22}. \tag{1.1.14}$$

In practical amplifiers the parameters y_{11} and y_{22} are always considered in connection with the tuned input and output circuits respectively. These tuned circuits are so designed that the imaginary parts of y_{11} and y_{22} are included in the tuning susceptances. The real parts g_{11} and g_{22} act as a damping on the tuned circuits due to the transistor. This explains why y_{11} and y_{22} have been expressed in the form of $(g + jb)$ in the above expressions.

The admittances y_{12} and y_{21} are transfer properties of the four-terminal network, and can most conveniently be expressed in terms of modulus and argument because their product must be evaluated in order to analyse the amplifier.

1.1.2 HYBRID-*H* PARAMETER REPRESENTATION

Analogous to the method of obtaining the admittance parameters of the transistor presented in the preceding subsection, we may derive from the combinations of functions (see Eq. (1.1.1) and Fig. 1.2):

$$\left.\begin{aligned} V_1 &= H_1(I_1,\ V_2)\ , \\ I_2 &= H_2(I_1,\ V_2)\ , \end{aligned}\right\} \tag{1.1.15}$$

the relations:

$$\left.\begin{aligned} v_1 &= h_{11}\, i_1 + h_{12}\, v_2\ , \\ i_2 &= h_{21}\, i_1 + h_{22}\, v_2\ . \end{aligned}\right\} \tag{1.1.16}$$

In these relations the quantities h_{11}, h_{12}, h_{21} and h_{22} are the small signal *hybrid-h parameters* or, shorter, *h-parameters* of the transistor which are defined as:

h_{11} is the small-signal input impedance of the transistor four-terminal network with the output terminals short-circuited (v_2 being zero);

h_{22} is the small-signal output admittance of the transistor four-terminal network with the input terminals open-circuited (i_1 being zero);

h_{12} is the small-signal reverse transfer voltage ratio of the transistor four-terminal network which equals the ratio of the open-circuited input voltage and the output voltage (i_1 being zero);

h_{21} is the small-signal forward transfer current ratio which equals the ratio of the short-circuited output current and the input current (v_2 being zero).

Summarizing:

$$\left.\begin{aligned} h_{11} &= \frac{v_1}{i_1}\bigg|_{v_2=0}\ , \qquad & h_{12} &= \frac{v_1}{v_2}\bigg|_{i_1=0}\ , \\ h_{21} &= \frac{i_2}{i_1}\bigg|_{v_2=0}\ , \qquad & h_{22} &= \frac{i_2}{v_2}\bigg|_{i_1=0}\ . \end{aligned}\right\} \tag{1.1.17}$$

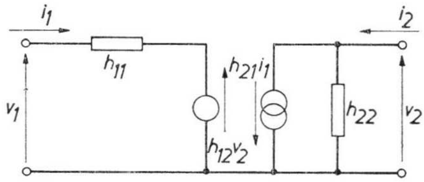


Fig. 1.4. Hybrid-H parameter equivalent circuit.

Fig. 1.4 represents an equivalent circuit based on Eqs. (1.1.16) and (1.1.17). Because of the arrangement of elements in this equivalent circuit the h-matrix is often referred to as the *series-parallel matrix*.

The quantities h_{11}, h_{12}, h_{21} and h_{22} thus depend on the biasing point of the transistor as well as on the signal frequency. Moreover, they are generally complex in character so that each should be split up into a real and an imaginary part:

$$h_{11} = Re\,(h_{11}) + \mathrm{j}\,Im(h_{11}), \tag{1.1.18}$$

$$h_{12} = |h_{12}| \cdot \exp(\mathrm{j} \cdot \arg h_{12}), \tag{1.1.19}$$

$$h_{21} = |h_{21}| \cdot \exp(\mathrm{j} \cdot \arg h_{21}), \tag{1.1.20}$$

$$h_{22} = Re(h_{22}) + \mathrm{j}Im\,(h_{22}). \tag{1.1.21}$$

The four h-parameters thus contain an impedance, an admittance and two dimensionless quantities, which explains the term "hybrid" used in connection with these parameters.

1.1.3 IMPEDANCE PARAMETER PRESENTATION

Considering the combination of functions:

$$\left.\begin{aligned} V_1 &= Z_1\,(I_1, I_2),\\ V_2 &= Z_2\,(I_1, I_2), \end{aligned}\right\} \tag{1.1.22}$$

from Eq. (1.1.1), we may obtain the relations:

$$\left.\begin{aligned} v_1 &= z_{11}\,i_1 + z_{12}\,i_2\,,\\ v_2 &= z_{21}\,i_1 + z_{22}\,i_2\,. \end{aligned}\right\} \tag{1.1.23}$$

The quantities z_{11}, z_{12}, z_{21} and z_{22} represent the small-signal *impedance parameters* or *z-parameters* of the transistor and are defined as:

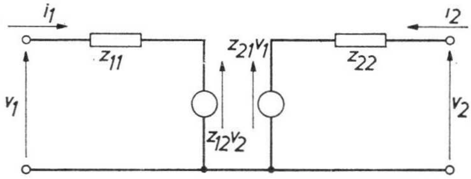


Fig. 1.5. Impedance parameter equivalent circuit.

$$\left.\begin{aligned} z_{11} &= \frac{v_1}{i_1}\bigg|_{i_2=0}, & z_{12} &= \frac{v_1}{i_2}\bigg|_{i_1=0}, \\ z_{21} &= \frac{v_2}{i_1}\bigg|_{i_2=0}, & z_{22} &= \frac{v_2}{i_2}\bigg|_{i_1=0}. \end{aligned}\right\} \tag{1.1.24}$$

Fig. 1.5 shows and equivalent circuit based on the relations (1.1.23) and (1.1.24).

1.1.4 HYBRID-*K* PARAMETERS [1]

From the combination of functions:

$$\left.\begin{aligned} I_1 &= K_1(V_1, I_2)\,, \\ V_2 &= K_2(V_1, I_2), \end{aligned}\right\} \tag{1.1.25}$$

from Eq. (1.1.1) it follows:

$$\left.\begin{aligned} i_1 &= k_{11}\,v_1 + k_{12}\,i_2\,, \\ v_2 &= k_{21}\,v_1 + k_{22}\,i_2\,. \end{aligned}\right\} \tag{1.1.26}$$

Here the quantities k_{11}, k_{12}, k_{21} and k_{22} are the *hybrid-k parameters* or, shorter, *k-parameters* of the transistor which are defined as:

$$\left.\begin{aligned} k_{11} &= \frac{i_1}{v_1}\bigg|_{i_2=0}, & k_{12} &= \frac{i_1}{i_2}\bigg|_{v_1=0}, \\ k_{21} &= \frac{v_2}{v_1}\bigg|_{i_2=0}, & k_{22} &= \frac{v_2}{i_2}\bigg|_{v_1=0}. \end{aligned}\right\} \tag{1.1.27}$$

In Fig. 1.6 an equivalent circuit based on these relations is shown. Because of the arrangement of elements in the equivalent circuit the k-matrix is referred to as the *parallel-series matrix*.

---

[1]) Often the symbol g is used for this set of parameters. In this book we prefer to use the symbol k because the symbol g is employed to denote conductances.

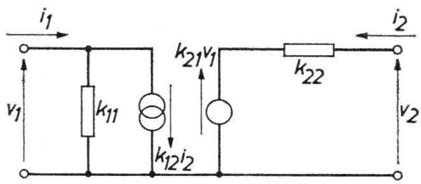


Fig. 1.6. Hybrid-K parameter equivalent circuit.

1.1.5 DEPENDENCE OF TRANSISTOR PARAMETERS ON TEMPERATURE

In the preceding sub-sections we have considered the transistor as a black box with two pairs of terminals, containing an unspecified electrical network and we have derived relations describing what happens when currents and voltages are applied to the terminals. Generalizing, it has been accepted that for small signals the relations are linear functions with parameters dependent on the biasing point and on frequency. So far it has not been necessary to consider the contents of the black box in more detail. Inside the black box, however, there is the transistor material in which the electrical phenomena are strongly dependent on temperature.

Hence, the electrical parameters measured at the terminals are also more or less dependent on this temperature. In Book II, Chapter I curves are given showing the dependency of the admittance parameters of a certain type of transistor on junction temperature.

1.2 Characteristic Matrices

As follows from Section 1.1 a transistor may be represented by the admittance matrix, the impedance matrix, the series-parallel matrix or the parallel-series matrix. In Figs. 1.3, 1.5, 1.4 and 1.6 four-terminal network equivalent circuits are shown for the various matrices. The choice of matrix to actually represent the transistor depends upon which equivalent fourpole forms the best equivalent representation of the electrical behaviour of the transistor. This might become apparent from the following considerations:

A transistor suitable for use in high-frequency bandpass amplifiers and connected in either the common base or the common emitter configuration will generally have an output impedance which is larger than or in the same order of magnitude as that of the tuned circuit connected to its output terminals. The output side of such a transistor can therefore best be characterized by a current source in parallel with an admittance (the output self-admittance) as is the case in the equivalent fourpole circuits for the y- and h-matrices.

Considering the input side of a transistor it follows that it may either be "voltage-driven" or "current-driven" depending on the relative magnitudes of the input impedance Z_i of the transistor and the internal impedance Z_s of the driving source. If the transistor is said to be current-driven the matrix most suitable for this case is the *h*-matrix in which the h_{21} parameter then relates the current through the load and the current determined by the driving source. If, on the other hand, $Z_s < Z_i$, the transistor is voltage-driven and the most representative matrix is the *y*-matrix.

It thus follows that, depending on the electrical behaviour of the transistor, the 21 parameter of a certain matrix gives a better description of the properties of the complete transistor than the 21 parameters of other matrices do. The matrix that gives the best description is called the *characteristic matrix* (see Bibliography [1.6] and [1.7]).

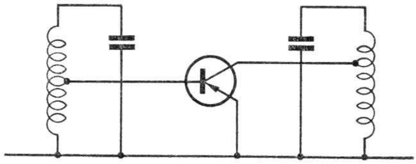
Taking into account the considerations regarding the output of transistors suitable for use in high-frequency bandpass amplifiers, the characteristic matrix will either be the *y*-matrix or the *h*-matrix. Whether the *y*-matrix or the *h*-matrix is characteristic depends on the properties of the input side of the transistor which in turn depend on the type of transistor, the frequency of operation, the transistor configuration (common-base or common-emitter) and on the circuitry at the input side.

Because no general conclusions can be drawn with respect to these points both matrices will be considered in this book.

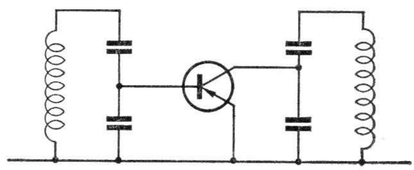
1.3 The *Y*- and *H*-Matrices of the Transistor and the External Circuitry

In bandpass amplifiers tuned circuits or combinations thereof are used as coupling elements between the various transistors. In most cases the transistor input and output terminals are connected to taps on these tuned circuits as shown in Figs. 1.7 a and b. Assuming that the inductive or capacitive taps on the tuned circuits behave as ideal transformers, the tuned circuits are effectively in parallel with the transistor terminals. In these cases it is very convenient to express the properties of transistors as well as those of the tuned circuits in terms of admittance parameters. The total admittance of the tuned circuit and the transistor admittances connected in parallel can then be evaluated by simply adding the individual admittances, taking into account the proper transformer ratios.

In practical amplifiers the output terminals of the transistors will in most cases be connected directly across the whole circuit so that no tap is necessary at all. If, however, a tap is required the tapping ratio will be such that it can easily be realized. At the input side of the transistor, however, large



a



b

Fig. 1.7. In practical amplifiers the transistor input- and output terminals are connected to taps on the tuned circuits which form (part of) the coupling element between the various stages of the amplifier. Fig. 1.7.a shows an inductive tap on the tuned circuits whereas Fig. 1.7.b shows the capacitive method of tapping.

tapping ratios are usually required because of the small input impedance of the transistors or for reasons of stability. As will be considered in detail in Chapter XII the required tapping ratios can easily be realized at rather low frequencies. At higher frequencies, however, considerable differences between the behaviour of such a tap and that of an ideal transformer may be found. These differences are due to the "spread-inductance" or "spread-capacitance" (see Chapter XII) of the tap and the heavy load presented to it by the transistor. In these cases it will often prove to be advantageous to use a series-tuned circuit at the input side of the transistor (provided the real part of the input impedance is sufficiently low to reach the required quality factor of the tuned circuit).

When a series-tuned circuit is used at the input side of the transistor (and a parallel-tuned circuit at the output side) it is convenient to express the properties of the transistors in terms of the series-parallel matrix (h-parameters). Then the impedances of the transistor can easily be combined with those of the tuned circuits. If the h-matrix is characteristic of the transistor to be used in the amplifier (see Section 1.3), application of a series-tuned input circuit is especially advantageous.

1.4 Further Considerations on the Choice of a Fourpole Parameter System for the Transistor

Apart from considerations regarding the matrix which is characteristic for the transistor and the matrix most suitable for the transistor in connection with its external circuitry, there are other aspects which might also influence the choice of a particular matrix. One of these aspects is that the fourpole parameters chosen to characterize the transistor must be measured on the device itself using a not too complicated measuring gear. The most suitable matrix in this respect is the admittance matrix because only short-circuits need to be provided at certain terminals to measure these parameters. Such short-circuits are easier to realize than the open circuits required at certain terminals for measuring other matrix parameters, see Bibliography [1.7] and [1.8].

Another aspect of the choice of a parameter system is its relation to the complete electrical equivalent circuit of the transistor derived from its physical operation. The parameters of the matrix chosen should preferably define single elements of the equivalent circuit as accurately as possible. This point will, however, not be dealt with further, because electrical equivalent circuits are considered to be beyond the scope of this book (see Bibliography [1.8]).

1.5. Transistor Parameter Nomenclature

At present it is customary to use for transistors the symbols according to the I.E.E.E.-standards, see Bibliography [1.9]. These symbols include an indication as to which of the three transistor terminals is common to both the input and output circuits. This may be either the base, the emitter or the collector. The indication is given by using the letter *b*, *e* or *c* respectively as the second suffix in the symbol denoting a given fourpole parameter. The first suffix of these symbols indicates which of the fourpole parameters is referred to, the input, reverse transfer, forward transfer and output parameters being denoted by the suffixes *i*, *r*, *f* and *o* respectively. The symbol y_{ie} thus denotes the input admittance parameter of a transistor in common emitter configuration, and so forth.

The table below gives a survey of the notations using *y*- and *h*-parameters.

Instead of using these symbols in this book, preference is given to the more general symbols y_{11}, y_{12}, y_{21} and y_{22} or h_{11}, h_{12}, h_{21} and h_{22}. In so doing, the results of the analyses are applicable to transistors irrespective of which terminal is chosen as the common one. In fact, these results may even be applied to circuits using electron tubes.

SYMBOLS OF TRANSISTOR PARAMETERS ACCORDING TO THE I.E.E.E.-STANDARDS

admittance parameters	common base	common emitter	common collector	general symbols
input parameter	y_{ib}	y_{ie}	y_{ic}	y_{11}
reverse transfer parameter	y_{rb}	y_{re}	y_{rc}	y_{12}
forward transfer parameter	y_{fb}	y_{fe}	y_{fc}	y_{21}
output parameter	y_{ob}	y_{oe}	y_{oc}	y_{22}
hybrid *h*-parameters				
input parameter	h_{ib}	h_{ie}	h_{ic}	h_{11}
reverse transfer parameter	h_{rb}	h_{re}	h_{rc}	h_{12}
forward transfer parameter	h_{fb}	h_{fe}	h_{fc}	h_{21}
output parameter	h_{ob}	h_{oe}	h_{oc}	h_{22}

1.6 Relations between the Fourpole Parameters of a Transistor in the Different Configurations

A transistor may be used in an amplifier either in common base, common emitter or common collector configuration. Obviously, if a set of fourpole parameters is specified for any of these configurations, the parameters for the other configurations can be calculated.

1.6.1 ADMITTANCE PARAMETERS

A transistor is basically a three-terminal device for which, according to Appendix I, the indefinite admittance matrix can be written as:

$$\begin{Vmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{Vmatrix} \tag{1.6.1}$$

Using the parameter nomenclature of the preceding section and the notation of Fig. 1.8 we may write for Eq. (1.6.1):

$$\begin{Vmatrix} y_{ie} = y_{ic} & y_{re} & y_{rc} \\ y_{fe} & y_{ob} = y_{oe} & y_{fb} \\ y_{fc} & y_{rb} & y_{ib} = y_{oc} \end{Vmatrix} \tag{1.6.2}$$

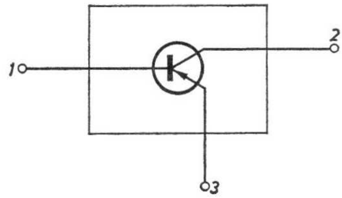


Fig. 1.8. The transistor as a three-terminal network.

An indefinite admittance matrix has the property that each row and each column adds to zero (see Appendix I). Applying this property to Eq. (1.6.2) the parameters of either of the three transistor configurations can be related to those of the others. Table 1.1 presents the relationships between these parameters interrelations.

TABLE 1.1 ADMITTANCE PARAMETER RELATIONSHIPS

	COMMON BASE b	COMMON EMITTER e	COMMON COLL. c
	y_{ib}	$y_{ie} = \Sigma y_b$	$y_{ic} = \Sigma y_b$
y_i	$y_{ib} = \Sigma y_e$	y_{ie}	$y_{ic} = y_{ie}$
	$y_{ib} = y_{oc}$	$y_{ie} = y_{ic}$	y_{ic}
	y_{rb}	$y_{re} = -(y_{rb} + y_{ob})$	$y_{rc} = -(y_{ib} + y_{fb})$
y_r	$y_{rb} = -(y_{re} + y_{oe})$	y_{re}	$y_{rc} = -(y_{ie} + y_{re})$
	$y_{rb} = -(y_{fc} + y_{oc})$	$y_{re} = -(y_{ic} + y_{rc})$	y_{rc}
	y_{fb}	$y_{fe} = -(y_{fb} + y_{ob})$	$y_{fc} = -(y_{ib} + y_{rb})$
y_f	$y_{fb} = -(y_{fe} + y_{oe})$	y_{fe}	$y_{fc} = -(y_{ie} + y_{fe})$
	$y_{fb} = -(y_{rc} + y_{oc})$	$y_{fe} = -(y_{ic} + y_{fc})$	y_{fc}
	y_{ob}	$y_{oe} = y_{ob}$	$y_{oc} = y_{ib}$
y_o	$y_{ob} = y_{oe}$	y_{oe}	$y_{oc} = \Sigma y_e$
	$y_{ob} = \Sigma y_c$	$y_{oe} = \Sigma y_c$	y_{oc}
	$\Delta y_b = y_{ib}\, y_{ob} - y_{rb} y_{fb}$	$\Delta y_e = y_{ie}\, y_{oe} - y_{re}\, y_{fe}$	$\Delta y_c = y_{ic}\, y_{oc} - y_{rc}\, y_{fc}$
	$\Sigma y_b = y_{ib} + y_{rb} + y_{fb} + y_{ob}$	$\Sigma y_e = y_{ie} + y_{re} + y_{fe} + y_{oe}$	$\Sigma y_c = y_{ic} + y_{rc} + y_{fc} + y_{oc}$

1.6.2 HYBRID *H*-PARAMETERS

The relationships between the h-parameters for a transistor in the three transistor configurations can be obtained by calculating the required set of parameters of another (given) set of parameters using the equations (see Fig. 1.9): for the common base configuraton:

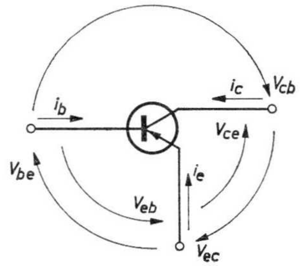


Fig. 1.9. Various currents and voltages at the three terminals of a transistor. Taking into account these currents and voltages the parameters of either the common base, common emitter or common collector configuration can be calculated if these parameters for one configuration are given.

$$\left.\begin{aligned} v_{eb} &= h_{ib} \cdot i_e + h_{rb} \cdot v_{cb}\,, \\ i_c &= h_{fb} \cdot i_e + h_{ob} \cdot v_{cb}\,, \end{aligned}\right\} \tag{1.6.3}$$

for the common emitter configuration:

$$\left.\begin{aligned} v_{be} &= h_{ie} \cdot i_b + h_{re} \cdot v_{ce}\,, \\ i_c &= h_{fe} \cdot i_b + h_{oe} \cdot v_{ce}\,, \end{aligned}\right\} \tag{1.6.4}$$

and for the common collector configuration:

$$\left.\begin{aligned} v_{bc} &= h_{ic} \cdot i_b + h_{rc} \cdot v_{ec}\,, \\ i_e &= h_{fc} \cdot i_b + h_{oc} \cdot v_{ec}\,. \end{aligned}\right\} \tag{1.6.5}$$

Furthermore:

$$i_b + i_c + i_e = 0, \tag{1.6.6}$$

and:

$$v_{be} + v_{cb} + v_{ec} = 0. \tag{1.6.7}$$

The results of these calculations are compiled in Table 1.2.

1.7 Transistor Fourpole Parameters and Narrow Band Amplifier Analysis

In the preceding sections it has been shown that the fourpole parameters of a transistor are dependent on the frequency of operation. The bandpass amplifiers with relatively narrow bandwidth as analyzed in this book have frequency characteristics which are mainly controlled by tuned circuits external to the

TABLE 1.2 HYBRID-H PARAMETER RELATIONSHIPS

	COMMON BASE b	COMMON EMITTER e	COMMON COLLECT. c
	h_{ib}	$h_{ie} = \frac{h_{ib}}{H_b}$	$h_{ic} = \frac{h_{ib}}{H_b}$
h_i	$h_{ib} = \frac{h_{ie}}{H_e}$	h_{ie}	$h_{ic} = h_{ie}$
	$h_{ib} = \frac{h_{ic}}{H_c}$	$h_{ie} = h_{ic}$	h_{ic}
	h_{rb}	$h_{re} = \frac{\Delta h_b - h_{rb}}{H_b}$	$h_{rc} = \frac{1 + h_{fb}}{H_b}$
h_r	$h_{rb} = \frac{\Delta h_e - h_{re}}{H_e}$	h_{re}	$h_{rc} = 1 - h_{re}$
	$h_{rb} = \frac{1 + h_{fc}}{H_c}$	$h_{re} = 1 - h_{rc}$	h_{rc}
	h_{fb}	$h_{fe} = -\frac{h_{fb} + \Delta h_b}{H_b}$	$h_{fc} = \frac{h_{rb} - 1}{H_b}$
h_f	$h_{fb} = -\frac{h_{fe} + \Delta h_e}{H_e}$	h_{fe}	$h_{fc} = -(1 + h_{fe})$
	$h_{fb} = \frac{h_{rc} - 1}{H_c}$	$h_{fe} = -(1 + h_{fc})$	h_{fc}
	h_{ob}	$h_{oe} = \frac{h_{ob}}{H_b}$	$h_{oc} = \frac{h_{ob}}{H_b}$
h_o	$h_{ob} = \frac{h_{oe}}{H_e}$	h_{oe}	$h_{oc} = h_{oe}$
	$h_{ob} = \frac{h_{oc}}{H_c}$	$h_{oe} = h_{oc}$	h_{oc}
	$\Delta h_b = h_{ib}\, h_{ob} - h_{rb}\, h_{fb}$	$\Delta h_e = h_{ie}\, h_{oe} - h_{re}\, h_{fe}$	$\Delta h_c = h_{ic}\, h_{oc} - h_{rc}\, h_{fc}$
	$H_b = 1 + h_{fb} - h_{rb} + \Delta h_b$	$H_e = 1 + h_{fe} - h_{re} + \Delta h_e$	$H_c = 1 + h_{fc} - h_{rc} + \Delta h_c$

transistors. The variation with frequency of the transistor parameters is of minor significance in determining the performance of the amplifier. Therefore it will be assumed that the input and output parameters of the transistors are constant over the frequency range in which the amplifier gain is significant in so far as their dampings and capacitances are concerned. Moreover, the transfer parameters are assumed to be constant over this frequency range as far as their modulus and argument are concerned. Under these assumptions the amplifier analysis can be carried out in terms of circuit parameter values at the centre frequency.

1.8 General Parameter Notation

At some places in the following chapters it will be desirable not to restrict the amplifier analysis to a particular parameter system. In these cases a general notation will be used as shown in Eq. (1.8.1):

$$\left.\begin{aligned} \alpha_{11} &= \gamma_{11}\beta_1 + \gamma_{12}\beta_2 , \\ \alpha_{12} &= \gamma_{21}\beta_{11} + \gamma_{22}\beta_2 . \end{aligned}\right\} \qquad (1.8.1)$$

In these equations the general symbols γ refer to the parameters of either the y, z, h or k-matrix equations. Furthermore, the symbols β_1, and β_2 denote the independent variables and α_1 and α_2 the dependent variables of the general matrix equation.

BIBLIOGRAPHY

[1.1] W. W. GÄRTNER, *Transistors, Principles, Design and Applications,* D. van Nostrand Comp. Inc. New-York, 1960 Chapter 7.

[1.2] R. A. GREINER, *Semiconductor Devices and Applications,* McGraw-Hill Book Comp., Inc., New York, 1961, Chapter 12.

[1.3] W. W. HAPP, *Dynamic Characteristics of Four-Terminal Networks,* 1954, I.R.E. Conv. Rec., Pt **2**, p. 60-76.

[1.4] C. R. LAUGLIN, *The Indefinite Y Matrix,* Electronic Ind., **21**, p. 121-122, Dec. 1962.

[1.5] D. A. L. LINDEN, *A Note on Four-Pole Parameters,* Proc. I.R.E., **46**, p. 375, 1958.

[1.6] J. G. LINVILL and J. F. GIBBONS, *Transistors and Active Circuits,* McGraw-Hill Book Comp., Inc., New York, 1961, Chapter 9.

[1.7] G. MARTE, *Charakteristische Matrizen und Ersatzschaltbilder aktiver Vierpole,* A. E. Ü., **16**, p. 227-229, May 1962.

[1.8] G. MARTE, *Charakteristische Matrizen der Transistor-Grundschaltungen,* A. E. Ü., **16**, p. 343-346, July 1962.

[1.9] T. R. NISBET and W. W. HAPP, *Jacobians — a New Computational Tool,* Electronic Ind., **17**, p. 69-71, 1958.

[1.10] R. PAUL, *Die Messung der Kennwerte von Transistoren,* Nachrichtentechnik, **12**, p. 163-167 (part I), p. 213-217 (part II), p. 260-265 (part III), p. 308-313 (part IV), p. 340-348 (part V).

[1.11] R. L. PRITCHARD, J. B. ANGELL, R. B. ADLER, J. M. EARLY and W. M. WEBSTER, *Transistor Internal Parameters for Small-Signal Representation,* Proc. I.R.E., **49**, p. 725-738, April 1961.

[1.12] *I.R.E. Standards on Semiconductor Symbols* Proc. I.R.E. **44**, p. 934-937 (July 1956).

CHAPTER 2

THE VARIOUS ASPECTS OF BANDPASS AMPLIFIER DESIGN

This chapter deals with definitions and the interpretation of those terms and concepts which are essential for the design of transistorized bandpass amplifiers.

The general survey in the preface showed that when designing bandpass amplifiers a large number of problems must be faced. Of these problems, the most important is that of achieving sufficient protection against self-oscillation of the amplifier. In this chapter the investigation of this stability problem is confined to the comparatively simple case of a single-stage amplifier with single-tuned circuits, both at the input and at the output terminals. This case is dealt with in great detail, and a number of concepts regarding the stability problem are explained. This will facilitate a general understanding of this problem, which will prove to be of great advantage when dealing with similar problems in the more complex amplifiers to be discussed later.

Other problems encountered in bandpass amplifier design are those of gain, amplitude response and phase response. Since in the amplifiers dealt with there is a certain amount of feedback, that is to say a return of a portion of the amplifier output power to the input, the method of alignment must also be investigated in detail, since in such an amplifier the tuning of one resonant circuit influences the properties of all other circuits.

In most treatises on bandpass amplifiers the problem of how to align the amplifier is not discussed. It will become clear from the analyses given below that this omission is due to the fact that in these treatises it is generally assumed that the amplifiers are tuned according to one particular method, which we will refer to as method A.

There are, however two other methods of aligning an amplifier, which yield well-defined results. As a matter of fact, these methods — to be termed methods B and C — offer distinct advantages over method A, as regards both the performance of the amplifier and ease of alignment.

Because the method of alignment has considerable influence on the frequency-dependent properties of the tuned circuits of the amplifier, its amplitude and phase response must be investigated for each of these three methods of alignment.

Here again, these points are analyzed in detail for a single-stage amplifier. The various problems present themselves most fundamentally for such an amplifier, so that a clear picture is easily obtained. The following analysis may thus be considered as an introduction to the various aspects of practical bandpass amplifier design as applied to the analyses of more complex amplifiers.

As already referred to in Chapter 1, in order to analyze the amplifiers, the transistors and their associated circuitry will be expressed in an "*Y*-matrix environment"[1]) as well as an "*H*-matrix environment". Both matrix environments will prove to be very useful in analyzing practical amplifier configurations. To keep the analyses as practical as possible, both systems will be treated separately. The calculation based on *Y*-matrices will be carried out first. For the case of *H*-matrices the results of the calculations are derived by means of analogies.

2.1 Single-Stage Amplifier with Single-Tuned Circuits

2.1.1 GENERAL AMPLIFIER CIRCUIT

To analyze a single-stage amplifier (containing one active element) the properties of the active element as well as those of the passive elements can, according to Chapter 1, be expressed using either the *Y*, *Z*, *H* or *K*-matrices. In Fig. 2.1 the four basic matrices of the single-stage amplifier are shown.

The analysis can be based on each of the four matrices of the amplifier. For practical reasons, however, only the *Y*- and *H*-environments will be considered in detail.

2.1.2 AMPLIFIER CIRCUIT BASED ON ADMITTANCE PARAMETERS

Fig. 2.2 shows a schematic circuit diagram of a single-stage amplifier comprising two single-tuned circuits. This amplifier circuit can most readily be analyzed by means of the admittance matrix system. The current source which drives the amplifier is assumed to have an admittance Y_S, and the amplifier is loaded by an admittance Y_L. Usually, the latter admittance is formed by the input admittance of a following amplifier stage.

For the sake of simplicity the tappings on the tuned circuits, which are necessary in a practical amplifier for the impedance transformations, have been omitted.

The admittance of the tuned circuit formed by L_1^*, C_1^* and G_1^* will be denoted by Y_1^*, and that of the tuned circuit formed by L_2^*, C_2^* and G_2^* by Y_2^*.

[1]) The term "matrix environment" is used to express that the equivalent four-terminal network of the transistor together with the circuitry at its in out- and output side are arrranged in a manner inherent to the respective matrix.

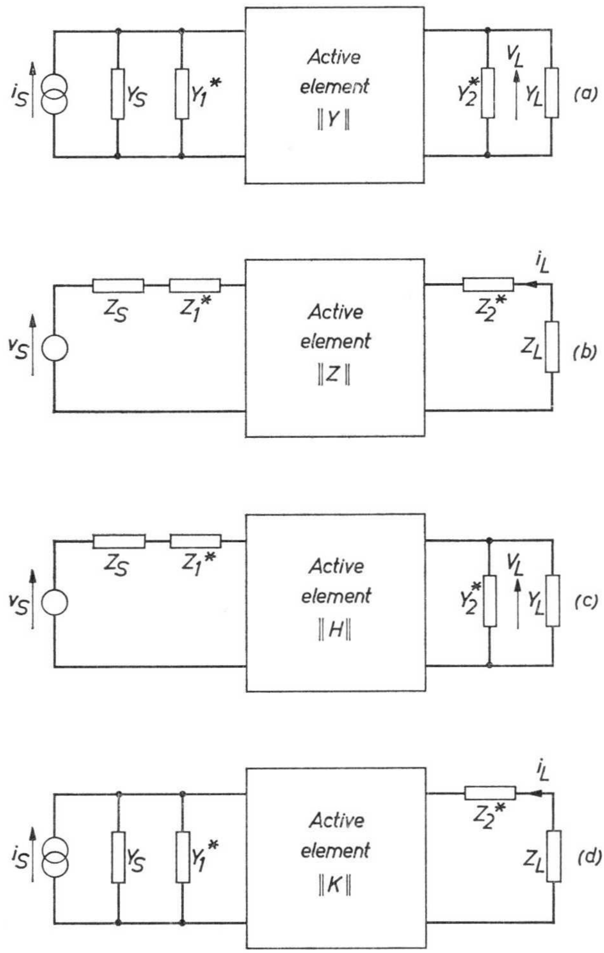


Fig. 2.1. Basic matrix environments of the single-stage amplifier. The respective environments will be referred to as
a. the Y-matrix environment (parallel-parallel matrix),
b. the Z-matrix environment (series-series matrix),
c. the H-matrix environment (series-parallel matrix),
d. the K-matrix environment (parallel-series matrix).

The following relations hold for the circuit of Fig. 2.2:

$$\left.\begin{aligned} i_1 &= (Y_S + Y_1{}^* + y_{11})v_1 + y_{12}v_2, \\ i_2 &= y_{21}v_1 + (y_{22} + Y_2{}^* + Y_L)v_2. \end{aligned}\right\} \tag{2.1.1}$$

By putting

and

$$\left.\begin{aligned} Y_1 &= Y_S + Y_1{}^* + y_{11}, \\ Y_2 &= y_{22} + Y_2{}^* + Y_L, \end{aligned}\right\} \tag{2.1.2}$$

Eq. (2.1.1) is simplified to:

$$\left.\begin{aligned} i_1 &= Y_1 v_1 + y_{12} v_2, \\ i_2 &= y_{21} v_1 + Y_2 v_2. \end{aligned}\right\} \tag{2.1.3}$$

Here Y_1 and Y_2 defined by Eq. (2.1.2) represent the admittances of single-tuned circuits. According to Appendix II:

$$Y = G(1 + jx), \tag{2.1.4}$$

in which x represents the normalized detuning of the circuit with respect to resonant frequency (at which $x = 0$) and equals

$$x = \beta Q. \tag{2.1.5}$$

In this expression β is the relative detuning of the circuit with respect to the resonant frequency f_0:

$$\beta = \frac{f}{f_0} - \frac{f_0}{f}, \tag{2.1.6}$$

and Q is the quality factor of the circuit. With Eq. (2.1.4), Eq. (2.1.3) becomes:

$$\left.\begin{aligned} i_1 &= G_1(1 + jx_1) v_1 + y_{12} v_2, \\ i_2 &= y_{21} v_1 + G_2(1 + jx_2) v_2, \end{aligned}\right\} \tag{2.1.7}$$

or, using a matrix notation:

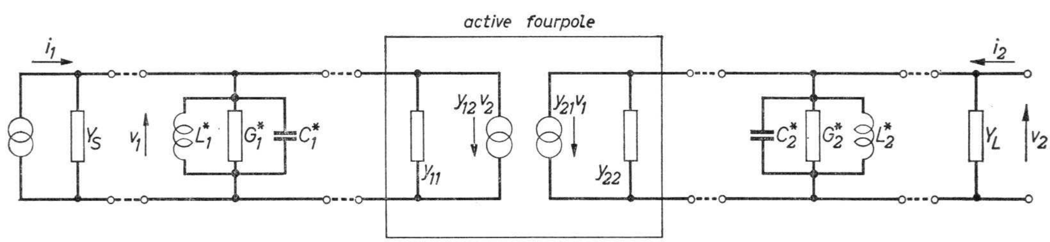


Fig. 2.2. Schematic diagram of a single-stage amplifier with single-tuned circuits at the input and output terminals. The active fourpole represents the transistor or electron tube; Y_S denotes the admittance of the current source which drives the amplifier, and Y_L the load admittance of the amplifier.

$$\begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} G_1(1+jx_1) & y_{12} \\ y_{21} & G_2(1+jx_2) \end{Vmatrix} \cdot \begin{Vmatrix} v_1 \\ v_2 \end{Vmatrix}. \tag{2.1.8}$$

The *determinant* of Eq. (2.1.8), to be denoted by Δ, can be simplified to:

$$\Delta = G_1G_2 \begin{vmatrix} 1+jx_1 & \dfrac{y_{12}y_{21}}{G_1G_2} \\ 1 & 1+jx_2 \end{vmatrix}. \tag{2.1.9}$$

The determinant in Eq. (2.1.9) will further be referred to as the *reduced determinant* δ, so:

$$\delta = \begin{vmatrix} 1+jx_1 & \dfrac{y_{12}y_{21}}{G_1G_2} \\ 1 & 1+jx_2 \end{vmatrix}. \tag{2.1.10}$$

Because both y_{12} and y_{21} are generally complex quantities, it will be useful to introduce:

$$T_y = \frac{|y_{12}y_{21}|}{G_1G_2}, \tag{2.1.11}$$

and:

$$\Theta_y = \arg y_{12} + \arg y_{21}. \tag{2.1.12}$$

The quantities T and Θ will be termed the *regeneration coefficient* and the *regeneration phase angle* of the amplifier stage respectively. The quantity δ_y can now be written:

$$\delta_y = \begin{vmatrix} 1+jx_1 & T_y \exp(j\Theta_y) \\ 1 & 1+jx_2 \end{vmatrix}. \tag{2.1.13}$$

Thus Eq. (2.1.8) becomes:

$$\begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = G_1G_2 \cdot \delta_y \cdot \begin{Vmatrix} v_1 \\ v_2 \end{Vmatrix}. \tag{2.1.14}$$

If the output terminals of the circuit according to Fig. 2.2 are open circuited (the load of the amplifier is already accounted for in Y_2), $i_2 = 0$. Since i_1 is also equal to the source current i_S, the output voltage may, according to Eqs. (2.1.8) to (2.1.13) be expressed by:

$$v_2 = -\frac{y_{21}\, i_S}{G_1 G_2 \cdot \delta_y} \tag{2.1.15}$$

Or, since the ratio v_2/i_S represents the transimpedance of the complete amplifier:

$$Z_t = -\frac{y_{21}}{G_1 G_2} \cdot \frac{1}{\delta_y} \,. \tag{2.1.16}$$

2.1.3 GENERAL AMPLIFIER CIRCUIT BASED ON HYBRID *H*-PARAMETERS

Fig. 2.3 shows a schematic circuit diagram of a single-stage amplifier with a series tuned circuit at its input side and a parallel tuned circuit at its output side. This amplifier circuit can most easily be analyzed using the *H*-parameter system.

The amplifier is driven from a voltage source with source impedance Z_S and is loaded by a load of admittance Y_L. The impedance of the tuned circuit formed by $L_1{}^*$,$C_1{}^*$ and $R_1{}^*$ will be denoted by $Z_1{}^*$ and that of the tuned circuit formed by $L_2{}^*$, $C_2{}^*$ and $G_2{}^*$ by $Y_2{}^*$.

By putting:

$$\left.\begin{aligned} Z_1 &= Z_S + Z_1{}^* + h_{11} \,, \\ Y_2 &= h_{22} + Y_2{}^* + Y_L \,, \end{aligned}\right\} \tag{2.1.17}$$

the following relations are obtained for this circuit:

$$\left.\begin{aligned} v_1 &= Z_1\, i_1 + h_{12} v_2 \,, \\ i_2 &= h_{21}\, i_1 + Y_2 v_2 \,. \end{aligned}\right\} \tag{2.1.18}$$

Considering that:

and

$$\left.\begin{aligned} Z_1 &= R_1(1 + \mathrm{j}x_1) \,, \\ Y_2 &= G_2(1 + \mathrm{j}x_2) \,, \end{aligned}\right\} \tag{2.1.19}$$

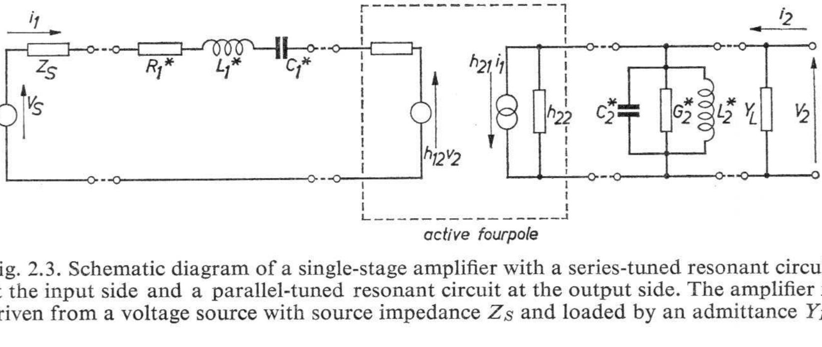


Fig. 2.3. Schematic diagram of a single-stage amplifier with a series-tuned resonant circuit at the input side and a parallel-tuned resonant circuit at the output side. The amplifier is driven from a voltage source with source impedance Z_S and loaded by an admittance Y_L.

in which:

$$R_1 = Re(Z_S) + R_1{}^* + Re(h_{11}), \qquad (2.1.20)$$

and

$$G_2 = Re(h_{22}) + G_2{}^* + Re(Y_L), \qquad (2.1.21)$$

Eq. (2.1.18) can be written, in analogy with the preceding sub-section:

$$\left\| \begin{matrix} v_1 \\ \\ i_1 \end{matrix} \right\| = R_1 G_2 \cdot \delta_h \cdot \left\| \begin{matrix} i_2 \\ \\ v_2 \end{matrix} \right\| . \qquad (2.1.22)$$

The reduced determinant δ_h equals:

$$\delta_h = \left| \begin{matrix} 1 + jx_1 & T_h \exp(j\Theta_h) \\ 1 & 1 + jx_2 \end{matrix} \right\|, \qquad (2.1.23)$$

in which:

$$T_h = \frac{|h_{12} \cdot h_{21}|}{R_1 G_2}, \qquad (2.1.24)$$

and

$$\Theta_h = \arg h_{12} + \arg h_{21}. \qquad (2.1.25)$$

According to Eqs. (2.1.22) to 2.1.25) the output voltage of the amplifier of Fig. 2.3 becomes, provided the output terminals are open-circuited:

$$v_2 = -\frac{h_{21} v_S}{R_1 G_2} \cdot \frac{1}{\delta_h}. \qquad (2.1.26)$$

The *forward voltage gain* of the amplifier then follows from:

$$K_t = \frac{v_2}{v_S} = -\frac{h_{21}}{R_1 G_2} \cdot \frac{1}{\delta_h}. \text{ [1]} \qquad (2.1.27)$$

2.1.4 THE TRANSFER FUNCTION OF THE AMPLIFIER

The forward transfer impedance, or transimpedance, of the amplifier circuit of Fig. 2.2 as derived in sub-section 2.1.2 and the forward transfer voltage ratio, or voltage gain, of the amplifier circuit of Fig. 2.3 as derived in sub-section 2.1.3 are important quantities. Investigation of these quantities leads to conclusions regarding the stability, the gain and the frequency

---

[1]) The symbol K_t for forward voltage transfer ratio (voltage gain) is chosen as analogous to the symbol Z_t which denotes the forward transfer impedance or transimpedance. Similarly, Y_t denotes the transadmittance and H_t denotes the current gain of an amplifying system.

response of the amplifier. These points are dealt with in succession in the following sections.

2.2 Stability

The problem of self-oscillations occurring in bandpass amplifiers is often encountered by designers. These oscillations are always due to some form of feedback from the output to the input. This feedback may arise from a common power supply; from coupling caused by "earth currents" when the chassis does not constitute an ideal mass; from stray capacitance and mutual inductance linkages between interstage coupling elements; or from reverse transmission occurring within the transistor or electron tube.

The latter cause of feedback is the most serious because, unlike the other causes, it cannot be avoided or reduced by careful layout of the amplifier; the feedback which exists within transistors or electron tubes being a property of the device itself. There is no possibility of remedying it simply by decoupling or shielding, so that steps must be taken in advance. In all amplifier designs it is therefore necessary to investigate the internal feedback of the transistors or electron tubes that are to be used, and to ascertain to what extent this internal feedback may affect the stability of the amplifier. The problem of securing satisfactory stable operation of the amplifier is of prime importance; an amplifier which is barely stable, that is to say not sufficiently stable, is useless.

2.2.1 STABILITY OF SINGLE-STAGE AMPLIFIERS

In a single-stage amplifier as discussed in Section 2.2 the output voltage v_2 becomes infinite for a finite value of i_S or v_S if the determinant δ becomes zero (cf. Eq. (2.1.15) and (2.1.26)).[1] The amplifier is then on the verge of oscillation. The condition $\delta = 0$ will therefore be considered as the boundary of stability of the amplifier.

It may thus be written that the amplifier is at the boundary of stability when

$$\delta = \begin{vmatrix} 1 + \mathrm{j}x_1 & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}x_2 \end{vmatrix} = 0 \qquad (2.2.1)$$

By writing out the determinant, the quantity δ can be written:

$$\delta = (1 + \mathrm{j}x_1)(1 + \mathrm{j}x_2) - T \exp(\mathrm{j}\Theta). \qquad (2.2.2)$$

---

[1]) Provided R_1, G_1 and G_2 (see Eqs. (2.1.15) and (2.1.26)) have positive values ($R_1 > 0$, $G_1 > 0$ and $G_2 > 0$).

This quantity is obviously composed of the two vectors

$$(1 + jx_1)(1 + jx_2),$$

and

$$T \exp (j\Theta),$$

so that it can be ascertained graphically by constructing these two vectors and determining their difference. This procedure, which gives a clear indication of the stability properties of the amplifier, will be illustrated by discussing in succession single-stage amplifiers with two identical synchronously tuned resonant circuits and with non-identical resonant circuits.

2.2.2 SINGLE-STAGE AMPLIFIER WITH TWO IDENTICAL SYNCHRONOUSLY TUNED RESONANT CIRCUITS

In the case of $x_1 = x_2 = x$ the first term of Eq. (2.2.2) becomes:

$$(1 + jx)^2 = 1 - x^2 + j2x. \tag{2.2.3}$$

It can be shown that the locus of $(1 + jx)^2$, plotted in the complex plane, is a parabola with its focus at the origin and a directrix perpendicular to the real (horizontal) axis in the point (2.0). Fig. 2.4 shows such a parabola.

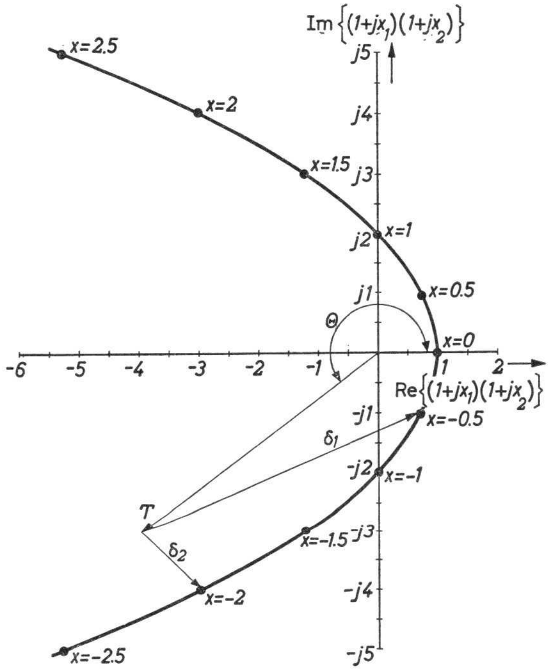


Fig. 2.4. Parabola representing $(1 + jx)^2$, and vector T, illustrating how δ can be determined.

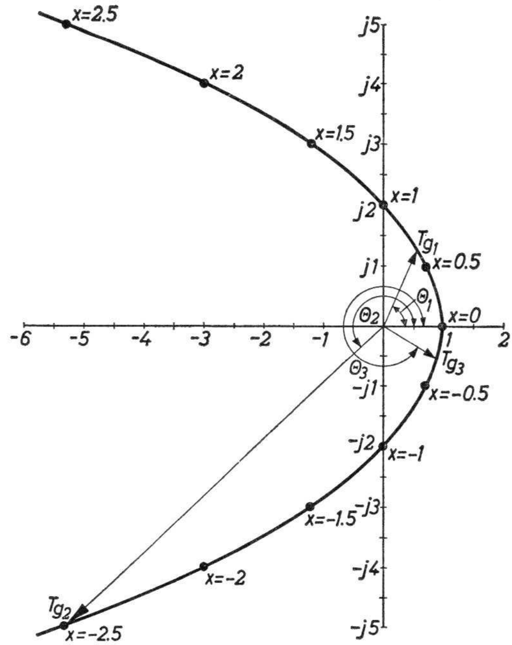


Fig. 2.5. Parabola representing $(1 + jx)^2$, and several vectors T_g applicable to three amplifiers with particular values of Θ on the boundary of stability.

The vector T is now drawn for an arbitrary angle Θ, and is constructed for the normalized frequencies $x = -0.5$ and $x = -2.0$.

It follows from Eq. (2.2.1) as well as from Fig. 2.4 that the boundary of stability of the amplifier will be reached when the top of the vector T coincides with the locus of $(1 + jx)^2$. This value of T will be denoted by T_g. By way of example three different values of T_g have been plotted in Fig. 2.5 for different angles Θ.

The locus of T_g as a function of Θ thus represents the boundary of stability. In following chapters dealing with more complicated amplifiers, the advantage of defining the stability boundary as the locus of T_g will become clear.

Eqs. (2.1.15) and (2.1.16) reveal that at a constant magnitude of the source current i_S the output voltage v_2 of the amplifier increases as δ decreases. At $\delta = 0$, v_2 becomes infinitely large. The region of the complex plane of Fig. 2.4 for which $\delta > 0$ thus corresponds to the region of stable operation of the amplifier. In the case under consideration this is the region within the parabola for which T is smaller than T_g.

Since at $\delta = 0$, that is at $T = T_g$, the amplifier is at the boundary of stability, the region outside the parabola corresponds to the region of unstable operation of the amplifier; in this region $\delta < 0$ or $T > T_g$. Since amplifiers must necessarily be stable, they must be so designed that T lies within the parabola.

By means of Eq. (2.2.2) it is possible to express T_g in terms of Θ. On the boundary of stability $\delta = 0$, whence:

$$(1 + jx_1)(1 + jx_2) - T_g \exp(j\Theta) = 0. \tag{2.2.4}$$

Putting $x_1 = x_2 = x$ and separating the real and the imaginary parts of this expression gives:

$$\left.\begin{aligned} 1 - x^2 &= T_g \cos \Theta , \\ 2x &= T_g \sin \Theta , \end{aligned}\right\} \tag{2.2.5}$$

and

whence, by eliminating x:

$$T_g = \frac{2}{(1 + \cos \Theta)}; \tag{2.2.6}$$

from which it follows that an amplifier is stable, provided:

$$T < \frac{2}{(1 + \cos \Theta)}. \tag{2.2.7}$$

2.2.3 SINGLE-STAGE AMPLIFIER WITH TWO NON-IDENTICAL RESONANT CIRCUITS

In an amplifier there may be differences between the tuned circuits either because they are not tuned to the same frequency or because they have different quality factors, or for both reasons. In the single-stage amplifier under consideration this results in x_1 and x_2 having different values ($x = \beta Q$). In discussing this case distinction will be made between the input and output circuits having different quality factors and/or different resonant frequencies.

2.2.3.1 Different Quality Factors, Equal Resonant Frequencies

When the quality factor Q_1 of the input circuit differs from the quality factor Q_2 of the output circuit, this may be expressed by (assuming a to differ from unity):

$$aQ_1 = Q_2, \tag{2.2.8}$$

whence:

$$(1 + j\beta Q_1)(1 + j\beta Q_2) = 1 - a(\beta Q_1)^2 + j\beta Q_1(1 + a).$$

By putting $\beta Q_1 = x_1$ and $\beta Q_2 = x_2$, this expression becomes:

$$(1 + jx_1)(1 + jx_2) = (1 + jx_1)(1 + jax_1) = 1 - ax_1^2 + jx_1(1 + a). \quad (2.2.9)$$

The locus of Eq. (2.2.9) is again a parabola; its vertex coincides with that of the parabola representing $(1+jx)^2$. Both these loci have been plottted in Fig. 2.6. For the parabola representing Eq. (2.2.9), a is given a value of either 0.5 or 2. This curve intersects the imaginary axis at the points $(0, \pm j(1 + a)/\sqrt{a})$. For $a \neq 1$ the curve thus lies outside the parabola representing $(1 + jx)^2$, except at the vertex where the two curves coincide. This implies that if a is given a value differing from unity, the vector $T \exp (j\Theta)$ remaining constant, the stability will slightly increase.

2.2.3.2 Different Resonant Frequencies, Equal Quality Factors

If the resonant frequency of the input and that of the output circuit of the single-stage amplifier differ, this may be expressed by putting:

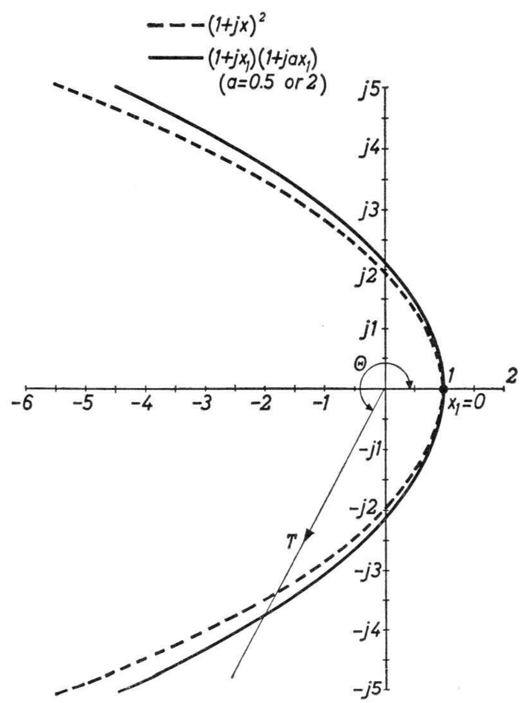


Fig. 2.6. Loci of Eq. (2.2.9) with $a = 0.5$ or $a = 2$ (fully drawn curve) and of the function $(1 + jx)^2$ (broken curve). At values of a differing from unity the former curve is always located outside the latter curve.

$$\beta_2 = \beta_1 + \frac{b}{Q}, \tag{2.2.10}$$

whence:

$$(1 + j\beta_1 Q)(1 + j\beta_2 Q) = 1 - (\beta_1 Q)^2 - \beta_1 Q b + j(2\beta_1 Q + b).$$

By putting $\beta_1 Q = x_1$, this expression becomes:

$$(1 + jx_1)\{1 + j(x_1 + b)\} = 1 - x_1^2 - x_1 b + j(2x_1 + b). \tag{2.2.11}$$

The locus of Eq. (2.2.11) is also a parabola lying outside the parabola representing $(1 + jx)^2$. It intersects the positive real axis in the point $((1 + b^2/4), 0)$ and the imaginary axis in the points $(0, \pm j\sqrt{b^2 + 4})$. Two of these loci, namely those for $b = \pm 0.5$ and for $b = \pm 1.0$, have been plotted in Fig. 2.7. It is seen that due to the tuning frequencies of the circuits being

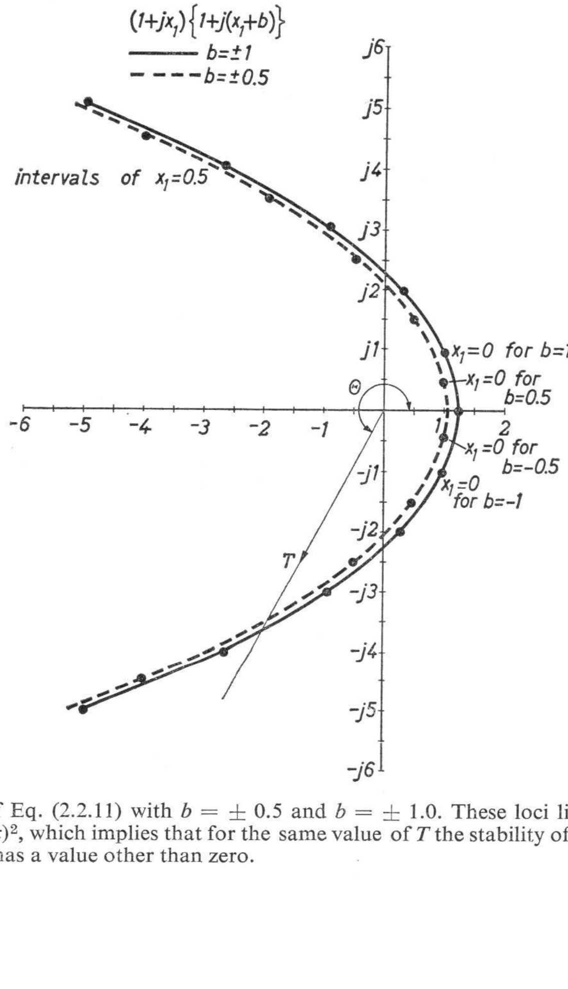


Fig. 2.7. Loci of Eq. (2.2.11) with $b = \pm\, 0.5$ and $b = \pm\, 1.0$. These loci lie outside the parabola $(1 + jx)^2$, which implies that for the same value of T the stability of the amplifier is increased if b has a value other than zero.

different, the stability of the amplifier has slightly increased in this case also. The greater the difference in tuning frequencies the greater will be the increase in stability.

The case of the resonant frequencies of the two tuned circuits being different is of particular importance in connection with the method of aligning the amplifier, as will be shown in Section 2.3.

2.2.3.3 Different Quality Factors, Different Resonant Frequencies

When both the quality factors and the resonant frequencies of the tuned circuits of the single-stage amplifier differ, and the same notation is used as before, it may be written:

$$(1 + jx_1)\{1 + j(ax_1 + b)\} = 1 - ax_1^2 - bx_1 + j\{(a + 1)x_1 + b\}. \quad (2.2.12)$$

As shown in sub-sections 2.2.3.1 and 2.2.3.2 the curve representing $(1 + jx_1)(1 + jx_2)$ will be symmetrical with respect to the real axis if either only the quality factors or only the resonant frequencies differ. When, however, both the quality factors and the resonant frequencies differ the curve, representing Eq. (2.2.12), will be asymmetrical with respect to the real axis.

In Fig. 2.8 such curves have been plottted for $a = 2$ or $a = 0.5$ and $b = 1$ and $b = -1$.For the sake of comparison the parabola representing $(1 + jx)^2$, applicable to the condition $a = 1$ and $b = 0$, has also been plotted. Both curves according to Eq. (2.2.12) are seen to lie outside the latter parabola. This means that by suitably choosing a and b in a particular amplifier design, it is possible to improve the stability, which, in turn, influences other properties of the amplifier.

2.2.4 STABILITY FACTOR

Sub-section 2.2.1 indicated in which region of the complex plane T should be located to ensure stable operation of the amplifier. It was shown that T should lie within the parabola which represents the boundary of stability. It was further shown that for single-stage amplifiers with non-identical tuned circuits (or with identical circuits tuned to different frequencies) the boundary of stability is situated just outside the parabola which is valid for the case of two identical, synchronously tuned circuits. It is, however, generally required that alignable amplifiers do not become unstable over the entire alignment range. This means that stability of such an amplifier has to be considered at the worst possible conditions that might occur during alignment. Following from the considerations in sub-section 2.2.3, the worst possible condition in view of stability is the case of equal resonant frequencies

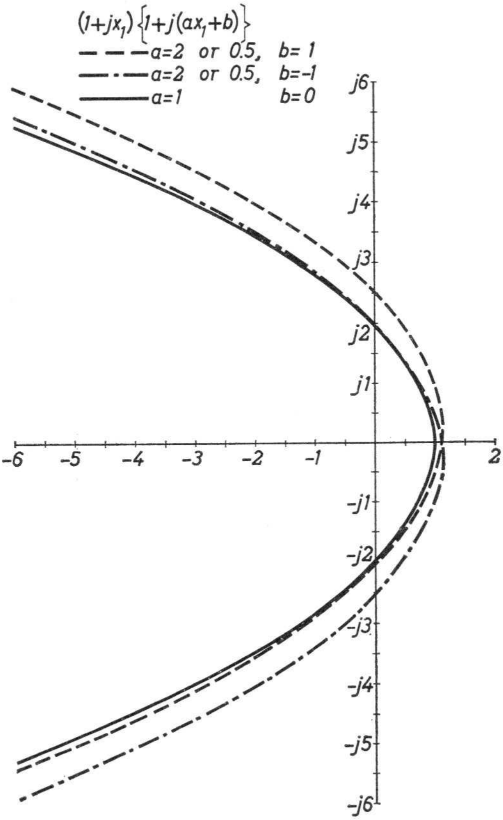


Fig. 2.8. Loci of Eq. (2.2.12). The curves are not symmetrical with respect to the real axis, in contrast to those applying to the case in which either a or b is zero.

and equal quality factors of both tuned circuits. If, therefore, an amplifier is designed such that for correct operation the tuned circuits have different resonant frequencies, the stability of the amplifier should be considered taking the resonant frequencies as equal because during alignment the situation of equal resonant frequencies may actually occur. Further investigation of the stability of the single-stage amplifier will therefore be confined to the case of an amplifier with two identical tuned circuits. The boundary of stability which is valid in this case (cf. sub-section 2.2.2) will be considered as the *basic boundary of stability*. In dealing with more complex amplifiers later, it will be seen that this boundary of stability is indeed very basic.

Hence, if the top of the vector T is situated within the parabola $(1 + jx)^2$ the amplifier will be stable or, in other words, it will not oscillate by itself. The location of T inside this parabola is, however, not a sufficient condition for the stability of a practical amplifier. This may be illustrated by Fig. 2.9 in which the vector T' is applicable to an amplifier which is barely stable because T' is only slightly smaller than T_g. If due to variations of temperature, supply voltage or other conditions, T' is slightly increased so that it approaches T_g more closely, or even becomes equal to it, the stable amplifier will have become unstable as a result of environmental conditions. To ensure that the amplifier remains stable over a wide range of environmental conditions, T should be given a sufficiently small value. It is also necessary to keep T small with respect to T_g in order to make allowance for spreads in transistor parameters. For this purpose the amplifier is normally designed for a transistor having average values of parameters, but it must also remain reasonably stable when equipped with a transistor of the same type having the most unfavourable combination of parameters.

It may thus be concluded that in order to avoid the risk of instability, practical amplifiers should be so designed that T is smaller than T_g by a certain factor, the stability factor. In a practical design this factor, defined as:

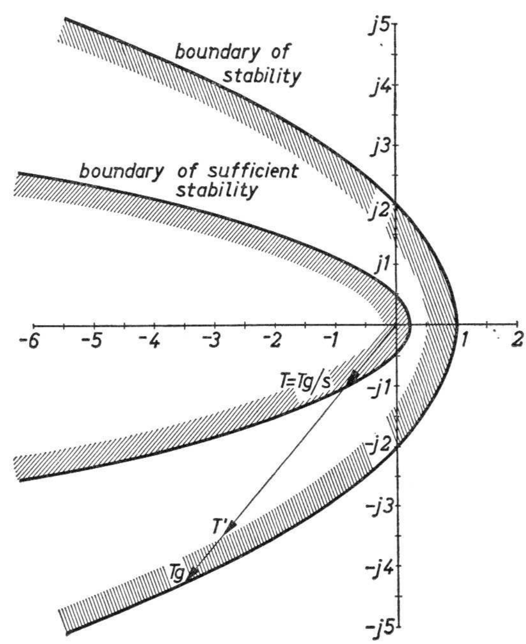


Fig. 2.9. The vector T' applies to an amplifier which, due to a small change in environmental conditions, may become unstable, revealing that the design was not adequate. To ensure that the amplifier is sufficiently stable, T should be located within or on the parabola representing the boundary of sufficient stability.

$$s = \frac{T_g}{T}, \tag{2.2.13}$$

should be so chosen that not only will the amplifier be sufficiently protected against self-oscillation under the most unfavourable conditions, but certain requirements concerning the response curve are also satisfied. This point will be dealt with in sub-section 2.5.2.

In Fig. 2.9 the curve representing T_g/s as a function of Θ has been plotted together with the locus for T_g. It can be shown that the curve $T_g/s = f(\Theta)$ is also a parabola, confocal with the parabola for T_g, its vertex lying at the point $(1/s, 0)$.

The region within the parabola for T_g/s may thus be considered as the region of sufficiently stable operation of the amplifier, the parabola itself representing the boundary of sufficient stability of the amplifier.

2.2.5 POTENTIAL UNSTABILITY AND INHERENT STABILITY

In the preceding comments it was investigated under what conditions instability might occur in a single-stage amplifier. It was shown that it depends on the magnitudes of T and Θ whether or not an amplifier is stable. The stability of an amplifier can therefore be governed by modifying T and/or Θ.

Eqs. (2.1.11) and 2.1.12) reveal that both T and Θ can be modified by changing the product $y_{12}y_{21}$. This can be achieved by different d.c. biasing of the transistor or by applying neutralization. Moreover, it is possible to modify T, as appears from Eq. (2.1.11) by controlling the product G_1G_2.

According to Fig. 2.2 the product G_1G_2 can be varied by modifying either the dampings G^* of the tuned circuits, or the source and load dampings G_S and G_L respectively. If y_{12} and y_{21} are left unchanged T will reach an upper limit when Y_S, Y_1^*, Y_2^* and Y_L are made purely susceptive (G_S, G_1^*, G_2^* and G_L being made zero). It may then be written:

$$\left.\begin{aligned} G_1 &= g_{11}, \\ G_2 &= g_{22}. \end{aligned}\right\} \tag{2.2.14}$$

Identical comments apply to an amplifier in the H-matrix environment. If in the circuit of Fig. 2.3 the impedance Z_S and Z_1^* and the admittances Y_2^* and Y_L are made purely susceptive, we obtain according to Eqs. (2.1.20) and (2.1.21):

$$\left.\begin{aligned} R_1 &= R_e(h_{11}), \\ G_2 &= R_e(h_{22}) \cdot \end{aligned}\right\} \tag{2.2.15}$$

Combining Eq. (2.2.14) with Eq. (2.1.11) and Eq. (2.2.15) with Eq. (2.1.24) it follows [1]:

$$\frac{|y_{12}y_{21}|}{g_{11}g_{22}} = \frac{|h_{12}h_{21}|}{R_e(h_{11}) \cdot R_e(h_{22})} = t\,. \tag{2.2.16}$$

The quantity t will be called the *intrinsic regeneration coefficient* of the transistor. The value of t is independent of the matrix in which the properties of the transistor are expressed. This might be seen either from considering the physical operation of the transistor or from inspecting a matrix conversion table. This coefficient has been plotted in Fig. 2.10 for a particular case. In this graph the regeneration coefficient T of the amplifier stage and the boundary of stability T_g have also been drawn. In all practical cases t will obviously exceed T.

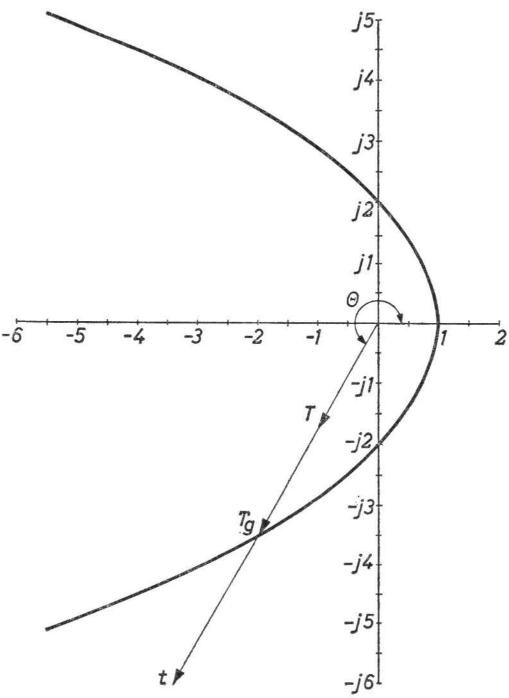


Fig. 2.10. A transistor is said to be potentially unstable when its intrinsic regeneration coefficient t lies outside the parabola representing the boundary of stability T_g: The regeneration coefficient T of a practical amplifier stage equipped with this transistor is generally much smaller than t (and also smaller than T_g).

[1]) Assuming $g_{11} > 0$, $g_{22} > 0$, $R_e(h_{11}) > 0$ and $R_e(h_{22}) > 0$.

It will be clear that there is no risk of instability occurring in an amplifier unless the intrinsic regeneration coefficient t is of such magnitude that the transistor with purely susceptive terminations may itself become unstable. This will be the case when t is located outside the parabola which represents the boundary of stability. The transistor is then said to be *potentially unstable*. According to Fig. 2.10 potential instability occurs when:

$$t > T_g, \tag{2.2.17}$$

whereas an amplifier cannot become unstable when:

$$t < T_g. \tag{2.2.18}$$

In the latter case the transistor is said to be *inherently stable*.

It thus depends on the four-terminal network parameters of the transistor whether it is potentially unstable or inherently stable. Since these parameters are frequency-dependent, a transistor may be potentially unstable over a certain range of frequencies (usually the mid-range) and inherently stable over other ranges (usually the very low and very high ranges). A statement that a transistor or any other active device is potentially unstable is therefore incomplete unless the frequency range to which this statement applies is also specified.

It may thus be concluded that if an amplifier is designed for using potentially unstable transistors (or electron tubes), provision must be made for the ultimate amplifier design to be sufficiently stable. It is only when the transistors are inherently stable to such an extent that t is located within the region of sufficient stability that no stability considerations are required. This will thus be the case when:

$$t < T_g/s. \tag{2.2.19}$$

2.3 Tuning Procedure of the Amplifier

The frequency response of an amplifier depends on the properties of its tuned circuits, whilst the method of alignment largely determines the influence of these frequency-dependent properties on the performance of the amplifier. It is therefore necessary to investigate the various methods of aligning an amplifier insofar as they may lead to different results.

Three methods of tuning [1]) amplifiers with feedback will be discussed.

---

[1]) A resonant circuit is said to be tuned when it gives the excepted response at the desired frequency, i.e. at the tuning frequency. Note that the term "expected response" does not necessarily imply "maximum response".

These tuning methods are closely related to the matrix environment in which the properties of the amplifiers are expressed and the way in which the various tuning methods are carried out in practice agrees with definitions of the four-terminal network parameters used to analyze the amplifier (see sub-section 2.3.6).
Although the advantages and disadvantages of the tuning methods become more apparent in amplifiers which comprise a large number of tuned circuits, the different methods will be investigated here with reference to a single-stage amplifier with two single-tuned circuits. In this way the consequences of the tuning method on the mathematical analysis of the amplifier performance can easily be explained. The results thus obtained can readily be applied to more complex amplifiers, as will be shown later.

To ascertain the influences of the various methods of tuning an amplifier the admittance parameter representation will be considered. In a later sub-section the consequences of the tuning methods are derived for an amplifier using a hybrid-H parameter representation.

2.3.1 GENERAL CONSIDERATIONS REGARDING THE METHOD OF TUNING AN AMPLIFIER IN THE Y-MATRIX ENVIRONMENT

The transimpedance function of the single-stage amplifier based on admittance parameters according to Eq. (2.1.16) was obtained by assuming both x_1 and x_2 to be zero at the tuning frequency; for in that case, by definition, $x = \beta Q$ and $\beta = 0$ at resonance (cf. Appendix II).

To appreciate the consequences of these assumptions, which were made without taking into consideration other factors introduced by the transistors, it is necessary to ascertain the admittances presented by the input and output terminals of the transistor in the amplifier.

It will be assumed that a transistor fourpole, defined by the parameters y_{11}, y_{12}, y_{21} and y_{22}, together with its tuned output circuit and load admittance, is connected as shown in Fig. 2.11. In this circuit the following relations apply:

$$\left.\begin{aligned} i_1 &= y_{11}v_1 + y_{12}v_2\,, \\ i_2 &= y_{21}v_1 + (y_{22} + Y_2{}^* + Y_L)v_2. \end{aligned}\right\} \qquad (2.3.1)$$

When $i_2 = 0$ (i.e. when the output terminals are open-circuited) the input admittance is:

$$y_{in} = \frac{v_1}{i_1} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_2{}^* + Y_L}\,,$$

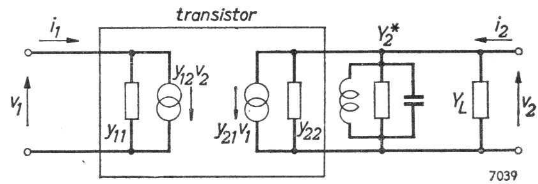


Fig. 2.11. Circuit arrangement for calculating the admittance presented by the transistor input terminals.

or, from Eq. (2.1.2):

$$y_{in} = y_{11} - \frac{y_{12}y_{21}}{Y_2}. \qquad (2.3.2)$$

The first term of this equation represents the transistor self-admittance y_{11}, whilst the second admittance term $-y_{12}y_{21}/Y_2$ accounts for the presence of the feedback parameter y_{12} of the transistor.

The output admittance presented to the transistor output terminals in the single-stage amplifier can be calculated in an analogous way. The admittance of the tuned input circuit, $Y_1{}^*$, and the source admittance Y_S are now connected to the transistor input terminals in the normal way as shown by Fig. 2.2. The output admittance then is:

$$y_{out} = y_{22} - \frac{y_{12}y_{21}}{Y_1}. \qquad (2.3.3)$$

Apart from the first term y_{22}, representing the transistor self-admittance parameter, the output admittance contains a term $-y_{12}y_{21}/Y_1$.

Eqs. (2.3.2) and (2.3.3) reveal that the input and output admittance of the transistor in the amplifier depend on the complex values of Y_2 and Y_1 respectively, and hence on the tuning of the amplifier.

Now, according to Eq. (2.1.2):

$$\left.\begin{aligned} Y_1 &= y_{11} + Y_1{}^* + Y_S, \\ \text{and} \qquad Y_1 &= y_{22} + Y_2{}^* + Y_L. \end{aligned}\right\} \qquad (2.3.4)$$

It is thus seen that the admittance Y_1, as defined here, only contains the part y_{11} of the transistor input admittance, whereas the input admittance itself comprises, apart from y_{11}, a term $-y_{12}y_{21}/Y_2$. The latter term can, however, be reduced to such an extent as to become negligible by making Y_2 very large. This can be achieved either by heavily damping or detuning the output circuit of the amplifier.

According to Eq. (2.1.4):

$$Y_1 = G_1(1 + jx_1),$$

in which x_1 must disappear at the tuning frequency, as stated earlier. This will therefore be the case only when the influence of the term $- y_{12}y_{21}/Y_2$ on the input admittance is negligible. It may thus be concluded that Y_2 should be made very large in order to align the input circuit of the amplifier, or, in other words, to adjust its circuit elements so that $x_1 = 0$ at the tuning frequency (The expected response of the tuned circuit is that which is obtained when $x_1 = 0$).

The same argument holds for the output tuned circuit. In this case:

$$Y_2 = G_2(1 + jx_2),$$

in which the condition $x_2 = 0$ at the tuning frequency cannot be satisfied unless the output circuit is tuned with the input circuit heavily damped or detuned.

Stringent requirements are therefore imposed on the method of tuning the amplifier if the properties of the resonant circuits are to be defined by $G_1(1 + jx_1)$ and $G_2(1 + jx_2)$.

2.3.2 TUNING METHOD A

One way of tuning the single-stage amplifier — to be termed "method A" — therefore consists in tuning each of the two tuned circuits with the other circuit heavily damped or detuned.

In practice, the tuning can be carried out by feeding a signal of the tuning frequency to the circuit to be tuned, via a high impedance and measuring the voltage produced across the circuit by means of a vacuum tube voltmeter. The circuit elements are then so adjusted that the voltmeter reading is at a maximum. Care should be taken that the instruments used for tuning this circuit do not introduce any noticeable damping or detuning.

Both tuned circuits of the single-stage amplifier can easily be adjusted in this way. However, if the amplifier contains a considerable number of tuned circuits, this procedure is rather laborious and takes much time. In fact, each circuit must be aligned separately, and during this operation at least the preceding and the following circuits must be heavily damped or detuned. This method was assumed to be applied in the single-stage amplifier considered hitherto. The results thus obtained remain valid, however, provided tuning method A is applied.

2.3.3 TUNING METHOD B

Another method of tuning the single-stage amplifier, termed "method B", consists in first tuning the output circuit with the input circuit heavily damped. The admittance of the output circuit as a function of the frequency can then be expressed by:

$$Y_2 = G_2(1 + jx_2).$$

Subsequently the input circuit is tuned so that the reading of a vacuum tube voltmeter connected to the output terminals is at a maximum, the tuning of the output circuit being left unchanged.

According to Eq. (2.3.2) the input admittance of the transistor at the tuning frequency is:

$$y_{in} = y_{11} - \frac{y_{12}y_{21}}{G_2},$$

provided the output circuit has already been tuned ($x_2 = 0$).

The total admittance at the input terminals of the transistor, including the tuned circuit admittance $Y_1{}^*$ and the source admittance Y_S, then becomes :

$$y_{in\ tot} = Y_S + Y_1{}^* + y_{11} - \frac{y_{12}y_{21}}{G_2}, \qquad (2.3.5)$$

which can also be written:

$$y_{in\ tot} = Y_1 - \frac{y_{12}y_{21}}{G_2}, \qquad (2.3.6)$$

or

$$Y_{in\ tot} = G_1\left(1 + jx_1 - \frac{y_{12}y_{21}}{G_1G_2}\right). \qquad (2.3.7)$$

Substitution of Eqs (2.1.11) and (2.1.12) gives:

$$Y_{in\ tot} = G_1(1 + jx_1 - T\cos\Theta - jT\sin\Theta). \qquad (2.3.8)$$

When the input circuit is being tuned the reading of the voltmeter will be at a maximum if the admittance of this circuit as a function of the normalized detuning, as defined by Eq. (2.3.8), is at a minimum. This will be the case when the imaginary part of this expression is zero, that is to say when:

$$x_1 - T\sin\Theta = 0,$$

or

$$x_1 = T\sin\Theta. \qquad (2.3.9)$$

When the input circuit is tuned in this way it is essential for x_1 to have the value given by Eq. (2.3.9) at the tuning frequency so that, at this frequency, the total susceptance of the circuit is zero. The quantity $x = \beta Q$ has, however, already been so defined that x itself becomes zero at the tuning frequency. In order to keep this definition of x valid for the method of tuning described here, the quantity x_1' will be introduced:

$$x_1' = T \sin \Theta. \tag{2.3.10}$$

The total susceptance of the input tuned circuit will therefore be defined as:

$$G_1(x_1 + x_1'). \tag{2.3.11}$$

The input tuned circuit thus gives the expected response when at the tuning frequency its susceptance equals

$$G_1 x_1' = G_1 \cdot T \sin \Theta,$$

in accordance with Eq. (2.3.8).

If the single-stage amplifier is aligned according to method B, that is by firstly tuning the output circuit with the input circuit heavily damped and subsequently tuning the input circuit with the output circuit unchanged, the total admittance of the output tuned circuit will therefore be:

$$Y_{2\,tot} = G_2(1 + jx_2). \tag{2.3.12}$$

The total admittance of the input circuit will be:

$$Y_{1\,tot} = G_1\{1 + j(x_1 + x_1')\}. \tag{2.3.13}$$

It should be recognized that the conductive term of this admittance has remained unchanged. This is due to the fact that Eq. (2.3.13) clearly expresses the admittance of the input circuit without the influences of the feedback of the transistor on this circuit. The conductance therefore remains G_1, but the susceptance is increased by $G_1 x_1'$ in order to render the total susceptance, including that due to the transistor feedback, equal to zero at the tuning frequency, as required by the tuning procedure adopted. In other words, if tuning method B is followed the susceptance must be corrected by an amount $G_1 x_1'$. The term x_1' in Eq. (2.3.13) will therefore henceforth be referred to as the *tuning correction term.*

In practice, tuning method B is very convenient for amplifiers containing a large number of tuned circuits. An output voltmeter having a very high impedance is connected across the output circuit and a low-impedance signal generator feeds a signal having the desired frequency to the penultimate tuned circuit of the amplifier, after which the output circuit is tuned to maximum deflection of the output meter. All other tuned circuits of the amplifier are then aligned in succession in the same way, the output circuit having been tuned first. The signal generator is always connected to the circuit that precedes the one to be tuned.

Compared with method A, this method of tuning is therefore more convenient and time-saving. Moreover, the consequences of the tuning procedure on the performance of the amplifier can easily be taken into account in a mathematical analysis. This has already been shown in the foregoing analyses, and it will be seen later that similar results are obtained for more complex amplifiers. Another point is that, with tuning method B, better amplitude response and envelope delay curves are usually obtained, which will also be discussed later.

2.3.4 TUNING METHOD C

An alternative method of tuning, "method C", is as follows. First the input circuit is tuned to the desired frequency with the output circuit heavily damped or detuned. This damping or detuning of the output circuit can conveniently be achieved by using a low-impedance output meter, for example a vacuum-tube voltmeter the input probe of which is shunted by a large capacitance. Under these conditions the input admittance of the transistor is y_{11}, which implies that the total admittance of the input circuit is:

$$Y_{1\,tot} = G_1(1 + jx_1).$$

After the input circuit has been tuned, the output circuit is aligned, the input circuit remaining unaffected. The signal required for tuning the output circuit is thus obtained from the current source connected to the input circuit. A tube voltmeter, which must not introduce any noticeable damping in this case, is connected to the output terminals.

In analogy with the comments in sub-section 2.3.3, the total admittance of the output circuit, with the input circuit correctly tuned, is:

$$Y_{out\,tot} = y_{22} + Y_2{}^* + Y_L - \frac{y_{12}y_{21}}{G_1},$$

or

$$Y_{out\,tot} = Y_2 - \frac{y_{12}y_{21}}{G_1} = G_2\left(1 + jx_2 - \frac{y_{12}y_{21}}{G_1G_2}\right).$$

Substitution of Eqs. (2.1.11) and (2.1.12) gives:

$$Y_{out\,tot} = G_2(1 + jx_2 - T\cos\Theta - jT\sin\Theta). \qquad (2.3.14)$$

It can be shown in a similar way as in sub-section 2.3.3 that with this method of tuning the total admittance of the output circuit is

$$Y_{2\,tot} = G_2\{1 + j(x_2 + x_2'')\}, \tag{2.3.15}$$

in which the tuning correction term x'' equals [1])

$$x_2'' = T \sin \Theta \tag{2.3.16}$$

Similar to tuning method B, method C is particularly useful for aligning amplifiers (with feedback) which comprise a large number of tuned circuits. In this case, too, the circuits are tuned in succession, but here the input circuit is tuned first.

2.3.5 BASIC DEFINITIONS OF THE VARIOUS TUNING METHODS

The various tuning methods of selective amplifiers with feedback are considered in the preceding paragraph by analyzing an amplifier in the Y-matrix environment. As already referred to, the tuning methods and their practical execution are closely related to the matrix environment of the amplifier. This means that there may be differences between corresponding tuning methods for amplifiers in the Y- or H-matrix environments although the basic definitions are the same. This will become apparent by considering these basic definitions which will therefore be stated here explicitely.

2.3.5.1 Tuning Method A

The resonant circuits of a single-stage amplifier are said to be tuned according to method A when, during alignment, the total immittance of the circuit at the input terminals of a transistor only contains the input self-immittance (either y_{11} or h_{11}) of this transistor and the total immittance of the circuit at the output terminals of this transistor only contains its output self-immittance (either y_{22} or h_{22}).

2.3.5.2 Tuning Method B

A single-stage amplifier is said to be tuned according to method B when the resonant circuits are tuned in succession starting at the output side and the tuning is carried out such that, during alignment, the total immittance of the circuit connected to the output terminals of a transistor only contains the output self-immittance (y_{22} or h_{22}) of this transistor.

2.3.5.3 Tuning Method C

A single-stage amplifier is said to be tuned according to method C when the resonant circuits are tuned in succession starting at the input side and

---

[1]) The double dash which is used here distinguishes the tuning correction term from that used for tuning method B.

the tuning is carried out such that, during alignment, the total immittance of the circuit connected to the input terminals of a transistor only contains the input self-immittance (either y_{11} or h_{11}) of this transistor.

Due to the different definitions of y_{22} and h_{22} (input terminals of the transistor short-circuited or open-circuited respectively), the way in which tuning methods A en B must be carried out in amplifiers in either the Y or H-matrix environment is completely different.

2.3.6 VARIOUS TUNING METHODS FOR AN AMPLIFIER IN THE *H*-MATRIX ENVIRONMENT

For the single-stage amplifier in the H-matrix environment the total input self-immittance is the impedance Z_1 and the total output self-admittance is the admittance Y_2. In analogy with the preceding paragraphs, tuning correction terms are required for tuning methods B and C.

2.3.6.1 Tuning Method A

For the single-stage amplifier tuned according to method A we may write, see Eq. (2.1.19):

$$\left.\begin{aligned} Z_1 &= R_1(1 + \mathrm{j}x_1)\,, \\ \text{and} \qquad Y_2 &= G_2(1 + \mathrm{j}x_2)\,. \end{aligned}\right\} \tag{2.3.17}$$

From the basic definition it follows that for an amplifier in the H-matrix environment the output circuit should be short-circuited to tune the input circuit and the input circuit should be open-circuited to tune the output circuit. The latter condition is sometimes difficult to fulfil in a practical amplifier because the tuning signal should be supplied to the transistor via an impedance which is large compared with h_{11} [1]).

2.3.6.2 Tuning Method B

For the same reasons as tuning method *A*, tuning method B is less practical for amplifiers in the H-matrix environment [2]). For completeness of the theoretical analysis, however, suppose that an amplifier is tuned according to this method. Thus, in analogy with sub-section 2.3.3, a tuning correction term x' appears in the equation for the total input impedance Z_1 of the amplifier whereas no tuning correction term appears in the total output admittance Y_2. With Eqs. (2.1.17) and (2.1.19):

---

[1]) The same conclusions can be drawn for amplifiers in the Z- and K-matrix environments.

[2]) The same applies to amplifiers in the Z-matrix environment. For amplifiers in the K-matrix environment, however, tuning method B will prove to be the most practical method.

and
$$\left.\begin{aligned} Z_1 &= R_1\{1 + \mathrm{j}(x_1 + x_1')\}, \\ Y_2 &= G_2(1 + \mathrm{j}x_2)\,. \end{aligned}\right\} \tag{2.3.18}$$

2.3.6.3 Tuning Method C

According to the basic definition of tuning method C in sub-section 2.3.5 a single-stage amplifier in the H-matrix environment should be tuned as follows:

Firstly the input circuit is tuned. Because h_{11} is defined with the output terminals of the transistor short-circuited, the output circuit of the amplifier must be heavily damped or detuned for tuning the input circuit. Next, the output circuit is tuned with the (already tuned) input circuit left operative. With this tuning operation no practical difficulties are encountered and it may thus be concluded that tuning method C is very useful for amplifiers in the H-matrix environment [1]).

In analogy with sub-section 2.3.4 a tuning correction term x_2'' appears in the total output admittance Y_2 of the amplifier whereas no tuning correction term appears in the total input impedance Z_1. With Eqs. (2.1.17) and (2.1.19) we then obtain:

$$\left.\begin{aligned} Z_1 &= R_1(1 + \mathrm{j}x_1), \\ Y_2 &= G_2\{1 + \mathrm{j}(x_2 + x_2'')\}\,. \end{aligned}\right\} \tag{2.3.19}$$

2.3.7 INFLUENCE OF THE METHODS OF TUNING ON THE REDUCED DETERMINANT

It has been shown that for tuning method B a tuning correction term x_1' must be introduced for the immittance of the input circuit whereas for the output circuit no correction term is required. For tuning method C the reverse applies, and for tuning method A no tuning correction term is necessary at all.

The three tuning methods may be represented by one set of equations giving the immittances of the input and output circuits:

$$Y_1 = G_1\{1 + \mathrm{j}(x_1 + p_1x_1' + p_2x_1'')\}, \tag{2.3.20.a}$$

$$Z_1 = R_1\{1 + \mathrm{j}(x_1 + p_1x_1' + p_2x_1'')\}, \tag{2.3.20.b}$$

$$Y_2 = G_2\{1 + \mathrm{j}(x_2 + p_1x_2' + p_2x_2'')\}. \tag{2.3.20.c}$$

Equations a and c are valid for amplifiers in the Y-matrix environment

---

[1]) Further consideration of tuning method C leads to the conclusion that it is less practical for amplifiers in the Z- and K-matrix environment.

whereas equations *b* and *c* apply to amplifiers in the *H*-matrix environment. In Eq. (2.3.20) the tuning correction terms are:

$$\left.\begin{aligned} x_1' &= T \sin \Theta\,, & x_1'' &= 0, \\ x_2' &= 0, & x_2'' &= T \sin \Theta\,, \end{aligned}\right\} \qquad (2.3.21)$$

while p_1 and p_2 would furthermore be given the values tabulated below:

Table 2.1	tuning method A	tuning method B	tuning method C
p_1	0	1	0
p_2	0	0	1

The reduced determinant of the single-stage amplifier becomes with Eqs. (2.3.20) and (2.3.21):

$$\delta = \begin{vmatrix} 1 + \mathrm{j}(x_1 + p_1 x_1' + p_2 x_1'') & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}(x_2 + p_1 x_2' + p_2 x_2'') \end{vmatrix}. \qquad (2.3.22)$$

Since δ contains all frequency-dependent terms of the transfer function of the amplifier, see Eqs. (2.1.16) and (2.1.27), Eq. (2.3.22) can be used universally to investigate the gain, the amplitude response and the phase response of the single-stage amplifier, the table above being employed to account for the different methods of tuning.

2.3.8 INFLUENCE OF THE METHODS OF TUNING ON THE STABILITY OF THE AMPLIFIER

With tuning methods B or C the input and output circuits of the amplifier are detuned with respect to each other by an amount [1])

$$x_1' = x_2'' = T \sin \Theta.$$

At $x_1 = x_2 = x$ the reduced determinant, as given by Eq. (2.3.22), becomes:

$$(1 + \mathrm{j}x)(1 + \mathrm{j}x + \mathrm{j}T \sin \Theta) - T \exp(\mathrm{j}\Theta). \qquad (2.3.23)$$

---

[1]) It should be recognized that, although the amplifier is synchronously tuned, which means that a signal of the same frequency is used for aligning both circuits, it is inherent to tuning methods B and C that the circuits resonate at different frequencies.

At the boundary of stability this determinant becomes zero. Denoting the value of T at this boundary by T_g' gives:

$$(1 + jx)^2 + (1 + jx) \cdot jT \sin \Theta - T_g' \exp (j\Theta) = 0. \qquad (2.3.24)$$

The real and imaginary parts of this expression can be separated, which gives:

$$1 - x^2 - xT \sin \Theta = T_g' \cos \Theta, \qquad (2.3.25)$$

and

$$2x + T \sin \Theta = T_g' \sin \Theta, \qquad (2.3.26)$$

or, since $T = \dfrac{T_g'}{s}$,

$$2x = \left(1 - \frac{1}{s}\right) T_g' \sin \Theta. \qquad (2.3.27)$$

This equation is now substituted for x in Eq. (2.3.25), so that after some rearrangement a quadratic expression for T_g' is obtained of which only the (largest) positive root has significance:

$$T_g' = \frac{2}{\cos \Theta + \sqrt{\cos^2 \Theta + \left(1 - \dfrac{1}{s^2}\right) \sin^2 \Theta}}. \qquad (2.3.28)$$

With tuning method A the value of T_g becomes:

$$T_g = \frac{2}{\cos \Theta - \sqrt{\cos^2 \Theta - \sin^2 \Theta}}. \qquad (2.3.29)$$

(cf. sub-section 2.2.2, Eq. (2.2.6).

Comparison of Eqs. (2.3.28) and (2.3.29) reveals that T_g' is slightly larger than T_g. This confirms that, due to the tuning methods B and C, the stability of the amplifier is slightly improved, as was already shown in sub-section 2.2.3.2 and in Fig. 2.7 (now $b = T \sin \Theta$).

Considering that in a practical amplifier only the value of $T = T_g/s$ is of importance, this small increase in stability will be neglected henceforth. Moreover, the condition that during alignment the tuned circuits may resonate at the same frequency (see sub-section 2.2.4) must also be taken into account. As a matter of fact the tuning procedure affects only the stability factor s, the exact value of which is of secondary importance.

2.4 Gain

The method of expressing the gain of amplifiers equipped with electron tubes in terms of voltage gain has been abandoned for transistor amplifiers, the gain of which is as a rule expressed in terms of power gain. The reason for this is the difference in input admittance of the two types of amplifying devices.

Provided the frequency of the signal to be amplified is not extremely high, the real part of the input admittance of electron tubes is negligible, so that no signal-frequency power is required to produce a certain signal across the impedance in the output circuit, it being sufficient to apply a certain voltage. In bandpass amplifiers the relevant stage equipped with an electron tube is usually followed by another stage also provided with an electron tube, which again requires only a voltage to drive it. This explains why the gain of a tube amplifier is expressed in terms of voltage gain.

On the other hand, the real part of the input admittance of transistors is by no means negligible even at low frequencies. A certain amount of input power is therefore required to drive the transistor.

Furthermore, the impedance levels may be different at various points in the amplifier between which the gain is to be specified. It is therefore convenient to express the gain of transistor amplifiers in terms of power gain, it then being superfluous to state the different impedance levels. Expressing the gain in terms of power gain is, moreover, very useful because the following transistor also requires some driving power.

2.4.1 TRANSDUCER GAIN

The definition of the power gain of an amplifying circuit will now be discussed. It should be such that the gain figure gives a proper indication of the function of the circuit.

Fig. 2.12 represents an amplifier which delivers power into a load having an admittance $Y_L = G_L + B_L$. The amplifier is driven by a current source i_S having an admittance $Y_S = G_S + jB_S$.

The power supplied to the load by the amplifier is:

$$P_0 = |v_0|^2 \cdot G_L. \tag{2.4.1}$$

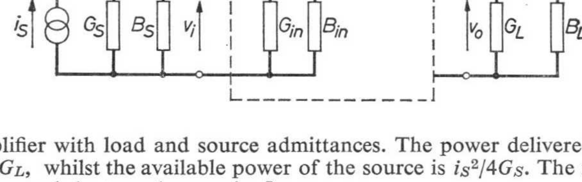


Fig. 2.12. Amplifier with load and source admittances. The power delivered to the load amounts to $v_0{}^2G_L$, whilst the available power of the source is $i_S{}^2/4G_S$. The ratio of these two powers is termed the transducer gain Φ_t.

The power supplied to the input admittance of the amplifier by the current source depends on the matching between the source and the amplifier.

Optimum matching can always be achieved by means of an impedance transforming network incorporated in the input circuit of the amplifier, so that its gain can best be related to the power that the source can deliver under matched conditions. This power, termed the available power of the source, equals:

$$P_{Sa} = \left(\frac{|i_S|}{2}\right)^2 \cdot \frac{1}{G_S} = |i_S|^2 \cdot \frac{1}{4G_S}. \tag{2.4.2}$$

The ratio P_0/P_{Sa} is termed the *transducer gain* of the amplifier. Hence, from Eqs. (2.4.1) and (2.4.2):

$$\Phi_t = 4G_S G_L \cdot \left|\frac{v_0}{i_S}\right|^2, \tag{2.4.3}$$

or, as the ratio v_0/i_S represents the transimpedance Z_t of the complete amplifier circuit:

$$\Phi_t = 4G_S G_L \cdot |Z_t|^2. \tag{2.4.4}$$

By means of the Thevenin-Norton theorem, the current source of Fig. 2.12 with admittance Y_S can be replaced by a voltage source with impedance Z_S, see Fig. 2.13.
If

$$Z_S = R_S + \mathrm{j}\, X_S, \tag{2.4.5}$$

the available power from the source is:

$$P_{Sa} = \frac{|v_S|^2}{4R_S}. \tag{2.4.6}$$

With Eq. (2.4.1) the transducer gain then becomes:

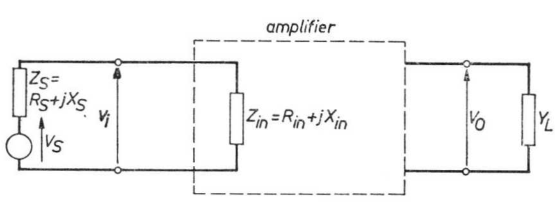


Fig. 2.13. Alternative amplifier arrangement for defining transducer gain. The current source i_S with admittance Y_S of Fig. 2.12 has been replaced by a voltage source with an impedance Z_S.

$$\Phi_t = 4R_S G_L \left| \frac{v_0}{v_S} \right|^2 = 4R_S G_L \cdot |K_t|^2 . \tag{2.4.7}$$

It will be obvious that Fig. 2.12 and Eq. (2.4.4) apply especially to amplifiers in the Y-matrix environment whereas Fig. 2.13 and Eq. (2.4.7) are applicable to amplifiers in the H-matrix environment.

Eqs. (2.4.4) and (2.4.7) thus give the power gain of an amplifier between a given load admittance having a damping G_L and a given signal source having a damping G_S or resistance R_S respectively. This gain figure is the best indication of the properties of the amplifier, provided that G_L and G_S or R_S are independent of the design of the amplifier. Definitions of transducer gain and other methods of expressing gain in power are presented in Appendix IV. Furthermore, the relative merits of the various methods of defining gain in power will be considered in Chapter 4.

2.4.2 POWER GAIN

It has been shown that the transducer gain is a very useful measure of the amplification of an amplifier as a whole. However, it is very often necessary to express the gain of the individual stages of an amplifier in order to judge their amplifying properties. For this purpose another gain figure, the power gain per stage of the amplifier, is used. This power gain is defined as the ratio of the amount of power fed to the load of the stage (usually the input impedance of the following stage) to the amount of power fed to the input impedance of the stage itself (see also Chapter 4). In the circuit of Fig. 2.12 the power gain is therefore given by:

$$\Phi = \frac{|v_0|^2 \cdot G_L}{|v_i|^2 \cdot G_{in}}, \tag{2.4.8}$$

and in the circuit of Fig. 2.13:

$$\Phi = \frac{|v_0|^2 \cdot G_L}{\dfrac{|v_i|^2}{R_{in}}}. \tag{2.4.9}$$

2.4.3 TRANSDUCER GAIN OF THE SINGLE-STAGE AMPLIFIER

Eqs. (2.4.4) and (2.4.7) give the transducer gain of an amplifier in the Y- or the H-matrix environment respectively. Together with Eq. (2.3.22) and Eqs. (2.1.16) resp. (2.1.27) the transducer gain of the single-stage amplifier at $x = 0$ becomes:

$$\left.\begin{aligned}\Phi_t &= 4G_SG_L\cdot\frac{|y_{21}|^2}{G_1{}^2G_2{}^2}\cdot\frac{1}{|\delta_0|^2},\\ \text{or}\qquad \Phi_t &= 4R_SR_L\cdot\frac{|h_{21}|^2}{R_1{}^2G_2{}^2}\cdot\frac{1}{|\delta_0|^2},\end{aligned}\right\}\tag{2.4.10}$$

in which:

$$\delta_0 = \begin{vmatrix} 1+\mathrm{j}(p_1x_1'+p_2x_1'') & T\exp(\mathrm{j}\Theta) \\ 1 & 1+\mathrm{j}(p_1x_2'+p_2x_2'') \end{vmatrix}.\tag{2.4.11}$$

When tuning method A is applied, δ_0 becomes:

$$\delta_0 = 1 - T(\cos\Theta + \mathrm{j}\sin\Theta),\tag{2.4.12}$$

whilst with tuning methods B and C:

$$\delta_0 = 1 - T\cos\Theta.\tag{2.4.13}$$

These expressions reveal that (because of the term $T\sin\Theta$ in Eq. (2.4.12)) tuning methods B and C usually yield a slightly higher transducer gain at $x = 0$ than method A.

Eq. (2.4.10) can also be written:

$$\Phi_t = \frac{|y_{21}|^2}{4g_{11}g_{22}}\cdot\frac{4G_Sg_{11}}{(G_S+g_{11})^2}\cdot\frac{4g_{22}G_L}{(g_{22}+G_L)^2}\cdot \frac{(G_S+g_{11})^2}{G_1{}^2}\cdot\frac{(g_{22}+G_L)^2}{G_2{}^2}\cdot\frac{1}{|\delta_0|^2}\tag{2.4.14.a}$$

or:

$$\Phi_t = \frac{|h_{21}|^2}{4R_e(h_{11})\cdot R_e(h_{22})}\cdot\frac{4R_S\cdot R_e(h_{11})}{\{R_S+R_e(h_{11})\}^2}\cdot\frac{4R_e(h_{22})\cdot G_L}{\{R_e(h_{22})+G_L\}^2}\cdot \frac{\{R_S+R_e(h_{11})\}^2}{R_1{}^2}\cdot\frac{\{R_e(h_{22})+G_L\}^2}{G_2{}^2}\cdot\frac{1}{|\delta_0|^2}\tag{2.4.14.b}$$

According to Appendix V, the first factor of these expressions denotes the maximum unilateralized power gain of the transistor, which will be denoted by Φ_{uM}. The second and third factors denote the mismatch losses [1]) between the

---

[1]) Strictly speaking, the term "mismatch losses" is used here incorrectly because the input and output self immitances are not the actual input and output immittances in case the transistor is non-unilateral. The influence of the transistor feedback on the transducer gain is, however, completely accounted for by the factor Φ_f. The remaining factors in the expression for the transducer gain (Eq. 2.4.14) only refer to a "unilateralized" transistor. This explains the term "mismatch losses" in the sense as used here.

real parts of the generator immittance and transistor input self-immittance and those between the real parts of transistor output self-immittance and load immittance respectively (see Appendix II); these mismatch losses will be denoted by Φ_{mm1} and Φ_{mm2} respectively. According to the same appendix, the last two factors of Eq. (2.4.14) represent the insertion losses of the first and second tuned circuit of the amplifier, to be denoted by Φ_{i1} and Φ_{i2} respectively. The factor $|1/\delta_0|^2$ represents the losses due to the feedback of the transistor at the tuning frequency. These losses will be denoted by Φ_f. Eq. (2.4.14) may thus be written:

$$\Phi_t = \Phi_{uM} \cdot \Phi_{mm1} \cdot \Phi_{mm2} \cdot \Phi_{i1} \cdot \Phi_{i2} \cdot \Phi_f. \qquad (2.4.15)$$

The quantity Φ_{uM} depends solely on the transistor properties and the chosen biasing point.

For tuning methods B and C, Φ_f can be written with Eq. (2.4.13) as:

$$\Phi_f = \frac{1}{(1 - T \cos \Theta)^2}, \qquad (2.4.16)$$

For amplifiers in the admittance matrix environment, Eq. (2.4.16) can be written as:

$$\Phi_f = \left\{ \frac{G_1}{G_1 - \dfrac{|y_{12}y_{21}| \cos \Theta}{G_2}} \right\}^2. \qquad (2.4.17)$$

The denominator of this expression represents the total damping at the input terminals of the transistor including the influences of the feedback whereas the numerator represents this damping *without* feedback influences. Now, for a certain output current of the transistor a certain voltage has to be produced at the input terminals ($i_0 = y_{21} \cdot v_i$). (Eq. 2.4.16)) then represents the square of the ratio of the sum of the input currents through the various dampings for the cases with and without feedback required to produced the same input voltage v_i. This squared ratio equals the influence due to the feedback on the gain in power of the amplifier.

For tuning method A, Φ_f can be expressed as:

$$\Phi_f = \frac{G_1^2}{\left\{G_1 - \dfrac{|y_{12}y_{21}| \cos \Theta}{G_2}\right\}^2 + \left\{\dfrac{|y_{12}y_{21}| \sin \Theta}{G_2}\right\}^2}, \qquad (2.4.18)$$

which reveals that the feedback losses are larger for tuning method A than for tuning methods B and C.

For amplifiers in the H-matrix environment similar arguments hold for the influence of the feedback.

The remaining factors constituting Φ_t depend on the design of the amplifier and are determined by the stability and response curve requirements.

Since it is not intended to discuss in this chapter an actual design of an amplifier of the type considered, but merely to define and explain several terms which are to be used later, the problem of obtaining maximum transducer gain will not be dealt with here. Chapter 4 will mainly be devoted to this problem together with some other points.

2.5 Frequency Response of the Amplifier

The only frequency-dependent term in the transfer function of the amplifier is the reduced determinant δ which, including the influences of the tuning procedure, is given by Eq. (2.3.22). From this reduced determinant the the complex response curve, the amplitude response curve and the phase response curve can be derived.

2.5.1 THE COMPLEX RESPONSE CURVE

The complex response curve of an amplifier is defined as the curve which gives the combined responses of the amplifier with respect to amplitude and phase, both as functions of frequency. This response corresponds to the transfer function of the single-stage amplifier, including influences of the tuning procedure, hence to the reciprocal of:

$$\delta = \begin{vmatrix} 1 + \mathrm{j}(x_1 + p_1 x_1' + p_2 x_1'') & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}(x_2 + p_1 x_2' + p_2 x_2'') \end{vmatrix}. \quad (2.5.1)$$

in which p_1 and p_2 follow from Table 2.1, and x' and x'' are the tuning correction terms (see sub-section 2.3).

Since x_1 is related in a simple manner to x_2, it is possible to express Z_t as a function of either x_1 or x_2 or, for example, as a function of the geometrical mean of the normalized detunings x_1 and x_2. Preference is given here to the latter method, and for this purpose a new normalized detuning

$$x = \sqrt{x_1 x_2}, \quad (2.5.2)$$

will be introduced. It is now possible to plot δ as a function of x in the complex plane. Fig. 2.14 shows such a graph for a single-stage amplifier having the following data: $x_1 = x_2$, $T = 2$, $\Theta = 225°$, $p_1 = 1$ and $p_2 = 0$ (tuning method B).

Both the amplitude and the phase response can be determined from this complex response curve. The length of the line $|\delta|$ is a measure for the reci-

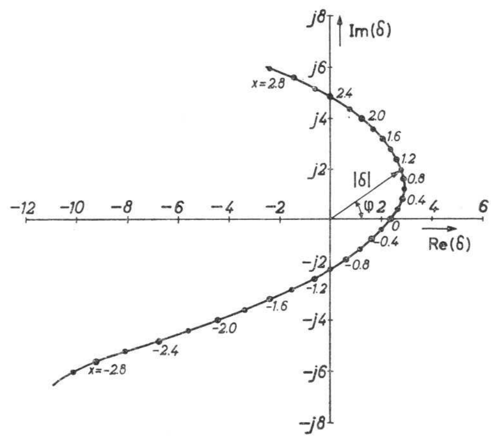


Fig. 2.14. Complex response curve of a single-stage amplifier with two single-tuned circuits having the following data: $x_1 = x_2 = x$, $T = 2$, $\Phi = 225°$, $p_1 = 1$ and $p_2 = 0$ (tuning method B). The length of the line $|\delta|$ is the reciprocal of the transimpedance at $x = 1$, whilst Φ denotes the phase angle of the transimpedance function at this frequency.

procal of the amplitude response for a normalized frequency $x = +1$, whilst angle φ represents the phase angle of δ at that frequency.

In most cases it is, however, more convenient to judge the amplitude and phase responses of the amplifier from separate curves. The amplitude response can then be obtained by determining the modulus of $1/\delta$, whilst the phase response follows either from the phase angle of $1/\delta$ or from a derived function of this phase angle.

2.5.2 THE AMPLITUDE RESPONSE CURVE

2.5.2.1 The Amplitude Response Curve of the Single-Stage Amplifier

The amplitude response of the single-stage amplifier as a function of the normalised detuning x follows from the modulus of $1/\delta$. The most important information to be given by an amplitude response curve is the ratio of the gain of the amplifier at a certain normalized detuning x to the gain at $x = 0$. This ratio is expressed by the *relative transfer function* [1]) a of the amplifier

$$a = \left|\frac{\delta_0}{\delta}\right|, \tag{2.5.3}$$

in which δ_0 is the magnitude of δ at $x = 0$.

The amplitude response can be determined from the parabola which forms a geometrical representation of the frequency-dependent part of δ.

[1]) By this term is understood the magnitude of the transfer function of the amplifier, relative to that at $x = 0$.

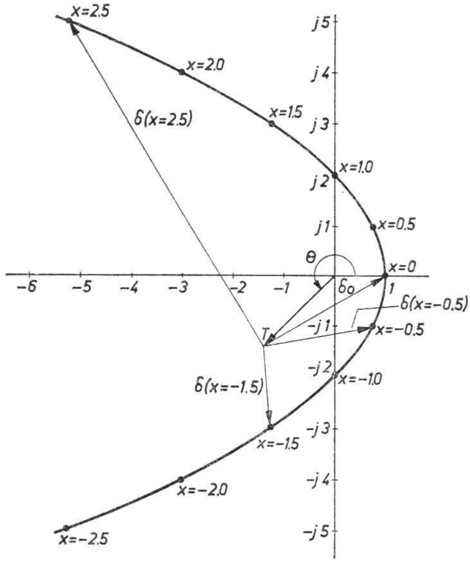


Fig. 2.15. The amplitude response of the single-stage amplifier is proportional to the reciprocal of the distance between the top of the vector T and the parabola (valid for the case of tuning method A).

In Fig. 2.15 such a parabola is shown for a single-stage amplifier tuned according to method A with $x_1 = x_2 = x$. The amplitude response of the amplifier is proportional to the reciprocal of the distance between the top of the vector T and the parabola. In Fig. 2.15 the amplitude responses for $x = 0$ (which equals δ_0) and $x = 2.5$, $x = -0.5$ and $x = -1.5$ are indicated. It appears that the amplitude response curve as a function of the normalized detuning x will be asymmetrical because the extremity of T is not located on the symmetry axis of the parabola.

The parabolic presentation is also very useful to illustrate the influences of the various methods of tuning on the amplitude response of the amplifier. In Fig. 2.16 parabolas applicable to tuning methods A (curve I), B (curve II) and C (curve III) for an amplifier for which $T = 2$, $\Theta = 225°$ and $x_2 = 2x_1$ are given. These parabolas are based on the relation

$$\{1 + j(x_1 + p_1x_1' + p_2x_1'')\}\{1 + j(x_2 + p_1x_2' + p_2x_2'')\}, \qquad (2.5.4)$$

which is the frequency-dependent part of Eq. (2.3.22). Inspection of Fig. 2.16 shows that curve III gives a less asymmetrical amplitude response curve than curves I and II because the top of the vector T lies closest to the symme-

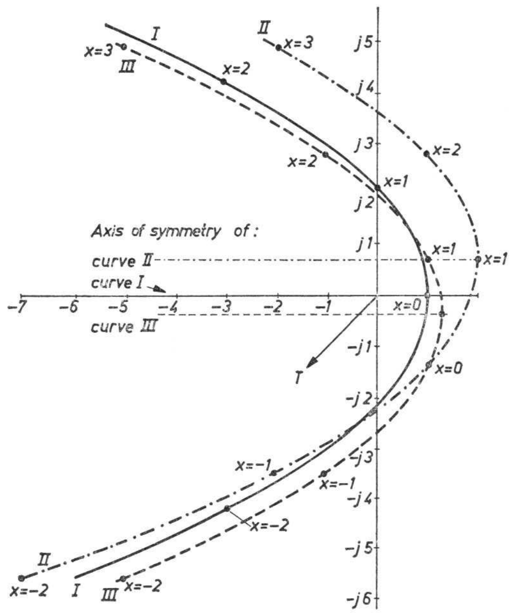


Fig. 2.16. Parabolas for single-stage amplifier with $T = 2$, $\Theta = 225°$ and $x_2 = 2x_1$. Curves I, II and III are valid for tuning methods A, B and C respectively. The axis of symmetry of curve III lies closest to the top of the vector T. Hence the amplitude response curve obtained with tuning method C has the less asymmetrical from, see also Fig. 2.17.

try axis of the first parabola. The amplitude response curve can be calculated from Eqs. (2.3.22), (2.4.11) and (2.5.3) from which:

$$a = |\delta_0/\delta| = \left| \frac{\begin{vmatrix} 1 + \mathrm{j}(p_1 x_1' + p_2 x_1'') & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}(p_1 x_2' + p_2 x_2'') \end{vmatrix}}{\begin{vmatrix} 1 + \mathrm{j}(x_1 + p_1 x_1' + p_2 x_1'') & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}(x_2 + p_1 x_2' + p_2 x_2'') \end{vmatrix}} \right| . \quad (2.5.5)$$

According to Table 2.1 the relative amplitude response curve will, in the case of tuning method A, assume the form:

$$\left| \frac{\delta_0}{\delta} \right| = \left| \frac{1 - T \exp(\mathrm{j}\Theta)}{(1 + \mathrm{j}x_1)(1 + \mathrm{j}x_2) - T \cdot \exp(\mathrm{j}\Theta)} \right| , \quad (2.5.6)$$

and in the case of tuning method B:

$$\left| \frac{\delta_0}{\delta} \right| = \left| \frac{1 - T \cos \Theta}{\{(1 + \mathrm{j}(x_1 + T \sin \Theta)\}(1 + \mathrm{j}x_2) - T \exp(\mathrm{j}\Theta)} \right| , \quad (2.5.7)$$

whilst in the case of tuning method C:

$$\left|\frac{\delta_0}{\delta}\right| = \left|\frac{1 - T\cos\Theta}{(1 + jx_1)\{(1 + j(x_2 + T\sin\Theta)\} - T\exp(j\Theta)}\right|. \qquad (2.5.8)$$

Calculated amplitude response curves of a single-stage amplifier have been plotted in Fig. 2.17 for the three different methods of tuning. It was assumed for this case that the regeneration coefficient $T = 2$ and the regeneration phase angle $\Theta = 225°$; the quality factor of the output tuned circuit was assumed to be twice the value of that of the input tuned circuit. In so doing, the different results of tuning methods B and C clearly stand out. If x_1 had been chosen equal to x_2 the curves representing the results of tuning methods B and C would coincide.

In this graph the normalized amplitude response curve for an amplifier without feedback ($T = 0$) has also been drawn. This curve is obviously symmetrical.

The curves in Fig. 2.17 are all plotted as functions of $x = \sqrt{x_1 x_2}$.

The various parameters of the four curves plotted in Fig. 2.14 are tabulated below.

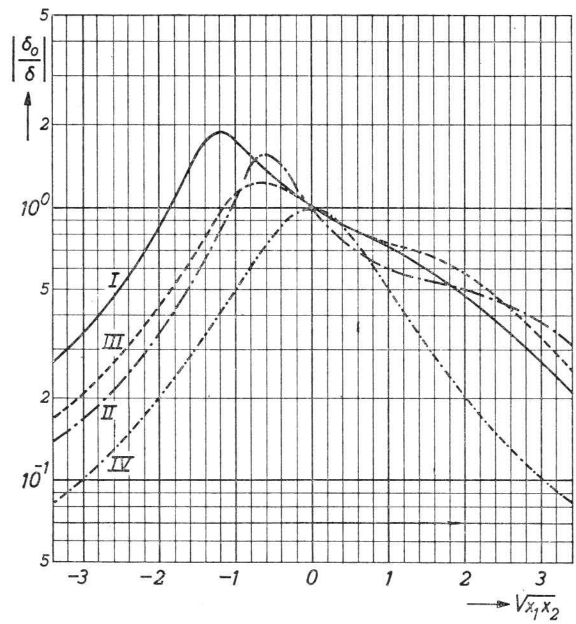


Fig. 2.17. Normalized amplitude response curves for a single-stage amplifier with two single-tuned circuits having the following data: $\Theta = 225°$, $T = 2$, and $Q_2 = 2Q_1$. Curves I, II and III show the results obtained with tuning methods A, B and C respectively. Curve IV represents the curve applicable to $T = 0$.

curve	tuning method	p_1	p_2	transfer function
I	A	0	0	(2.5.6)
II	B	1	0	(2.5.7)
III	C	0	1	(2.5.8)
IV	A, B or C	—	—	—

Comparison of curves I, II and III with curve IV reveals that the presence of feedback in the amplifier has considerable influence on the response curve. Comparison of curves I, II and III also reveals that tuning methods B and C result in less asymmetry of the response curve than tuning method A. This is to be attributed to the more symmetrical location of the extremity of the vector T for tuning methods B and C, see Fig. 2.16.

It is seen that the best results are obtained with tuning method C (curve III) in which the circuit having the smallest quality factor is tuned first. The tuning correction term $T \sin \Theta$ is then applied to the output tuned circuit on which the extra susceptance due to this tuning correction term is half as large as that occurring on the input circuit with tuning method B because

$$Q_2 = 2Q_1 \quad \text{or} \quad 2G_2 = G_1,$$

and hence,

$$G_2 x'' = \frac{1}{2} \cdot \frac{1}{G_1} \cdot \frac{1}{x_1'},$$

see sub-sections 2.3.3 and 2.3.4.

2.5.2.2 Conditions for Symmetrical Amplitude Response Curve

As pointed out in the preceding sub-section the amplitude response curve of the single-stage amplifier for $T \neq 0$ will generally be assymmetrical. For a particular combination of the tuning frequencies and the quality factors of the input and output circuits a symmetrical response curve can be obtained. By putting:

$$Q_2 = aQ_1, \tag{2.5.9}$$

$$\left.\begin{aligned} \beta_1 &= \beta + \frac{b_1}{Q_1}, \\ \beta_2 &= \beta + \frac{b_2}{Q_2}, \end{aligned}\right\} \tag{2.5.10}$$

and

$$\beta = \frac{f}{f_0} - \frac{f_0}{f}, \tag{2.5.11}$$

we obtain for the normalized detunings of the tuned circuits:

$$\left. \begin{aligned} x_1 &= \beta_1 Q_1 = \beta Q_1 + b_1 = x + b_1, \\ x_2 &= \beta_2 Q_2 = \beta\, a Q_1 + b_2 = ax + b_2. \end{aligned} \right\} \tag{2.5.12}$$

Here the quality x is the normalised frequency with respect to the centre frequency f_0. Now, the reduced determinant δ may be written:

$$\delta = \{1 + \mathrm{j}(x + b_1)\}\{1 + \mathrm{j}(ax + b_2)\} - T \exp(\mathrm{j}\Theta). \tag{2.5.13}$$

Then:

$$|\delta|^2 = \{1 - b_1 b_2 - T \cos \Theta - x(ab_1 + b_2) - ax^2\}^2 + \{b_1 + b_2 - T \sin \Theta + (a + 1)x\}^2 . \tag{2.5.14}$$

Now $|\delta|^2$ is a measure for the amplitude response curve of the amplifier which will be symmetrical with respect to $x = 0$ ($f = f_0$) when the terms with x and x^3 vanish from expression (2.5.14). That is when:

$$\{2(1 + a)(b_1 + b_2 - T \sin \Theta) - 2(ab_1 + b_2)(1 - b_1 b_2 - T \cos \Theta)\}x + + 2a(ab_1 + b_2)x^3 = 0 . \tag{2.5.15}$$

A symmetrical response curve is thus obtained for:

$$b_1 = \frac{T \sin \Theta}{1 - a}, \tag{2.5.16}$$

and

$$b_2 = \frac{T \sin \Theta}{1 - \dfrac{1}{a}} . \tag{2.5.17}$$

It follows that a symmetrical response curve is only possible for $a \neq 1$, i.e. for $Q_1 \neq Q_2$. To achieve symmetry the tuned circuits of the amplifier must each be tuned to such a frequency that b_1 and b_2 have values given by Eqs. (2.5.16) and (2.5.17). This can readily be accomplished by means of tuning method A but also tuning methods *B* or *C* lead to the desired result provided the circuit to be tuned first is given the proper value of b and the second circuit is tuned with the signal generator adjusted at f_0. This will be shown for tuning method B.

Firstly the output circuit is tuned to such a frequency that has the value given by Eq. (2.5.17). Then the input circuit is adjusted, according to method B, to such a frequency that $x = 0$, that is, to a frequency f_0.
The tuning correction term x_1' then follows from:

$$I_m \begin{vmatrix} 1 + \mathrm{j}x_1' & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j}\dfrac{aT \sin \Theta}{a - 1} \end{vmatrix} = 0,$$

or

$$x_1' = \frac{T \sin \Theta}{1 - a}. \tag{2.5.18}$$

which equals the value of b_1 given in Eq. (2.5.16).

2.5.2.3 *Influence of the Stability Factor on the Amplitude Response Curve*

As pointed out in sub-section 2.2.4, the value of the regeneration coefficient T of a practical amplifier should be so chosen that it ensures a certain stability factor. This stability factor should be sufficiently large to accommodate possible changes in environmental conditions and spreads in transistor parameters. However, the response curve of the amplifier also imposes certain requirements on the minimum value of the stability factor.

In Fig. 2.18 a set of amplitude response curves of the single-stage amplifier (valid in the case of tuning method A being applied) has been plotted for several values of T. For the amplifier under consideration $T_g = 5.07$ so that instability occurs at $T > 5.07$. This is evidenced by the curves for $T = 6$. The curve for $T = 0$ represents the idealized case in which no feedback is present. The curves for $T = 1$, 2 and 4 show an increasing departure from the symmetrical curve for $T = 0$.

Now the designer of an amplifier must base his design on such a value of T that the requirements regarding the symmetry of the response curve are fulfilled. The value of T that fulfils these requirements best can most easily be ascertained by means of a family of curves for various values of T, as, for example, that shown in Fig. 2.18.

Since the asymmetry of the response curves increases with increasing value of T, that is to say with decreasing value of the stability factor, it is possible to define a lower limit of s at which the asymmetry in a particular case is still acceptable. It should, however, be kept in mind that the same value of s will generally give a different amount of asymmetry in different amplifier arrangements.

This point will be discussed later.

It may thus be concluded that, although the value of s gives a rough

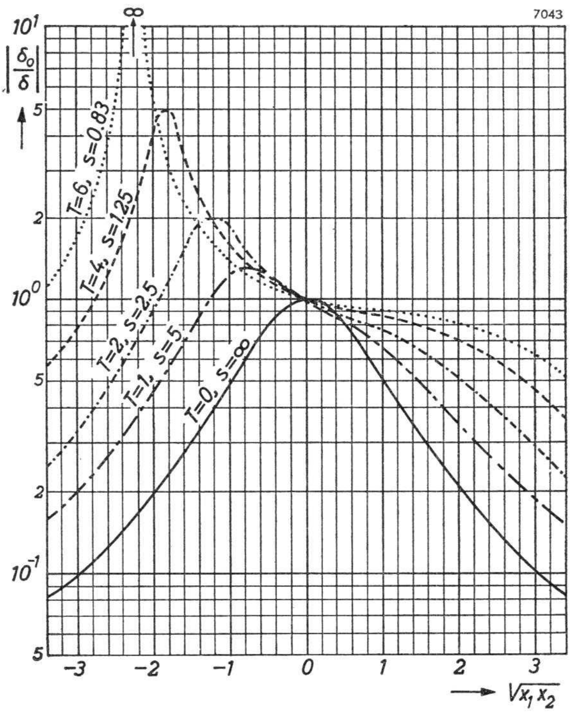


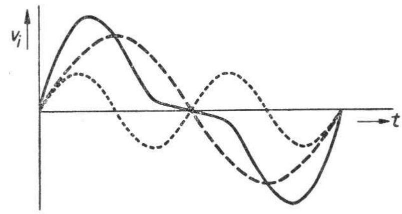
Fig. 2.18. Amplitude response curves of an amplifier with the value of T as parameter. At $T = T_g = 5.07$ this particular amplifier ($\Theta = 225°$, $x_2 = 2x_1$) becomes unstable. This graph clearly shows that the stability factor $s = T_g/T$ has great influence on the asymmetry of the curves.

indication of the amount of asymmetry that may be expected in a certain amplifier design, the acceptability or otherwise of this asymmetry can be judged only by plotting the response curve of the amplifier for the chosen value of T. The latter method will therefore be used in this book, especially for the more complex amplifiers.

2.5.3 PHASE RESPONSE CURVE

2.5.3.1 General

In radio receivers for amplitude modulated signals it is important that every frequency component of the audio signal is amplified to the same extent, but phase shifts in the components of different frequencies of the signal have little influence on the quality of reproduction. This is because the human ear is sensitive to the amplitudes of the various frequency components which, together, constitute the signal, but not to the phase of these components.



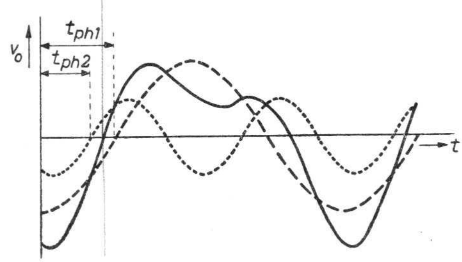


Fig. 2.19. Example of phase distortion.
a) Input signal of a network. The signal is composed of two components of angular frequency ω and 2ω, as shown by the broken lines.
b) Output signal of the same network. The phase delay t_{ph1} of the component of angular frequency ω is assumed to exceed the phase delay t_{ph2} of the component of angular frequency 2ω, as a result of which considerable (phase) distortion occurs in the composite waveform.

In radio receivers for frequency modulated signals, phase shifts occurring in the signal before detection are of importance because this detector is principally a phase-sensitive device. Phase shifts may therefore lead to a distorted output of the detector.

Furthermore, in television receivers phase shifts occurring in the video circuits have an important effect on picture quality. This is because the human eye is sensitive to the instantaneous amplitude of the complete signal. This means that stringent requirements are imposed on both the amplitude and the phase responses of the amplifiers and, in fact, of the whole network through which the video signal is transmitted. The video signal, which contains pulse-shaped intelligence, can be resolved by means of Fourier analysis into a large number of sinusoidal components. For a faithful transmission of the video signal through a network it is therefore essential that neither the relative amplitudes nor the relative phases of these components are distorted by the network.

This is illustrated in Fig. 2.19. The upper oscillogram represents the input signal of a network. This signal can be resolved into two components of angular frequencies ω and 2ω. The lower oscillogram represents the output signal. The two components which constitute the signal are shifted by different amounts along the time axis. It is clearly seen that due to these differing time delays serious distortion is introduced.

It is obvious that no phase distortion will occur in the network if the time delay is independent of the frequency. The delay involved in the phase distortion considerations may either be normal phase delay or modulation phase delay, depending on whether the signal passes through the network directly or in the form of a modulated carrier.

It is the purpose of the following sub-sections to illustrate that the envelope delay characteristic of a network is a very good measure of the phase distortion occurring in that network.

2.5.3.2 Phase Delay

If a sinusoidal signal of angular frequency ω is applied to the input terminals of a network the phase of the output will be delayed by a certain time t_{ph}, see Fig. 2.20. The output voltage of the amplifier thus lags with respect to the input voltage by an angle:

$$\varphi = - t_{ph} \cdot \omega. \tag{2.5.19}$$

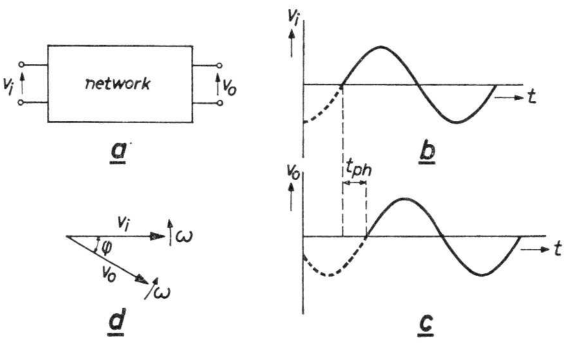


Fig. 2.20. Phase delay of a network (*a*) to which a sinusoidal input signal v_i is applied (*b*). The output signal v_0 (see *c*) is delayed in phase by the phase delay time t_{ph}, which corresponds to a lagging phase angle $\varphi = \omega t_{ph}$ (see *d*).

Fig. 2.21 shows the phase characteristic and the phase delay characteristic of a network in which phase distortion occurs. Since the phase characteristic is not a linear function of the frequency, the phase delay characteristic is not a horizontal line. In an amplifier which introduces no phase distortion the phase delay is independent of the frequency, as shown in Fig. 2.22*a*.

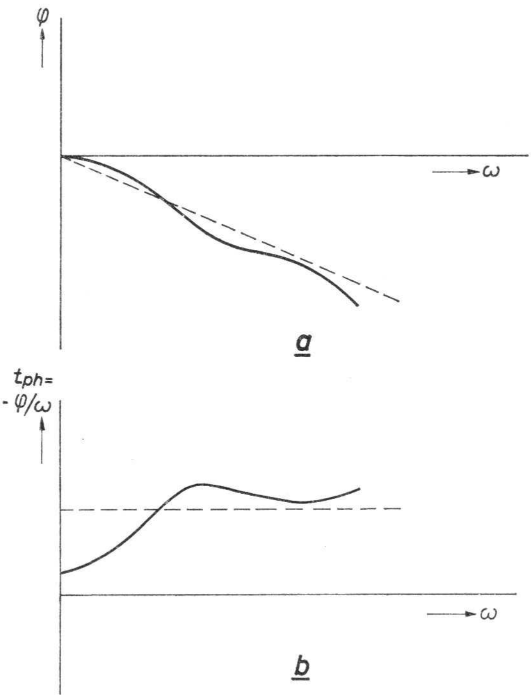


Fig. 2.21. (*a*) Phase characteristic of a network introducing phase distortion (fully drawn curve) and that of a network introducing no phase distortion (broken line). (*b*) Phase delay characteristics derived from the phase characteristics drawn in (*a*).

The phase delay characteristic is then a straight line passing through the origin or a straight line with a zero frequency intercept equal to an integral multiple of π radians, as shown in Fig. 2.22*b*. This zero frequency intercept is caused by the phase reversals occurring in the signal when the output signal current of the transistors or tubes in the network is converted into a voltage across their load; it does not introduce any phase distortion because the phase reversals do not require any time.

The phase shifts which occur in the various frequency components of the signal during their transmission through the network may be quite considerable. It is therefore difficult to determine small discrepancies of the phase characteristic from the linear phase versus frequency relation required for an undistorted transmission of the signal. A much better method of judging the phase distortion of the network therefore consists in determining its envelope delay, which will be dealt with in the following sub-sections.

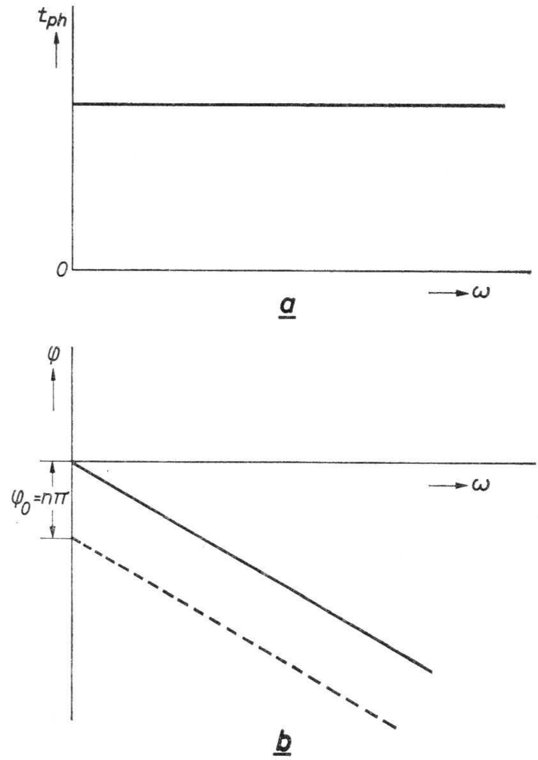


Fig. 2.22. At a constant phase delay time t_{ph} the phase angle φ is proportional to the frequency (see *a*). The phase characteristic then assumes the form of a straight line through the origin or a line parallel to it, intersecting the vertical axis at $-\varphi = n\pi$ (see *b*). The constant phase angle φ_0 is the result of the frequency-independent phase reversals occurring in the network under consideration.

2.5.3.3 Envelope Delay

It follows from the above that a network introduces no phase distortion when its phase characteristic has a constant slope. An obvious method of judging the phase response of a network therefore consists in determining the slope of the phase characteristic as a function of the frequency. Small discrepancies from the linear characteristic of phase versus frequency result in large differences of slope.

This phase slope, which is usually referred to as the envelope delay or group delay of the network, is defined as:

$$t_e = -\,\mathrm{d}\varphi/\mathrm{d}\omega. \tag{2.5.20}$$

The difference between the phase delay and the envelope delay is shown

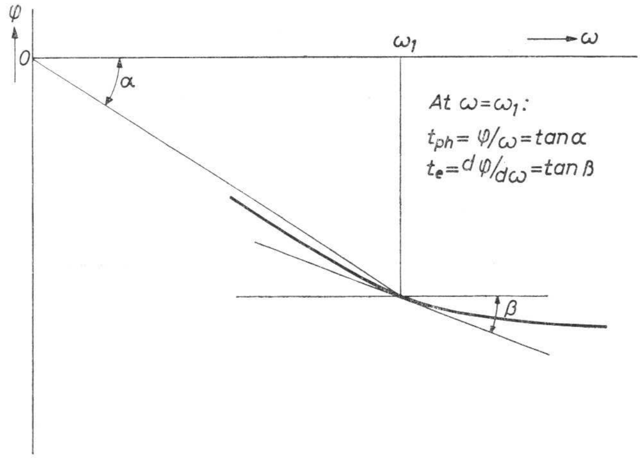


Fig. 2.23. Geometrical significance of the phase and envelope delays. At an angular frequency β the phase delay is determined by tan α and the envelope delay by tan β.

by Fig. 2.23. The phase delay of the network is determined by tan α, whereas the envelope delay is equal to tan β.

To illustrate the meaning of the term envelope delay, it will be assumed that the input signal v_i of the network consists of a carrier ω_0, modulated in amplitude by a signal of angular frequency ω_m. Then:

$$v_i = \hat{V}_i(1 + m \cos \omega_m t) \cdot \cos \omega_0 t, \tag{2.5.21}$$

in which m denotes the modulation depth.

If $\omega_m \ll \omega_0$ the phase characteristic of the network in the range of $(\omega_0 - \omega_m)$ to $(\omega_0 + \omega_m)$ may be considered as being linear, see Fig. 2.24. Therefore, if the network causes a phase lag equal to $-\varphi$ for the carrier frequency and phase lags equal to $-(\varphi + \Delta\varphi)$ and $-(\varphi - \Delta\varphi)$ for the upper and lower side bands respectively, the output signal will be:

$$v_1 = \hat{V}_0 \left[\cos(\omega_0 t - \varphi) + \frac{m}{2} \cos\{(\omega_0 - \omega_m)t - (\varphi - \Delta\varphi)\} + \right.$$

$$\left. + \frac{m}{2} \cos\{(\omega_0 + \omega_m)t - (\varphi + \Delta\varphi)\} \right], \tag{2.5.22}$$

$$= \hat{V}_0\{1 + m \cos(\omega_m t - \Delta\varphi)\} \cos(\omega_0 t - \varphi), \tag{2.5.23}$$

whence: $$v_0 = \hat{V}_0\{1 + m \cos \omega_m(t - \Delta\varphi/\omega_m)\} \cos \omega_0(t - t_{ph}). \tag{2.5.24}$$

This equation shows that the carrier is subject to a phase delay $t_{ph} = -\varphi/\omega_0$,

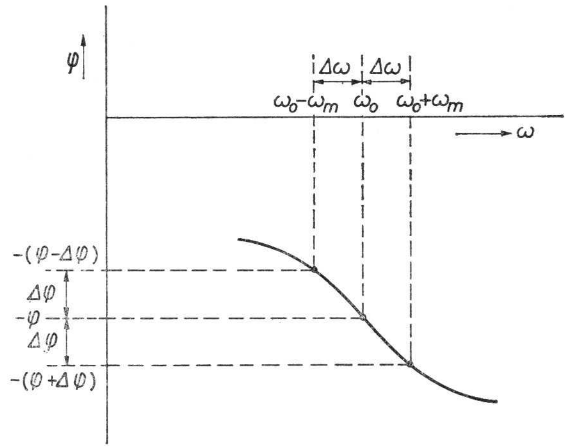


Fig. 2.24. Phase characteristic of a network. Provided $\Delta\omega \ll \omega_0$, the phase characteristic may be considered to be linear over the range from $\omega_0 - \omega_m$ to $\omega_0 + \omega_m$.

whilst the modulation signal is delayed in phase by $\Delta\varphi/\omega_m$; this means that the envelope of the modulated carrier is delayed by a time $\Delta\varphi/\omega_m = \Delta\varphi/\Delta\omega$. Now, according to the definition of envelope delay:

$$t_e = \frac{\mathrm{d}\varphi}{\mathrm{d}\omega} = \lim_{\Delta\omega \to 0} \frac{\Delta\varphi}{\Delta\omega}. \tag{2.5.25}$$

The phase diagrams of the input and output signals according to Eqs. (2.5.21) and (2.5.23) for the point T of the envelope curve of the input signal have been plotted in Fig. 2.25. These phase diagrams, together with the modulated carriers, also illustrate the meaning of envelope delay; this may thus be interpreted physically as the time required for a point T stituated on a sinusoidal envelope curve of a modulated carrier to travel through the network.

2.5.3.4 Relation between Phase Delay and Envelope Delay

It has been shown that the phase delay of a network is given by:

$$t_{ph} = \frac{\varphi}{\omega}, \tag{2.5.26}$$

whence:

$$\varphi = t_{ph}\omega. \tag{2.5.27}$$

Furthermore, the envelope delay of the network was defined by:

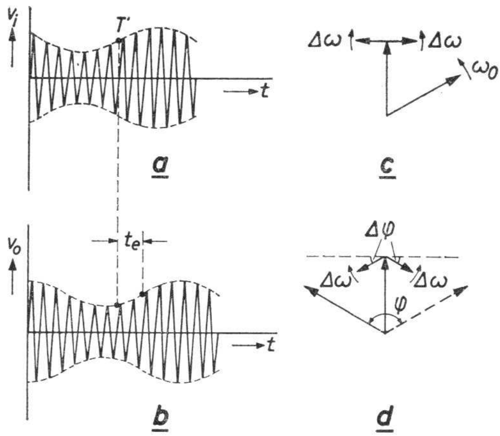


Fig. 2.25. The envelope of a modulated signal (a) fed to a network undergoes a delay in this network equal to t_e, as shown in b, representing the output signal. The phase diagrams of the signals corresponding to point T in a and b are shown in c and d respectively. The sideband phasors are both delayed in phase by an angle $\Delta\varphi = t_e\, \Delta\omega$.

$$t_e = \frac{\mathrm{d}\varphi}{\mathrm{d}\omega}, \tag{2.5.28}$$

or

$$t_e = \frac{\mathrm{d}}{\mathrm{d}\omega} \cdot (t_{ph} \cdot \omega),$$

whence:

$$t_e = t_{ph} + \omega \cdot \frac{\mathrm{d}t_{ph}}{\mathrm{d}\omega}. \tag{2.5.29}$$

Comparison of Eqs. (2.5.26) and (2.5.29) reveals that the envelope delay differs from the phase delay of the network only by the term which accounts for the speed with which the phase delay varies with the frequency. This illustrates once again why the envelope delay is a more accurate measure of the phase response of a network than the phase delay.

It is true that the phase response of a network can conveniently be expressed in terms of envelope delay, but to judge the performance of the amplifier it is important to know the phase response itself. Now, the envelope delay is the derivative of the phase response, which implies that the zero frequency intercept of the phase/frequency characteristic does not occur in the envelope delay curve. This zero frequency intercept should be zero or an integral multiple of π radians to ensure faithful transmission of a signal through the network. Therefore, if the phase characteristic is specified only in terms of envelope delay the assumption is tacitly made that the phase intercept distortion for the range of significant frequencies is either zero or negligible. For a signal passing through the network as a modulation on a carrier, there is, however, no need for the phase characteristic to fulfil this phase intercept requirement. In fact, only the phase characteristic over the band occupied by the modulated signal is then of importance.

For the bandpass amplifiers under consideration, therefore, the envelope delay fully characterizes that part of the phase response of the amplifier which is of interest. For low-pass amplifiers, for example, this would not be the case.

Hitherto the term "envelope delay" has been used for $\mathrm{d}\varphi/\mathrm{d}\omega$. This term suggests that the signals under consideration are modulated in amplitude, and although it has a main significance in this field, the definition $t_e = \mathrm{d}\varphi/\mathrm{d}\omega$ refers only to the slope of the phase/frequency characteristic. The envelope delay is therefore very frequently referred to as "group delay". This term indeed seems to be more appropriate because group delay — physically to be interpreted as the time delay of a small *group* $\Delta\omega$ of frequencies situated around ω_0 in passing through a network — is a more general term which refers neither to modulated nor to unmodulated signals. The present treatise, however, is confined to bandpass amplifiers in which the transmitted signals are normally modulated, so that there is no reason why the term envelope delay should not be used in the context of this book.

2.5.3.5 Envelope Delay as a Function of the Normalized Detuning

The envelope delay $t_e = \mathrm{d}\varphi/\mathrm{d}\omega$ of a network may also be written:

$$t_e = \frac{\mathrm{d}\varphi}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}\omega}, \tag{2.5.30}$$

in which x denotes the normalized detuning. Now $\mathrm{d}x/\mathrm{d}\omega$ is a constant equal to:

$$\frac{\mathrm{d}}{\mathrm{d}\omega} \cdot \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) Q = \left(\frac{1}{\omega_0} + \frac{\omega_0}{\omega^2}\right) Q,$$

whence:

$$\frac{\mathrm{d}x}{\mathrm{d}\omega} = \frac{Q}{\omega_0} \left\{ 1 + \left(\frac{\omega_0}{\omega}\right)^2 \right\}. \tag{2.5.31}$$

The envelope delay is then:

$$t_e = \frac{Q}{\omega_0}\left\{1 + \left(\frac{\omega_0}{\omega}\right)^2\right\}\frac{d\varphi}{dx}. \tag{2.5.32}$$

Provided $\omega_0/\omega \simeq 1$, this expression reduces to:

$$t_e = \frac{2Q}{\omega_0}\cdot\frac{d\varphi}{dx}. \tag{2.5.33}$$

The envelope delay of a network can thus be ascertained by determining $d\varphi/dx$, which is in turn given by the transfer function of the network. In order to investigate the envelope delay of a bandpass amplifier, it is thus sufficient to consider the differential quotient $d\varphi/dx$, the factor

$$\frac{Q}{\omega_0}\left\{1 + \left(\frac{\omega_0}{\omega}\right)^2\right\} \simeq \frac{2Q}{\omega_0}$$

merely influencing the envelope delay as a scale factor.

For a single-tuned circuit, for example, the phase angle equals:

$$\varphi = \tan^{-1}(-x). \tag{2.5.34}$$

Hence:

$$\frac{d\varphi}{dx} = \frac{1}{1 + x^2}, \tag{2.5.35}$$

and

$$t_e = \frac{2Q}{\omega_0}\cdot\frac{1}{1 + x^2}. \tag{2.5.36}$$

For amplifiers or networks, the transimpedance function of which has a complex character, $d\varphi/dx$ would become even more complex.
This differential quotient can then be approximated by $\Delta\varphi/\Delta x$, in which $\Delta\varphi$ is derived from the phase versus x characteristic. For a given value of x:

$$\varphi = \tan^{-1}[I_m\{Z_t(x)\}/R_e\{Z_t(x)\}]. \tag{2.5.37}$$

Provided φ is determined with sufficient accuracy and the intervals Δx are chosen small enough, $\Delta\varphi/\Delta x$ will very closely approximate $d\varphi/dx$.

Assuming this to be the case, and putting:

$$\frac{\Delta\varphi}{\Delta x} = \tau_e, \tag{2.5.38}$$

Eq. (2.5.20) becomes:

$$t_e = \tau_e \cdot \frac{dx}{d\omega}, \tag{2.5.39}$$

or:

$$t_e = \tau_e \cdot \frac{2Q}{\omega_0}. \tag{2.5.40}$$

It is thus seen that a very simple relation exists between the quantity τ_e (expressed in terms of radians) and the actual envelope delay t_e of the amplifier. Now τ_e can be evaluated as a function of the normalized detuning x by means of Eqs. (2.5.37) and (2.5.38), so that it is possible to plot curves which represent the envelope delay of the amplifier, except for a scale factor $dx/d\omega$. These curves can then be used universally for various values of ω, ω_0 and Q.

2.5.3.6 Envelope Delay of the Single-Stage Amplifier

The quantity τ_e can now be calculated according to the method outlined above for the single-stage amplifier with two single-tuned circuits, it being assumed that tuning is achieved by method A, B or C. The parameters of the amplifier are taken to be identical to those mentioned in sub-section 2.5.2.1 (Fig. 2.14).

Fig. 2.26 shows the results thus obtained, curves I, II, and III being applicable to tuning methods A, B and C respectively. For the sake of comparison

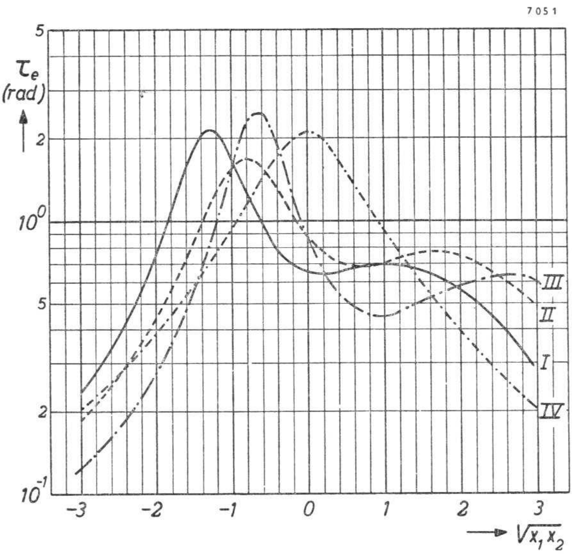


Fig. 2.26. Curves representing the envelope delay, except for a scale factor, of a single-stage amplifier with two single-tuned circuits ($T = 2$, $\Theta = 225°$, $x_2 = 2x_1$). The curves show the great influence of the method of tuning on the trend of these curves (curves I, II and III apply to tuning methods A, B and C respectively, curve IV to $T = 0$.)

the curve for $T = 0$ (curve IV) has also been plotted. It is seen that, so far as the envelope delay is concerned, tuning method C gives the best results for this amplifier (see curve III). As previously shown (sub-section 2.5.2.1) this also applies to the amplitude response curve.

BIBLIOGRAPHY

[2.1] G. S. BAHRS, *Stable Amplifiers employing Potentially Unstable Elements*, 1957, I.R.E Wescon Conv. Record, **1**, Pt. 2, p. 185-189.

[2.2] A. R. BOOTHROYD, *The Transistor as an Active Twoport Network*, Scientia Electrica, Band VII, 1961, Heft 1, p. 3-15.

[2.3] W. BUSSCHER, *Transfer Properties of a Network consisting of two Resonant Circuits coupled by a Fourpole*, Electronic Appl. Bull., **16**, p. 81-101, 1955.

[2.4] C. LE CAN, *High-Frequency Selective Fourpole with Complex Feedback*, parts I and II, Unpublished reports of Philips Semiconductor Electrical Development Laboratory, Nijmegen (Netherlands).

[2.5] D. D. HOLMES and T. D. STANLEY, *Stability Considerations in Transistor Intermediate Frequency Amplifiers*, Transistor I, R.C.A. Laboratories, p. 403-431,1956.

[2.6] E. KETTEL, *Hochfrequenzverstärkung mit Transistoren*, Telefunken Z., **106** p. 235-251, 1954.

[2.7] J. G. LINVILL and J. F. GIBBONS, *Transistors and Active Circuits*, McGraw-Hill Book Comp., Inc., New York, 1961, Chapter 9.

[2.8] S. J. MASON, *Power Gain in Feedback Amplifiers*, I.R.E. Trans. Circuit Theory, **CT-1**, p. 20-25, 1954.

[2.9] D. F. PAGE and A. R. BOOTHROYD, *Instability in Two-port Active Networks*, I.R.E. Trans. on Circuit Theory, **CT-5**, p. 133-139, 1958.

[2.10] R. PAUL, *Zur Stabilität von Transistorbestückten Schmalbandverstärkerstufen*, Nachrichtent., **11**, p. 295-302, 1961.

[2.11] G. PHILIPS-JONES, *Stability Conditions in Tuned Common-emitter Transistor Amplifiers*, Trans. I.E.E., Part C., p. 505-517, Jan. 1960.

[2.12] J. M. ROLLETT, *Stability and Power-Gain Invariants of Linear Twoports*, I.R.E. Trans. Circuit Theory, **CT-8**, p. 29-32, 1962.

[2.13] A. P. STERN, *Considerations on the Stability of Active Elements and Applications to Transistors*, 1956 I.R.E. Conv. Record, **Pt** 2, p. 46-52.

[2.14] A. P. STERN, *Stability and Power Gain of Tuned Transistor Amplifiers*, Proc. I.R.E., **45**, p. 335-343, 1957.

[2.15] S. VENKATESWARAN and A. R. BOOTHROYD, *Power Gain and Bandwidth of Tuned Transistor Amplifier Stages*, Trans. I.E.E., Part C, p. 518-529, 1960.

[2.16] S. VENKATESWARAN, *An Invariant Stability Factor and its Physical Significance*, I.E.E. Monograph No. 468E, September 1961.

[2.17] W. WOLFENDALE and L. E. JANSSON, *The Junction Transistor and its Applications*, Heywood 1958, in particular pp. 101-120 and 161-211.

CHAPTER 3

NEUTRALIZATION

As was seen from the analysis of the single-stage amplifier in Chapter 2, the internal feedback of the transistor has a large influence upon the performance of the amplifier. In the case of a potentially unstable transistor, its feedback may lead to instability of the amplifier unless special measures are taken. If the internal feedback does not result in instability, it may have a detrimental effect upon the amplitude and phase response of the amplifier. This may even occur when the transistor is inherently stable.

In many cases it will therefore be desirable to eliminate this feedback. This may be achieved by applying a technique referred to as neutralization.

Considering the transistor as a four-terminal network, this amounts to eliminating the reverse transfer parameter. This process is known as unilateralization.

By definition, a four-terminal network is unilateral if an excitation applied to one of its pairs of terminals produces a response at the second pair, whereas an excitation applied to the second pair does not produce a response at the first pair, or vice-versa.

Worded differently: in a unilateral network, no "backward transmission" is possible. This implies that only a perfectly neutralized transistor may be said to be unilateral.

Because in practical amplifiers the neutralization will often not be perfect, preference is given to the term "neutralization" [1]) to describe the technique to reduce or to cancel the internal feedback of the transistor.

3.1 Principle of Neutralization

If the relation between input and output currents and voltages of the equivalent transistor four-terminal network are expressed in terms of either the Y, Z, H or K-matrices, we can generally write:

---

1) Unilateralization is always a kind of neutralization but the reverse need not necessarily be the case.

$$\begin{Vmatrix} \alpha_1 \\ \alpha_2 \end{Vmatrix} = \begin{Vmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{Vmatrix} \cdot \begin{Vmatrix} \beta_1 \\ \beta_2 \end{Vmatrix}, \tag{3.1.1}$$

in which the symbols α and β depend on the matrix environment chosen.

Furthermore, we assume that another network in the same parameter system may be described by:

$$\begin{Vmatrix} \alpha_1' \\ \alpha_2' \end{Vmatrix} = \begin{Vmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{Vmatrix} \cdot \begin{Vmatrix} \beta_1' \\ \beta_2' \end{Vmatrix}. \tag{3.1.2}$$

If these networks are interconnected in such a way that $\beta_1' = \beta_1$ and $\beta_2' = \beta_2$, without disturbing the relationships between currents and voltages in the original networks, see Appendix I, the combined network can be described by:

$$\begin{Vmatrix} \alpha_1 + \alpha_1' \\ \alpha_2 + \alpha_2' \end{Vmatrix} = \begin{Vmatrix} \gamma_{11} + \Gamma_{11} & \gamma_{12} + \Gamma_{12} \\ \gamma_{21} + \Gamma_{21} & \gamma_{22} + \Gamma_{22} \end{Vmatrix} \cdot \begin{Vmatrix} \beta_1 \\ \beta_2 \end{Vmatrix}. \tag{3.1.3}$$

It is now said that the combined network is unilateral or perfectly neutralized if:

$$\gamma_{12} + \Gamma_{12} = 0. \tag{3.1.4}$$

This means that a transistor can be perfectly neutralized by correctly connecting to the transistor a second network with properties such that Eq. (3.1.4) is satisfied.

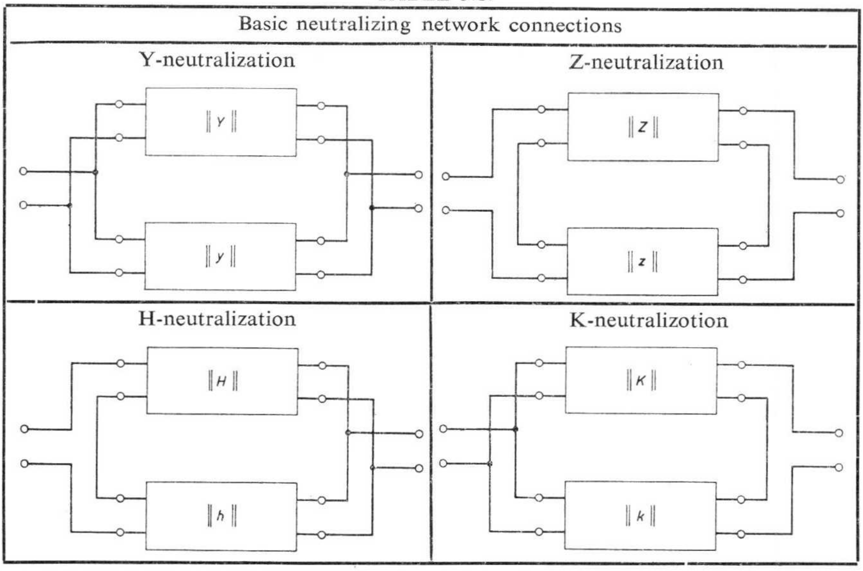
3.2 Basic Neutralizing Network Connections

There are four basic methods of connecting a neutralizing four-terminal network to the transistor. These methods differ in the way in which the input and output terminal pairs of both networks are interconnected. This may be either in

parallel — parallel,
series — series,
series — parallel,
or parallel — series.

To satisfy the condition imposed in the preceding section on the independent variable (β) of the matrix equations, it is required that both networks

TABLE 3.1.



are expressed in the correct parameter system depending on the method of interconnection. According to Appendix I, this is the Y, Z, H or K-parameter system respectively.

In Table 3.1 the four methods of interconnection are shown. These methods will further be referred to as

Y-neutralization, Z-neutralization, H-neutralization,
and K-neutralization.

3.3 *Y*-Neutralization

3.3.1 GENERAL

For a Y-type neutralization, both transistor and neutralizing networks are connected in parallel at the respective input- and output terminals. If the transistor parameters are indicated by lower case y's and those of the neutralizing network by capital Y's, for perfect neutralization (see Eq. (3.1.4)):

$$y_{12} + Y_{12} = 0.$$

Since the y_{12} parameter of a transistor suitable for use in I.F. amplifiers lies in the 3[rd] or the 4[th] quadrant, (see Book II, Chapter 2) Y_{12} must be situated in the 1[st] and 2[nd] quadrants to enable Eq. (3.1.4) to be satisfied (see Fig. 3.1). In practice it is required that the neutralizing network should consist of passive elements only. Because of the sign conventions adopted the Y_{12} parameter of such a network always lies in the 2[nd] or 3[rd] quadrant. Therefore

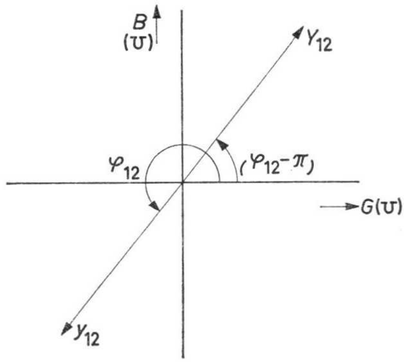


Fig. 3.1. Location of the y_{12} parameter of the transistor and of the Y_{12} parameter of the neutralizing network required to achieve neutralization.

a phase inverting transformer is necessary between the neutralizing network and the transistor in case the latter has its y_{12} parameter situated in the 3rd quadrant. No transformer is required for transistors in which the y_{12} parameter lies in the 4th quadrant.

For further considerations on Y-neutralization we will confine ourselves to transistors having y_{12} parameters in the 3rd quadrant. The complete Y-neutralizing circuit then becomes as shown in Fig. 3.2. The polarity of the phase inverting transformer is indicated by means of dots.

Furthermore, this transformer which is assumed to be ideal, has a transformer ratio of 1 : n. This implies that for perfect neutralization:

$$nY_{12} + y_{12} = 0. \qquad (3.3.1)$$

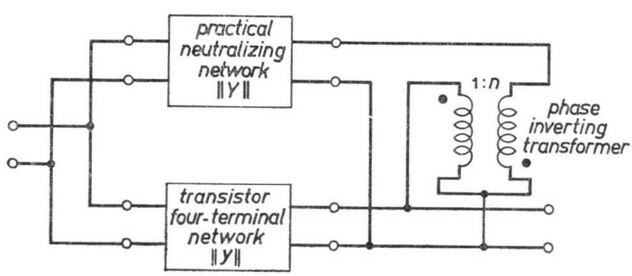


Fig. 3.2. Interconnection of a transistor four-terminal network of which it is assumed that the y_{12} parameter is situated in the 3rd quadrant and of a practical neutralizing network. The phase inverting transformer enables that the neutralizing network consists of passive elements only. The dots indicate the polarity of primary and secondary windings.

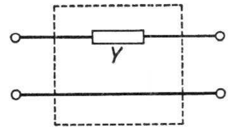


Fig. 3.3. *Y*-neutralizing network.

3.3.2 NEUTRALIZING CIRCUIT

The *Y*-neutralizing network may consist of a single admittance *Y* as shown in Fig. 3.3. It is required that the Y_{12} parameter of this fourpole should be in the 3rd quadrant; so *Y* may consist of either a series or a parallel combination of a capacitance and a resistance. In practice, the series combination is used in most cases because then the capacitor also separates the d.c. circuits at the transistor input- and output terminals.

Including an ideal phase-inverting transformer, the admittance parameters of the neutralizing circuit become (see Fig. 3.4):

$$\left.\begin{aligned} Y_{11} &= Y, & Y_{12} &= nY, \\ Y_{21} &= nY, & Y_{22} &= n^2Y, \end{aligned}\right\} \tag{3.3.1}$$

With the condition for perfect neutralization

$$y_{12} + nY = 0, \tag{3.3.2}$$

the admittance matrix of the combined network becomes:

$$\left\| y \right\| = \left\| \begin{matrix} y_{11} - \dfrac{y_{12}}{n} & 0 \\ y_{21} - y_{12} & y_{22} - ny_{12} \end{matrix} \right\|, \tag{3.3.3}$$

from which we find the maximum unilateralized power gain:

$$\Phi_{uM}' = \frac{|y_{21} - y_{12}|^2}{4\,Re\left(y_{11} - \dfrac{y_{12}}{n}\right) \cdot Re(y_{22} - ny_{12})}\ ^{1)}.$$

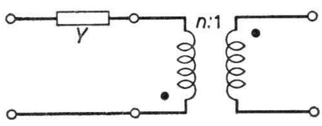


Fig. 3.4. *Y*-neutralizing circuit including ideal transformer.

---

1) The quantity Φ_{uM}' indicates the maximum unilateralized power gain of a transistor when perfectly neutralized by means of a practical network. The quantity Φ_{uM} (without dash) refers to the case of unilateralization by means of an ideal (loss-free) network, see Appendix V.

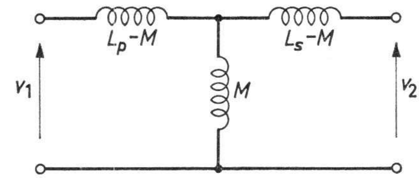


Fig. 3.5. Equivalent circuit diagram of a practical transformer.

For a certain value of n, Φ_{uM}' becomes maximal. This value is found by putting:

$$\frac{\mathrm{d}}{\mathrm{d}n}\,(\Phi_{uM}') = 0\,,$$

and is equal to:

$$n = \sqrt{\frac{g_{22}}{g_{11}}}. \qquad (3.3.4)$$

3.3.3. NON-IDEAL TRANSFORMER

The transformer used in practical neutralizing circuits at high frequencies suffers from various defects which result in a performance different from that of an ideal transformer. These defects, which will be considered separately, are

a. non-unity coupling coefficient,
b. losses, and
c. stray capacitances.

3.3.3.1 Non-Unity Coupling Coefficient

A practical transformer, of which the primary and secondary open circuit inductances are denoted by L_p and L_s and the mutual inductance by M, can be represented by the basic equivalent circuit given in Fig. 3.5.

For this equivalent circuit, the open circuit voltage ratio follows from:

$$\frac{v_2}{v_1} = \frac{M}{L_p} = k\sqrt{\frac{L_s}{L_p}}, \qquad (3.3.5)$$

in which the coupling coefficient k is given by:

$$k = \frac{M}{\sqrt{L_p\,L_s}}. \qquad (3.3.6)$$

By putting:

$$n = \sqrt{\frac{L_p}{L_s}}, \qquad (3.3.7)$$

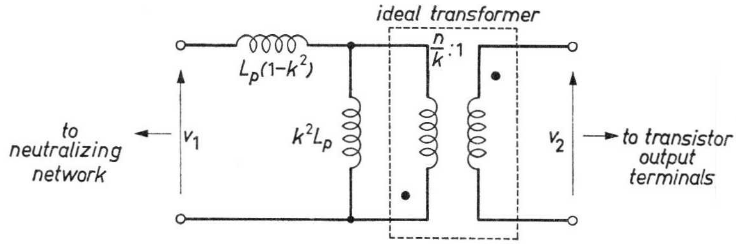


Fig. 3.6. Equivalent circuit of a practical transformer derived from that of Fig. 3.5.

the transformer ratio according to Eq. (3.3.5) becomes:

$$\frac{v_2}{v_1} = \frac{k}{n}. \tag{3.3.8}$$

Furthermore, the open circuit input inductance of this equivalent circuit equals L_p, whereas the short-circuited input inductance equals:

$$L_p - M + \frac{M(L_s - M)}{L_s} = L_p(1 - k^2). \tag{3.3.9}$$

These calculations reveal that another equivalent circuit, equal to that of Fig. 3.5 is as shown in Fig. 3.6. The inductance k^2L_p at the neutralizing network side of the transformer, see Fig. 3.6, may be transformed to the transistor side as shown in Fig. 3.7.

This inductance then becomes, using Eqs. (3.3.5) and (3.3.7):

$$k^2L_p \cdot \frac{n^2}{k^2} = L_s.$$

3.3.3.2 *Influence of Losses and Stray Capacitances*

The losses associated with the transformer merely consist of parallel dampings which can be represented by a single damping g_s across the transistor side. There are also losses associated with the spread-inductance

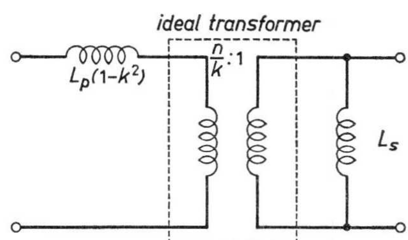


Fig. 3.7. Modification of the equivalent circuit of Fig. 3.6.

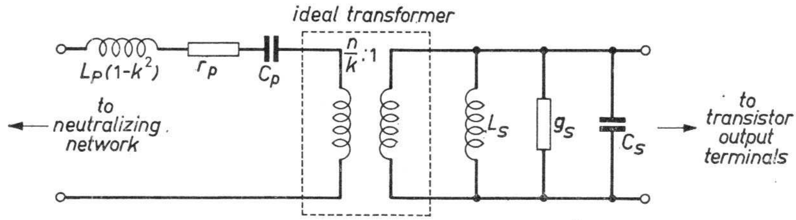


Fig. 3.8. Complete equivalent circuit of the transformer used in the *Y*-neutralizing circuit showing all parasitic effects.

$L_p(1-k^2)$ but these are negligibly small in most cases. For completeness, however, they may be represented by a resistance r_p in series with $L_p\,(1-k^2)$. Furthermore, the stray capacitances of the transformer may be represented by a capacitance C_s on the transistor side of the transformer and a capacitance C_p (the influence of which can be neglected) on the other side.

The complete equivalent circuit of the transformer then becomes as shown in Fig. 3.8.

3.3.4 PRACTICAL *Y*-NEUTRALIZED AMPLIFIER CIRCUIT

In practical amplifier circuits the output tuned circuit is used as the phase inverting transformer.

In Fig. 3.9 a *Y*-neutralized single-stage amplifier with two single-tuned circuits is shown.

It appears that the inductance L_s of Fig. 3.8 forms the tuning inductance of the output circuit whereas the parasitic effects of losses and stray capacitances may be included in its damping and tuning susceptance. This means that only the term $L_p(1-k^2)$ due to the non-unity coupling of the transformer need to be taken into account when designing the neutralizing circuit.

The effective admittance of the neutralizing circuit then becomes with Eq. (3.3.8) and Fig. 3.7:

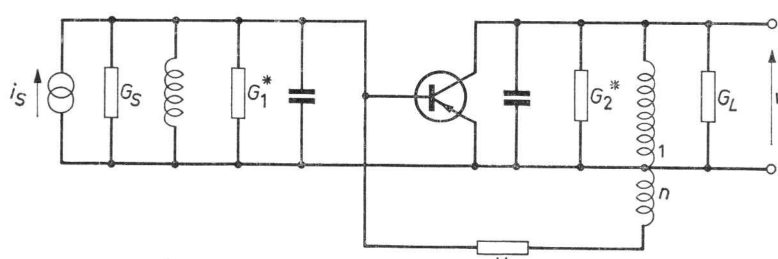


Fig. 3.9. Neutralized one-stage amplifier circuit. The quantities Y' and n' follow from Y and n taking into account the effects of a non-ideal transformer.

$$Y' = \frac{1}{\frac{1}{Y} + j\omega L_2 \cdot n^2(1-k^2)}, \tag{3.3.10}$$

in which L_2 is the tuning inductance of the output circuit of the amplifier. Furthermore the effective transformer ratio equals:

$$n' = \frac{n}{k}. \tag{3.3.11}$$

These values for Y' and n' substituted in Eqs. (3.3.1) to (3.3.4) give for perfect neutralization:

$$y_{12} + \frac{n}{k} Y' = 0, \tag{3.3.12}$$

and for optimum Φ_{uM}':

$$\frac{n}{k} = \sqrt{\frac{g_{22}}{g_{11}}}. \tag{3.3.13}$$

3.4 *H*-Neutralization

3.4.1 GENERAL

As appears from sub-section 3.2, with an *H*-type neutralization the input terminals of transistor and neutralizing networks are connected in series whereas the output terminals are connected in parallel, see Fig. 3.10. The elements contained in the neutralizing network should be arranged such that the interconnection with the transistor, which is in fact a three-terminal device, is permissible, see Appendix I.

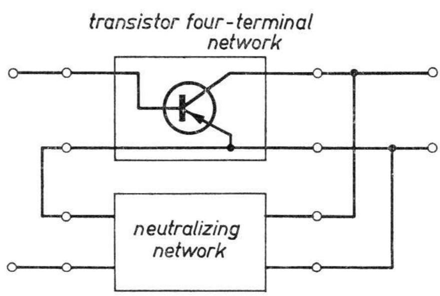


Fig. 3.10. Connection of a neutralizing network to a transistor for *H*-neutralization. Because the transistor is a three-terminal device care must be taken in arranging the elements of the neutralizing network in order that the interconnection of both networks is permissible.

Again capital letters are used to denote the parameters of the neutralizing network and lower case letters to denote those of the transistor.

For perfect neutralization:

$$h_{12} + H_{12} = 0. \tag{3.4.1}$$

3.4.2 NEUTRALIZING NETWORK

The H-neutralizing network consists of an impedance Z and an admittance Y as shown in Fig. 3.11. The elements of Z and Y are arranged such that a permissible connection is obtained when this neutralizing network is connected to the transistor as indicated in Fig. 3.10.

For this fourpole, the H-parameters are:

$$H_{11} = \frac{Z}{YZ+1}, \tag{3.4.2}$$

$$H_{12} = H_{21} = -\frac{YZ}{YZ+1}, \tag{3.4.3}$$

$$H_{22} = \frac{Y}{YZ+1}. \tag{3.4.4}$$

By suitably choosing Y and Z, the H_{12} parameter can be given any required phase angle. Hence, condition (3.4.1) can always be satisfied without the use of a phase inverting transformer as is necessary in the Y-neutralizing system.

For perfect neutralization the H-parameters of the combination of transistor and neutralizing network become, using a determinant notation:

$$\begin{vmatrix} h_{11} + \dfrac{h_{12}}{Y} & 0 \\ h_{21} + h_{12} & h_{22} + \dfrac{h_{12}}{Z} \end{vmatrix} \tag{3.4.5}$$

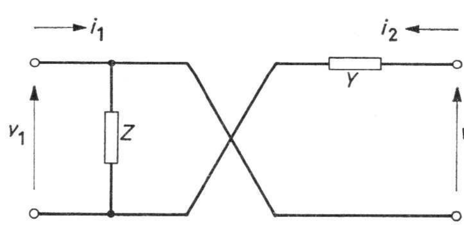


Fig. 3.11. H-neutralizing fourpole with elements Z and Y arranged in such a way that interconnection with the transistor as indicated in Fig. 3.10 is permissible.

According to Eq. (3.4.3), for perfect neutralization:

$$Y\,Z = \frac{h_{12}}{1 - h_{12}}. \tag{3.4.6}$$

By putting

$$h_{12} = |h_{12}|\{\cos(\arg h_{12}) + \mathrm{j}\sin(\arg h_{12})\},$$

Eq. (3.4.6) can be written as:

$$Re(Y\,Z) = \frac{|h_{12}|\cos(\arg h_{12}) - |h_{12}|^2}{1 - 2|h_{12}|\cos(\arg h_{12}) + |h_{12}|^2},$$

and

$$Im(Y\,Z) = \frac{|h_{12}|\sin(\arg h_{12})}{1 - 2|h_{12}|\cos(\arg h_{12}) + |h_{12}|^2}.$$

Because $|h_{12}| \ll 1$, these two equations can be written:

$$\left.\begin{aligned} Re(YZ) &= |h_{12}|\cos\arg(h_{12}) - |h_{12}|^2, \\ Im(YZ) &= |h_{12}|\sin(\arg h_{12}). \end{aligned}\right\} \tag{3.4.7}$$

If furthermore $|h_{12}|\cos(\arg h_{12}) \gg |h_{12}|^2$, Eq. (3.4.7) becomes:

$$\left.\begin{aligned} Re(YZ) &= |h_{12}|\cos(\arg h_{12}), \\ Im(YZ) &= |h_{12}|\sin(\arg h_{12}). \end{aligned}\right\} \tag{3.4.8}$$

There are two practical methods for realization of the *H*-neutralizing circuit, viz:

1. Z is made resistive and the necessary phase shift is obtained by means of Y, or
2. Y is made conductive and the phase shift is obtained by means of Z.

When, however, either Z or Y is made purely real the phase angle arg H_{12} of the neutralizing network becomes situated in the 2nd or 3rd quadrant, which implies that only transistors with the argument of h_{12} in the 1st or 4th quadrant can be neutralized.

3.4.2.1 Z is chosen to be purely resistive

If Z is made resistive and equal to R, the parameters of the combined fourpole become with Eqs. (3.4.5) and (3.4.6):

$$\begin{vmatrix} h_{11} + (1 - h_{12})R & 0 \\ h_{21} + h_{12} & h_{22} + \dfrac{h_{12}}{R} \end{vmatrix} . \tag{3.4.9}$$

The maximum unilateralized power gain of the combined fourpole reaches an optimum value for:

$$R_{\text{opt}} = \sqrt{\frac{Re(h_{11}) \cdot Re(h_{12})}{Re(1 - h_{12}) \cdot Re(h_{22})}}, \tag{3.4.10}$$

or, considering that $h_{12} \ll 1$:

$$R_{\text{opt}} = \sqrt{\frac{Re(h_{11})\ Re(h_{12})}{Re(h_{22})}} . \tag{3.4.11}$$

3.4.2.2 Y is chosen to be purely conductive

If Y is made purely conductive and equal to G the parameters of the combination transistor and neutralizing fourpole become with Eqs. (3.4.5) and (3.4.6):

$$\begin{vmatrix} h_{11} + \dfrac{h_{12}}{G} & 0 \\ h_{21} + h_{12} & h_{22} + (1 - h_{12})G \end{vmatrix} \tag{3.4.12}$$

The optimum in maximum unilateralized power gain is obtained for:

$$G_{\text{opt}} = \sqrt{\frac{Re(h_{22})\ Re(h_{12})}{Re(h_{11}) \cdot Re(1 - h_{12})}}, \tag{3.4.13}$$

or because $h_{12} \ll 1$:

$$G_{\text{opt}} = \sqrt{\frac{Re(h_{22})\ Re(h_{12})}{Re(h_{11})}} . \tag{3.4.14}$$

3.4.3 PRACTICAL *H*-NEUTRALIZED AMPLIFIER CIRCUIT

In Fig. 3.12 the circuit of a single-stage amplifier with an H-neutralization network is given.

It is assumed that the Z term of the neutralizing networks is a resistance R_{N1} whereas the Y term is composed of the series connection of R_{N2} and C_N, a combination of elements which has been chosen quite arbitrarily. The choice of the elements of which Z and Y are to be composed obviously depends on the phase angle of the h_{12} parameter.

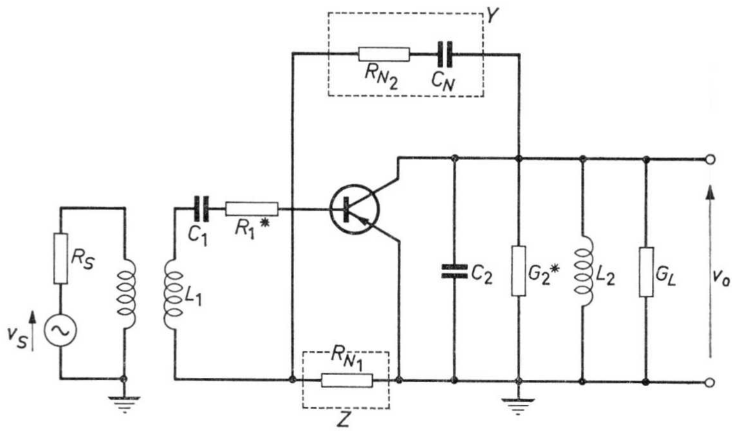


Fig. 3.12. Practical *H*-neutralized single-stage amplifier.

To allow one side of the voltage source to be connected to earth, an isolating transformer is used at the input of the amplifier.

An "*H*-type" neutralizing network may also be applied to an "*Y*-type" amplifier as shown in Fig. 3.13. Here the tuning capacitance of the input tuned circuit has been tapped such that the lower capacitance C_b forms (part of) the impedance Z of the neutralizing network. The design of the neutralizing network can most easily be carried out by first converting the y-parameter of the transistor to h-parameters with the aid of Table I.1 of Appendix I. Then:

$$h_{12} + H_{12} = 0.$$

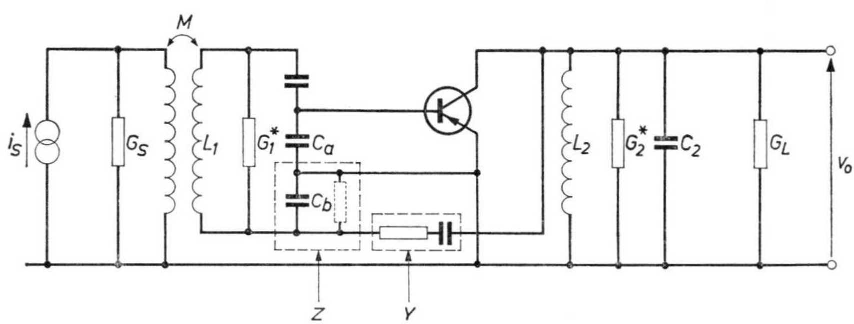


Fig. 3.13. *Y*-type of amplifier with *H*-type of neutralizing network. The capacitor C_b with an extra damping G_b connected in parallel forms the impedance Z of the neutralizing network.

3.5 *K*-Neutralization

3.5.1 GENERAL

In an amplifier with a *K*-type neutralizing network the input pairs of terminals of both four-terminal networks are connected in parallel whereas the output pairs are connected in series. This type of neutralizing circuit can be analyzed in an analogous way to the *H*-type of neutralization considered in Section 3.4. Similar results and conclusions will be found.

3.5.2 PRACTICAL *K*-NEUTRALIZED AMPLIFIER CIRCUIT

A practical form of a *K*-neutralized amplifier stage is the circuit presented in Fig. 3.14. Here the *K*-neutralizing circuit is applied to a *Y*-type amplifier.

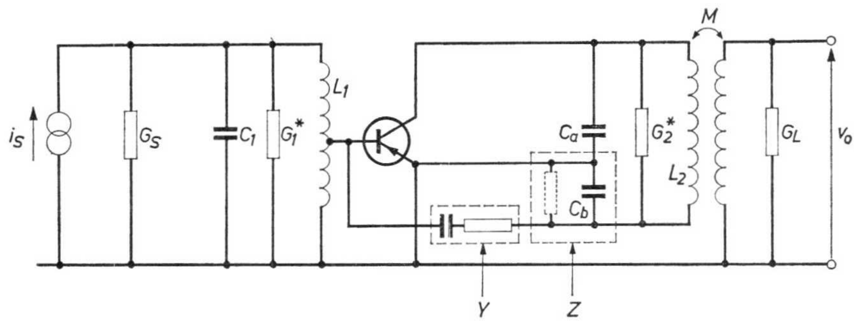


Fig. 3.14. *Y*-type of amplifier with *K*-type of neutralizing network. The capacitor C_b of the capacitive tap on the output tuned circuit forms part of the impedance Z of the neutralizing network.

The tuning capacitance of the output circuit is tapped and the tapping point is connected to earth. The lower capacitance C_b forms (part of) the impedance Z of the neutralizing network.

The most convenient way of designing the neutralizing circuit is to convert the transistor y-parameters to k-parameters, and putting

$$k_{12} + K_{12} = 0. \tag{3.5.1}$$

3.6 The Intermediate-Basis Circuit

The neutralizing circuits considered above employ a four-terminal network containing passive elements to achieve the neutralizing action.

Another type of neutralizing circuit which, basically, does not require any extra element, is the "intermediate-basis" circuit as it is referred to in literature (see Bibliography [3.1]). In this circuit, a tapping on either the tuned circuit between the two input terminals or that between the two output

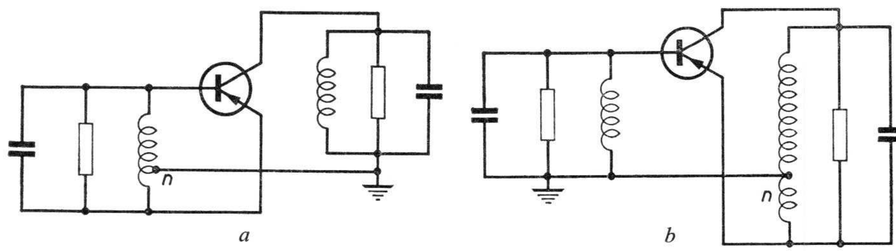


Fig. 3 15. Two possible forms of intermediate-basis circuit. By way of example, a transistor in common emitter connection is shown. Figures *a* and *b* present the *K* and *H*-intermediate-Basis circuits respectively.

terminals is connected to earth. In Fig. 3.15 the two possible forms of this terminals circuit are shown.

To analyze these circuits use can conveniently be made of the *K* and *H*-matrices respectively.

3.6.1 THE *K*-INTERMEDIATE-BASIS CIRCUIT

The *K*-intermediate-basis circuit or, as it is sometimes referred to, the "base-emitter drive" circuit, can be considered as a combination of two fourpoles connected in parallel at the input terminals and in series at the output terminals, see Fig. 3.16. For the two four-terminal networks the following equations may be written down, see Fig. 3.16.c:

$$\left.\begin{aligned} i_1 &= k_{11}v_1 + k_{12}i_2, \\ v_2 &= k_{21}v_1 + k_{22}i_2, \end{aligned}\right\} \tag{3.6.1}$$

and

$$\left.\begin{aligned} i_1' &= K_{11}v_1 - K_{12}i_2, \\ v_2' &= K_{21}v_1 - K_{22}i_2. \end{aligned}\right\} \tag{3.6.2}$$

The complete circuit is perfectly neutralized when:

$$k_{12} - K_{12} = 0. \tag{3.6.3}$$

If n is the tapping ratio of the input tuned circuit, K_{12} equals:

$$K_{12} = -\frac{n}{k}, \tag{3.6.4}$$

in which k is the coupling coefficient.

From the last two expressions:

$$\frac{n}{k} = -k_{12}. \tag{3.6.5}$$

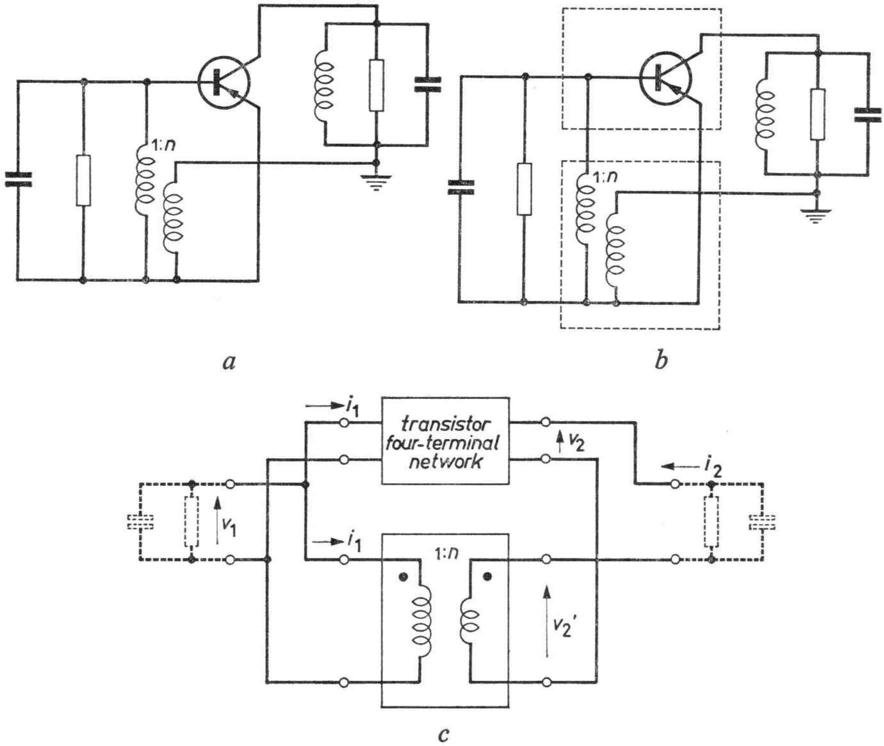


Fig. 3.16. *K*-intermediate-basis circuit. In Fig. 3.16a the tapping on the tuned circuit of Fig. 3.15.a has been replaced by a transformer with separate windings. In Fig. 3.16.b the same circuit has been drawn in a somewhat different form separating the transistor and the transformer. Fig. 3.16.c shows that the circuit may be considered as a *K*-combination of two fourpoles.

When the transistor properties are expressed in *y*-parameters, (Eq. 3.6.5) becomes, using a matrix conversion table (see Appendix I).

$$\frac{n}{k} = -\frac{y_{12}}{y_{22}}. \tag{3.6.6}$$

It follows from Eq. (3.6.5) that, since n/k is real, perfect neutralization is only possible if k_{12} is real. According to Eq. (3.6.6), this means that the phase angles of $-y_{12}$ and y_{22} must be equal. If the transistor proper does not fulfil this condition, $-\varphi_{12}$ and φ_{22} can be made identical by increasing artificially either the real or the imaginary part of y_{22} depending on whether φ_{22} must be made smaller or larger respectively. This is, however, only realizable in practice if the differences in the phase angles of $(-y_{12})$ and y_{22} are not too large.

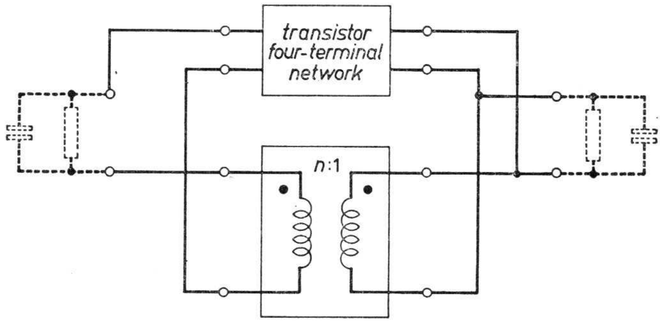


Fig. 3.17. H-intermediate-basis circuit with two fourpoles drawn separately.

3.6.2 THE H-INTERMEDIATE-BASIS CIRCUIT

The H-intermediate-basis circuit as given in Fig. 3.15.b may be considered as a combination of two four-terminal networks the input terminals of which are connected is series whereas the output terminals are connected in parallel, see Fig. 3.17. If the properties of the fourpoles are expressed in the H-parameter system, for perfect neutralization:

$$h_{12} - H_{12} = 0. \tag{3.6.7}$$

Taking into account the normal sign convention for currents and voltages, the H_{12} parameter of the transformer equals

$$H_{12} = -\frac{n}{k}, \tag{3.6.8}$$

in which n is the transformer ratio and k the coefficient of coupling. Combining the last two expressions:

$$h_{12} = -\frac{n}{k}. \tag{3.6.9}$$

Converting h-parameters to y-parameters gives:

$$h_{12} = -\frac{y_{12}}{y_{11}}, \tag{3.6.10}$$

and hence:

$$\frac{n}{k} = -\frac{y_{12}}{y_{11}}. \tag{3.6.11}$$

In analogy with the preceding sub-section, the phase angles of $-y_{12}$ and y_{11} must be equal. If the differences are not too large, this can be achieved by changing y_{11} artificially.

3.7 Fixed-Component Neutralization

In the preceding sections various cases of perfect neutralization have been considered. In each case the reverse transfer parameter of the neutralizing network has been chosen such that it completely cancels the reverse transfer parameter of the transistor under consideration.

In practice, most parameters of a transistor of a given type spread around certain average values. This is also the case for the reverse transfer parameter. It is therefore necessary, in order to achieve perfect neutralization, to adjust the elements of the neutralizing circuit separately for each transistor of a given type. In practical amplifiers it is, however, often required that fixed elements are used in the neutralizing circuit. The question then arises how to design the neutralizing circuit in order that good results are obtained for all transistors of the type given.

With fixed component neutralization, as this method will be referred to, perfect neutralization is achieved only for transistors which have a certain value of the feedback parameter. If the symbol γ_{12} is used to denote the reverse transfer parameter and Γ_{12} to denote that of the neutralizing network, perfect neutralization is obtained for:

$$\gamma_{12} + \Gamma_{12} = 0. \tag{3.7.1}$$

The amplifier is said to be "over-neutralized" if a transistor is used in the amplifier for which

$$|\gamma_{12}| < |\Gamma_{12}|, \tag{3.7.2}$$

and "under-neutralized" in case

$$|\gamma_{12}| > |\Gamma_{12}|. \tag{3.7.3}$$

In Fig. 3.18 the three cases of neutralization are shown. The resulting feedback in the case of over-neutralization as well as in the case of under-neutralization may lead to instability of the amplifier. To ensure stability the best values for the components of the neutralizing network are therefore those which yield equal stability factors for the over-neutralized and the under-neutralized cases. To determine these values it is required to investigate the spreads in the γ_{12} and γ_{21} parameters of the transistors and to find out which values of these parameters are most critical with respect to stability.

Furthermore, spreads occurring in the Γ_{12} parameter of the neutralizing network must also be incorporated in the design of this network.

In Chapter 11 an extensive treatment will be given of the influences of the various spreads in parameter of transistors and neutralizing networks on the stability of the amplifier. For the purpose of this chapter, in the following sub-sections only the spreads in the moduli of the transfer parameters are considered.

3.7.1 SPREAD IN THE TRANSISTOR REVERSE AND FORWARD TRANSFER PARAMETERS

As will be seen from Book II, Chapter 2, the spreads in the arguments of y_{12}, y_{21}, h_{12} and h_{21} are small compared with the spreads in the magnitudes of these quantities. To design the neutralizing circuit, it is therefore sufficient to consider the spreads in magnitude. Using a suffix M to indicate a maximum value of a quantity, a suffix m to indicate a minimum value and a suffix a to indicate an average value, Table 3.2 can be compiled [1]). To investigate the

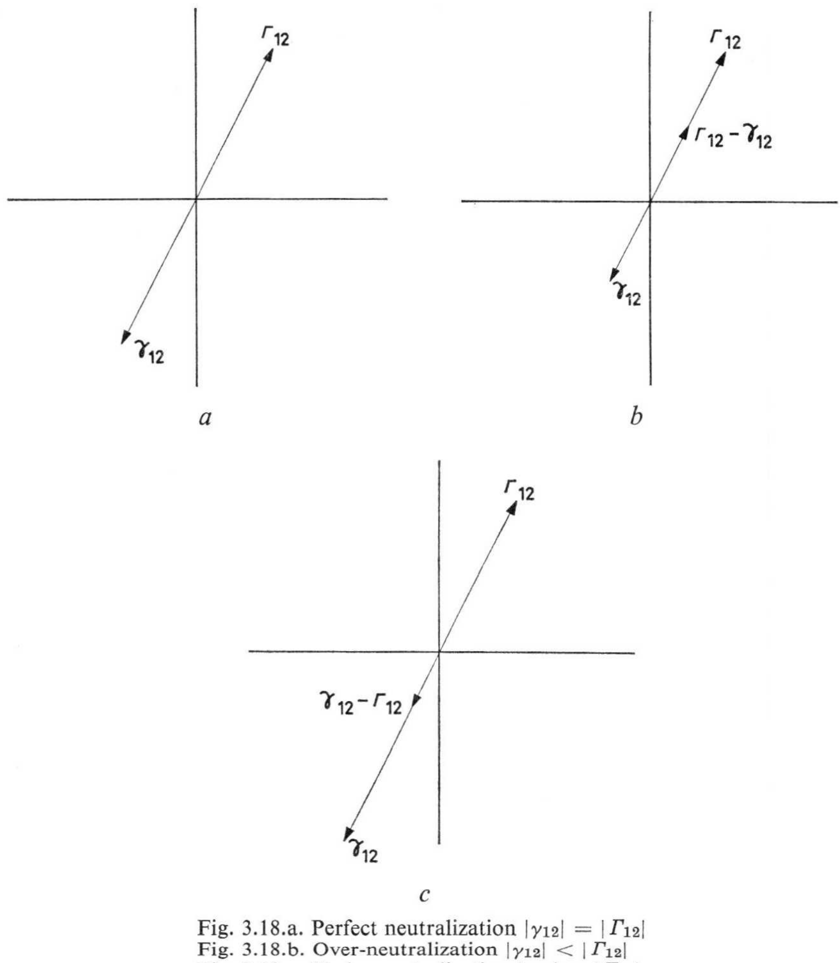


Fig. 3.18.a. Perfect neutralization $|\gamma_{12}| = |\Gamma_{12}|$
Fig. 3.18.b. Over-neutralization $|\gamma_{12}| < |\Gamma_{12}|$
Fig. 3.18.c. Under-neutralization $|\gamma_{12}| > |\Gamma_{12}|$

[1]) The 2σ values are usually quoted as the extreme values of a spreading parameter whereas the median is taken as the average value.

Table 3.2	reverse transfer parameter	forward transfer parameter	$M = \gamma_{12}\,\gamma_{21}$
Maximum value	$\gamma_{12\,M}$	$\gamma_{21\,M}$	M_M
Average value	$\gamma_{12\,a}$	$\gamma_{21\,a}$	M_a
Minimum value	$\gamma_{12\,m}$	$\gamma_{21\,m}$	M_m

stability of a neutralized amplifier for which the product of γ_{12} and γ_{21} is required, the most severe combination of these parameters must be taken into account. Although it is not very likely that in a single transistor the extreme maxima and the extreme minima occur simultaneously, the stability calculation will be based on such extreme combinations. In any case this produces a safe design and it is, moreover, the only possible approximation of the problem because of the lack of a reasonable correlation between the γ_{12} and γ_{21} parameters of the transistor of a certain type at a given frequency and biasing point.

3.7.2 CALCULATION OF Γ_{12}

As already stated the best value from the point of view of stability of Γ_{12} is that which renders the stability factors of extreme (with respect to γ_{12} and γ_{21}) transistors equal in the over-neutralized and the under-neutralized cases.

In the under-neutralized case, the regeneration phase angle of the transistor equals

$$\arg \gamma_{12} + \arg \gamma_{21} = \Theta, \tag{3.7.4}$$

and in the over-neutralized case this phase angle becomes (see Fig. 3.18)

$$\arg \gamma_{12} - \pi + \arg \gamma_{21} = \Theta - \pi. \tag{3.7.5}$$

If the stability factor equals s ($s = T_g/T$), the following condition is obtained (see sub-section 2.2.5):

$$s = \frac{\dfrac{2}{1 + \cos\Theta}}{\dfrac{\{|\gamma_{12\mathrm{M}}| - |\Gamma_{12}|\} \cdot |\gamma_{21\mathrm{M}}|}{A}}$$

$$= \frac{\dfrac{2}{1 + \cos(\Theta - \pi)}}{\dfrac{\{|\Gamma_{12}| - |\gamma_{12\mathrm{m}}|\} \cdot |\gamma_{21\mathrm{m}}|}{A}}, \tag{3.7.6}$$

or:

$$\frac{\{|\Gamma_{12}| - |\gamma_{12\mathrm{m}}|\} \cdot |\gamma_{21\mathrm{m}}|}{\{|\gamma_{12\mathrm{M}}| - |\Gamma_{12}|\} \cdot \gamma_{21\mathrm{M}}|} = \frac{1 + \cos\Theta}{1 - \cos\Theta}, \tag{3.7.7}$$

from which, after some calculation:

$$|\Gamma_{12}| = \frac{2M_a + \Delta M \cos\Theta}{2|\gamma_{21a}| + \Delta|\gamma_{21}| \cos\Theta}. \tag{3.7.8}$$

Furthermore, according to Eq. (3.6.1):

$$\arg\, \Gamma_{12} = \arg\, \lambda_{12} \pm \pi. \tag{3.7.9}$$

(Theoretically the arguments of Γ_{12} and λ_{12} may differ by $(2k + 1)\pi$ in which k is an integer. In practical amplifier circuits, however, $k = 0$.)

From the general expressions (3.7.8) and (3.7.9) corresponding expressions for admittance and hybrid parameters may be obtained by substitution. For the Y-neutralization system as well as the intermediate-basis circuit, the term π in Eq. (3.7.9) is contained in the phase inverting transformer.

BIBLIOGRAPHY

[3.1] H. BENEKING, *Aktive Vierpole in Zwischenbasisschaltung*, A. E. Ü., **9**, p. 519-527, 1956.

[3.2] A. J. COTE, JR, *Evaluation of Transistor Neutralization Networks*, I.R.E. Trans. on Circuit Theory, **CT-5**, p. 95-103, 1958.

[3.3] C. C. CHENG, *Neutralization and Unilateralization*, I.R.E. Trans. on Circuit Theory, **CT-2**, p. 138-145, 1955.

[3.4] C. Y. CHU, *Unilateralization of Junction Transistor Amplifiers at High Frequencies*, Proc. I.R.E., **43**, p. 1001-1006, 1955.

[3.5] L. GOHM, *Neutralisation über breite Frequenzbänder*, Nachrichten Technische Fachberichte **18**, p. 83-89, 1960.

[3.6] R. E. LESLIE and D. T. HESS, *Neutralizing Wide Band H.F. Transistor Amplifiers*, Electronic Industries, December 1959, p. 95-98.

[3.7] G. MEYER-BRÖTZ, *Die Neutralisierung im selektiven Transistorverstärker*, A. E. Ü., **10**, p. 391-397, 1956.

[3.8] YU. I. SIMONOV, *Calculation of the Y-Neutralization of Resonance Amplifiers with Transistors*, Elektrosvyaz, 1960, **10**, p. 35-38.

[3.9] A. P. STERN, C. A. ALDRIDGE and W. F. CHOW, *Internal Feedback and Neutralization of Transistor Amplifiers*, Proc. I.R.E. **43**, p. 838-847, 1955.

CHAPTER 4

SINGLE-STAGE AMPLIFIER WITH SINGLE-TUNED BANDPASS FILTERS OPTIMIZATION OF POWER GAIN

In Chapter 2 a single-stage amplifier with single-tuned bandpass filters was considered merely in order to present an introduction to the various aspects of the design of bandpass amplifiers. No attempt, however, was made to obtain an optimum design. The various design aspects of the single-stage amplifier are therefore considered again in this chapter but with a view to optimizing the amplifier with respect to power gain, taking into account the other design requirements.

4.1 The Various Kinds of Power Gain and their Significance

In Chapter 2 it was pointed out that the gain performance of an amplifier equipped with transistors can best be characterized by its *transducer gain.* The transducer gain, denoted by Φ_t is defined as (see Fig. 4.1):

$$\Phi_t = \frac{\text{power supplied to load}}{\text{power available from source}} = \frac{P_L}{P_{Sa}}. \qquad (4.1.1)$$

The transducer gain thus relates the power supplied by the amplifier to the load and the power that is delivered by the source when optimally terminated. This means that the transducer gain is a measure of the efficiency obtained by inserting the amplifier between source and load. Furthermore it follows that the transducer gain is a function of the source immittance, the load immittance and of the parameters of the amplifying network.

Other important quantities expressing gain in power are the power gain Φ, the available power gain Φ_a and the maximum available power gain Φ_{aM} [1]).

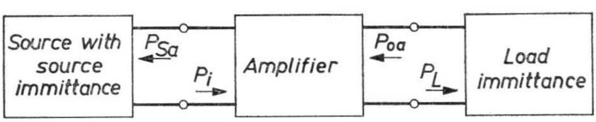


Fig. 4.1. Amplifier with source and load terminations defining various power quantities.

---

1) These and other quantities expressing gain in power are defined in Appendix IV.

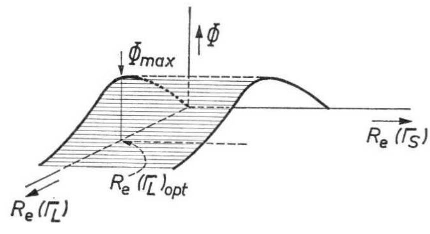


Fig. 4.2. The power gain of an amplifier becomes maximal for a certain value of the load immittance Γ_L as shown here for the case of an amplifier havin g purely real parameters.

The power gain Φ of an amplifier is defined as

$$\Phi = \frac{\text{power supplied to load}}{\text{power supplied to input of amplifier}} = \frac{P_L}{P_i}. \tag{4.1.2}$$

The power gain Φ, which is only defined if the input immittance of the amplifier has a positive real part, is a function of the load immittance and the properties of amplifying network whereas it is independent of the source immittance. This is illustrated in Fig. 4.2 for the case of an amplifier having purely real parameters. For a certain value of the (real) load immittance $Re(\Gamma_L)_{opt}$ the power gain becomes maximal.

When the source immittance Γ_S is selected such that it conjugately matches the input immittance of the amplifier, $P_i = P_{Sa}$ and $\Phi = \Phi_t$. Generally:

$$\Phi \geqslant \Phi_t. \tag{4.1.3}$$

The *available power gain* Φ_a of an amplifier is defined as:

$$\Phi_a = \frac{\text{power available from output of amplifier}}{\text{power available from source}} = \frac{P_{oa}}{P_{Sa}}. \tag{4.1.4}$$

The available power gain, which is only defined if the output immittance of the amplifier has a positive real part thus depends on the source immittance and on the parameters of the amplifying network. It is independent of the load immittance. This has been illustrated in Fig. 4.3 for the case that the parameters of the amplifier are purely real. For a certain value of the source immittance $Re(\Gamma_S)_{opt}$ the available power gain becomes maximal.

Furthermore, if a load immittance is selected which conjugately matches the output immittance of the amplifier, P_0 becomes equal to P_{oa} and $\Phi_a = \Phi_t$. Generally:

$$\Phi_a \geqslant \Phi_t \tag{4.1.5}$$

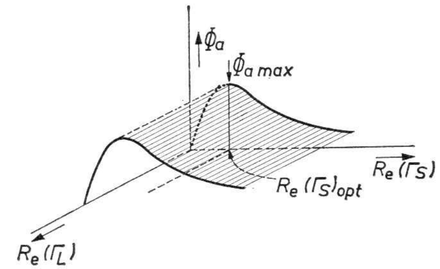


Fig. 4.3. The available power gain of an amplifier depends on the source immittance Γ_S as shown here for an amplifier with purely real parameters.

The *maximum available power gain* Φ_{aM} of an amplifying network is obtainable when both load and source immittances are selected such that maximum values are obtained for Φ and Φ_a. Then the source immittance and the load immittance are optimally matched to the input immittance and the output immittance of the amplifier respectively. We may write for this case:

$$\Phi_{aM} = \Phi_{t\ \max} = \Phi_{\max} = \Phi_{a,\max}. \tag{4.1.6}$$

The maximum available power gain is a very important property of the amplifier. If it is finite (i.e. if the amplifier is stable) source and load terminations can be selected such that the transducer gain becomes maximal and equal to Φ_{aM}.

Fig. 4.4 gives a geometrical presentation of the conditions under which the maximum available gain of an amplifier is achieved, again assuming purely real parameters for the amplifier (and hence, real optimal terminations). Intersections of the Φ plane of Fig. 4.2 for $Re(\Gamma_L)_{opt}$ and of the Φ_a plane of Fig. 4.3 for $Re(\Gamma_S)_{opt}$ are drawn in Fig. 4.4. The transducer gain becomes maximal and equal to Φ_{aM} at the point of crossing of these intersections. Optimizing an amplifier with respect to power gain thus means that such

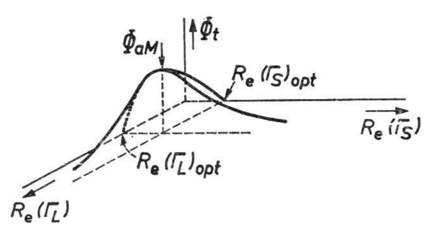


Fig. 4.4. If load and source immittances of the amplifier are so chosen that simultaneously the power gain and the available power gain are maximum it is said that the amplifier delivers its maximum available power gain. This is illustrated in the figure for an amplifier with purely real terminations.

terminations are selected which provide simultaneously conjugate matching at input- and output pairs of terminals. Such a power optimizing procedure is, however, only possible when the amplifier remains stable over a wide range of terminations which is the case for amplifiers employing inherently stable amplifying elements (see sub-section 2.2.5) as will be proved in a following section. Amplifiers employing potentially unstable amplifying elements are stable only over a restricted range of source and load terminations. For such amplifiers the conditions must be investigated under which the transducer gain becomes as large as possible thereby fulfilling the stability requirements.

Apart from the restrictions imposed upon the source and load terminations of the amplifier when a potentially unstable amplifying element is used, there may be other restrictions due to a prescribed method of tuning the amplifier. The latter restrictions affect only the imaginary parts of the source and load immittances. In this chapter, however, we will consider only those conditions of tuning which render the transducer gain maximal.

Furthermore there may be restrictions upon the real parts of source and load immittances due to requirements other than stability. These restrictions occur when it is attempted to achieve as large a value as possible of the transducer gain of an amplifier operating between a source and load having prescribed values of the real parts of the immittances.

In cases in which the design for optimum noise performance of the amplifier is of prime importance, special restrictions are also imposed on source terminations, see Section 4.4.

The various cases mentioned above will be dealt with in the following sections. Only amplifiers in the admittance matrix form will be considered in the analyses. When necessary, relations valid for amplifiers in the H matrix or any other environments may be derived analogously.

4.2 Single-Stage Amplifier with Variable Regeneration Coefficient

As follows from the preceding section, the maximum transducer gain of a single-stage amplifier is obtained for conjugate matching at input and output terminals of the transistor. Then source and load admittances depend on the transistor parameters. Because G_S and G_L are thus variable the regeneration coefficient T is also variable.

4.2.1 CONJUGATE MATCHING

Source and load admittances are simultaneously conjugately matched to the input and output admittances of the transistor respectively if:

$$Y_S = y_{in}^* = g_{in} - jb_{in}, \tag{4.2.1}$$

and

$$Y_L = y_{out}^* = g_{out} - jb_{out}. \tag{4.2.2}$$

The asterisk in y_{in}^* and y_{out}^* denotes the complex conjugate of y_{in} and y_{out} respectively.

4.2.2 INPUT AND OUTPUT ADMITTANCES OF AN AMPLIFIER UNDER CONJUGATELY MATCHED CONDITIONS

The input and output admittances of a fourpole are given by (see Fig. 4.5):

$$y_{in} = y_{11} - \frac{y_{12}y_{21}}{y_{22} + Y_L}, \tag{4.2.3}$$

$$y_{out} = y_{22} - \frac{y_{12}y_{21}}{y_{11} + Y_S}. \tag{4.2.4}$$

Combining these two equations:

$$(y_{in} - y_{11})(y_{22} + Y_L) = (y_{out} - y_{22})(y_{11} + Y_S). \tag{4.2.5}$$

By equating the real parts of this expression and with Eqs. (4.2.1) and (4.2.2) we obtain:

$$(g_{11} - g_{in})(g_{22} + g_{out}) + (b_{11} - b_{in})(b_{out} - b_{22})$$
$$= (g_{22} - g_{out})(g_{11} + g_{in}) + (b_{22} - b_{out})(b_{in} - b_{11}),$$

from which:

$$2g_{11}g_{out} - 2g_{in}g_{22} = 0,$$

or:

$$\frac{g_{11}}{g_{22}} = \frac{g_{in}}{g_{out}}. \tag{4.2.6}$$

Furthermore, from Eq. (4.2.3):

$$Y_L = \frac{y_{12}y_{21}}{y_{11} - y_{in}} - y_{22}. \tag{4.2.7}$$

From Eqs. (4.2.1) and (4.2.2):

$$Y_S + y_{in} = 2g_{in}, \tag{4.2.8}$$

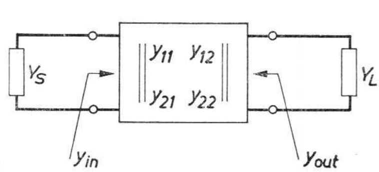


Fig. 4.5. Single-stage amplifier with terminations.

$$Y_L + y_{out} = 2g_{out}, \tag{4.2.9}$$

and by adding Eqs. (4.2.4) and (4.2.7):

$$-\frac{y_{12}y_{21}}{y_{11} + Y_S} + \frac{y_{12}y_{21}}{y_{11} - y_{in}} = 2g_{out},$$

or:

$$\frac{y_{12}y_{21}(Y_S + y_{in})}{(y_{11} + Y_S)(y_{11} - y_{in})} = 2g_{out}. \tag{4.2.10}$$

With Eq. (4.2.8), Eq. (4.2.10) becomes:

$$\frac{y_{12}y_{21}}{(y_{11} + Y_S)(y_{11} - y_{in})} = \frac{g_{out}}{g_{in}},$$

and with Eq. (4.2.6):

$$(y_{11} + Y_S)(y_{11} - y_{in}) = \frac{g_{11}}{g_{22}} \cdot y_{12}y_{21}. \tag{4.2.11}$$

By putting (see sub-section 2.2.5):

$$t = \frac{|y_{12}y_{21}|}{g_{11}g_{22}}, \tag{4.2.12}$$

and

$$\Theta = \arg y_{12} + \arg y_{21}, \tag{4.2.13}$$

in which t denotes the intrinsic regeneration coefficient of the transistor, Eq. (4.2.11) becomes:

$$(y_{11} + Y_S)(y_{11} - y_{in}) = g_{11}^2 t \cdot \exp(j\Theta). \tag{4.2.14}$$

Separating real and imaginary parts and using Eq. (4.2.1) we obtain:

$$g_{11}^2 - g_{in}^2 - (b_{11} - b_{in})^2 = g_{11}^2 t \cos \Theta, \tag{4.2.15}$$

and

$$2g_{11}(b_{11} - b_{in}) = g_{11}^2 t \sin \Theta. \tag{4.2.16}$$

Substituting $(b_{11} - b_{in})$ from Eq. (4.2.16) into Eq. (4.2.15) yields:

$$g_{in} = g_{11}[1 - t \cos \Theta - \tfrac{1}{4}t^2 \sin^2 \Theta]^{\frac{1}{2}}. \tag{4.2.17}$$

From Eq. (4.2.16):

$$b_{in} = b_{11} - \tfrac{1}{2}g_{11}t \sin \Theta. \tag{4.2.18}$$

For the output admittance we can derive in a similar manner:

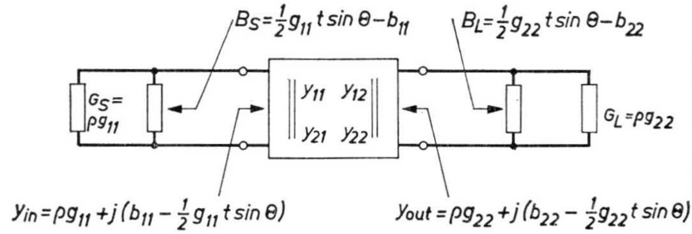


Fig. 4.6. Conjugately matched single-stage amplifier.

$$g_{out} = g_{22}[1 - t\cos\Theta - \tfrac{1}{4}t^2\sin^2\Theta]^{\frac{1}{2}}, \tag{4.2.19}$$

$$b_{out} = b_{22} - \tfrac{1}{2}g_{22}t\sin\Theta. \tag{4.2.20}$$

By putting:

$$\rho = [1 - t\cos\Theta - \tfrac{1}{4}t^2\sin^2\Theta]^{\frac{1}{2}}, \tag{4.2.21}$$

$$= [(1 - \tfrac{1}{2}t\cos\Theta)^2 - \tfrac{1}{4}t^2]^{\frac{1}{2}}, \tag{4.2.22}$$

Eqs. (4.2.17) and (4.2.19) become:

$$g_{in} = \rho g_{11}, \tag{4.2.23}$$

$$g_{out} = \rho g_{22}. \tag{4.2.24}$$

Obviously, only positive (real) values of ρ are significant for our amplifier analysis. When ρ becomes zero the amplifier is at the boundary of stability, see sub-section 4.2.5.1.

In Fig. 4.6 the conjugately matched amplifier is represented together with the calculated values of input and output terminations. Obviously transformers may be used to connect source and load to the transistor terminals. Then the transformer ratios must be incorporated in the values of Y_S and Y_L.

In Fig. 4.7, the quantity ρ has been plotted as a function of t with Θ as parameter.

4.2.3 MAXIMUM AVAILABLE POWER GAIN

The transducer gain of an amplifier is given by (see sub-section 2.4.1):

$$\Phi_t = 4G_S G_L |Z_{t0}|^2, \tag{4.2.25}$$

in which G_S and G_L denote the source and load conductances respectively and Z_{t0} denotes the transimpedance of the amplifier at the frequency at which the gain is required.

Now:

$$Z_{t0} = -\frac{y_{21}}{\Delta_0}, \tag{4.2.26}$$

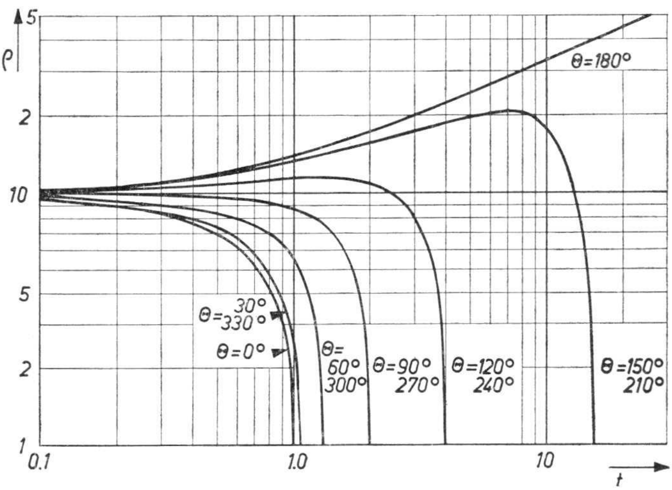


Fig. 4.7. The quantity ρ applicable to the conjugately matched amplifier as a function of the intrinsic regeneration coefficient t of the transistor and the regeneration phase angle Θ as parameter. The quantity ρ relates $g_{in} = G_S$ to g_{11} and $g_{out} = G_L$ to g_{22}.

$$\Delta = \begin{vmatrix} Y_1 & y_{12}y_{21} \\ 1 & Y_2 \end{vmatrix}, \tag{4.2.27}$$

$$Y_1 = Y_S + y_{11}, \tag{4.2.28}$$

$$Y_2 = Y_L + y_{22}. \tag{4.2.29}$$

If source and load admittances have the conjugate matching values we may write for Y_1:

$$Y_1 = g_{in} - \mathrm{j}b_{in} + g_{11} + \mathrm{j}b_{11}.$$

With Eqs. (4.2.23) and (4.2.18), Y_1 becomes:

$$Y_1 = g_{11}(1 + \rho + \mathrm{j}\tfrac{1}{2}t \sin \Theta). \tag{4.2.30}$$

Similarly:

$$Y_2 = g_{22}(1 + \rho + \mathrm{j}\tfrac{1}{2}t \sin \Theta). \tag{4.2.31}$$

With these expressions for Y_1 and Y_2 the determinant of Eq. (4.2.27) becomes at the frequency at which the maximum in transducer gain occurs:

$$\Delta_0 = g_{11}g_{22} \cdot \delta_0, \tag{4.2.32}$$

and
$$\delta_0 = \begin{vmatrix} 1 + \rho + j\tfrac{1}{2}t \sin\Theta & t \exp(j\Theta) \\ 1 & 1 + \rho + j\tfrac{1}{2}t \sin\Theta \end{vmatrix}. \tag{4.2.33}$$

Furthermore:

$$\left.\begin{aligned} G_S &= g_{in} = \rho g_{11}, \\ G_L &= g_{out} = \rho g_{22}. \end{aligned}\right\} \tag{4.2.34}$$

and

Then the transducer gain becomes:

$$\Phi_t = 4 g_{11} g_{22} \rho^2 \cdot \frac{|y_{21}|^2}{g_{11}{}^2 g_{22}{}^2 |\delta_0|^2},$$

or:

$$\Phi_t = \frac{|y_{21}|^2}{4 g_{11} g_{22}} \cdot \frac{16\rho^2}{|\delta_0|^2}. \tag{4.2.35}$$

According to Appendix V, the maximum unilateralized gain $\Phi_{u\mathrm{M}}$ of the transistor equals:

$$\Phi_{u\mathrm{M}} = \frac{|y_{21}|^2}{4 g_{11} g_{22}}.$$

This gives for Φ_t:

$$\Phi_t = \Phi_{u\mathrm{M}} \cdot \frac{16\rho^2}{|\delta_0|^2}. \tag{4.2.36}$$

Evaluating the reduced determinant δ_0, we obtain from Eq. (4.2.33):

$$|\delta_0|^2 = \{(1 + \rho)^2 - \tfrac{1}{4}t^2 \sin^2\Theta - t \cos\Theta\}^2 + \rho^2 t^2 \sin^2\Theta.$$

With Eq. (4.2.21) this reduces to:

$$|\delta_0|^2 = 8\rho^2(1 - \tfrac{1}{2}t \cos\Theta + \rho). \tag{4.2.37}$$

Then the transducer gain Φ_t, which under conjugately matched conditions equals the maximum available gain $\Phi_{a\mathrm{M}}$ becomes:

$$\Phi_{a\mathrm{M}} = \Phi_{u\mathrm{M}} \cdot \frac{2}{1 - \tfrac{1}{2}t \cos\Theta + \rho}. \tag{4.2.38}$$

Obviously, $\Phi_{a\mathrm{M}}$ has a significant value only if:

$$1 - \tfrac{1}{2}t \cos\Theta + \rho > 0, \tag{4.2.39}$$

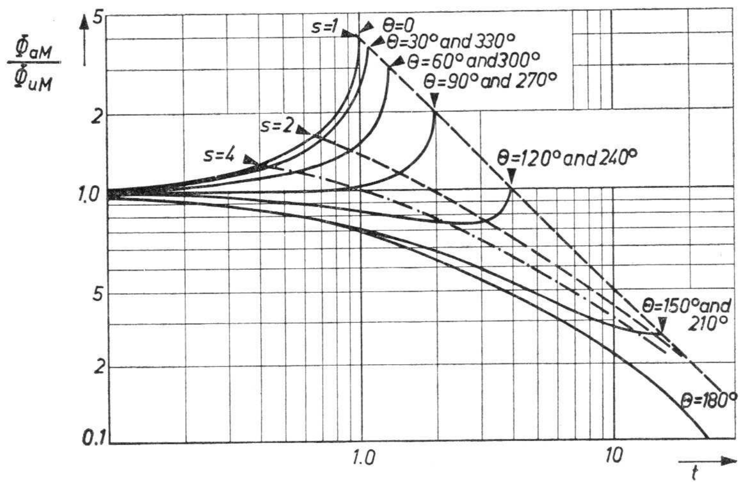


Fig. 4.8. The relation between the maximum available gain and the maximum unilateralized gain of a transistor is a function of t and Θ. This ratio is obviously maximum when the amplifier is at the boundary of stability which is indicated by the curve for $s = 1$.

and furthermore, ρ has a real value.
For an unilateral amplifier $t = 0$ and $\rho = 1$. Then:

$$\Phi_{aM} = \Phi_{uM}. \qquad (4.2.40)$$

The second factor of Eq. (4.2.38) is plotted in Fig. 4.8 as a function of t with Θ as parameter.

4.2.4 REGENERATION COEFFICIENT

The regeneration coefficient T of the amplifier is defined as (see sub-section 2.1.2):

$$T = \frac{|y_{12}y_{21}|}{G_1 G_2}, \qquad (4.2.41)$$

in which:

and

$$\left.\begin{aligned} G_1 &= G_S + g_{11}, \\ G_2 &= G_L + g_{22}. \end{aligned}\right\} \qquad (4.2.42)$$

With Eqs. (4.2.12), (4.2.23) and (4.2.24) we obtain from Eq. (4.2.41) for the conjugately matched amplifier:

$$T = \frac{t}{(1 + \rho)^2}. \qquad (4.2.43)$$

4.2.5 STABILITY

4.2.5.1 Boundary of Stability

In the conjugately matched amplifier under discussion the source and load dampings have fixed values (ρg_{11} and ρg_{22}) whereas the source and load susceptances are frequency-dependent. At the frequency at which the transducer gain is maximum these susceptances have values as required for conjugate matching. For such an amplifier it has been derived in Chapter 2 that the boundary of stability is given by:

$$T_g = \frac{2}{1 + \cos \Theta}. \qquad (4.2.44)$$

The boundary of stability may also be obtained by considering that the total input and output dampings of the amplifier become zero at this boundary. According to Eqs. (4.2.23) and (4.2.24) this is the case for $\rho = 0$. Then it follows from Eq. (4.2.21) that:

$$t_g = \frac{2}{1 + \cos \Theta}, \qquad (4.2.45)$$

in which t_g is the value of the intrinsic regeneration coefficient t which renders $\rho = 0$.

Hence:

$$T_g = t_g = \left(\frac{y_{12} y_{21}}{g_{11} g_{22}}\right)_g, \qquad (4.2.46)$$

and, apparently, conjugate matching in an amplifier is only possible if:

$$t < t_g,$$

or:

$$t < \frac{2}{1 + \cos \Theta}. \qquad (4.2.47)$$

This implies that conjugate matching in an amplifier is only possible if the transistor employed is inherently stable (see sub-section 2.2.5).

4.2.5.2 Stability Factor

The stability factor of an amplifier, defined as the reciprocal of the maximum real loopgain of the amplifier is given by:

$$s = \frac{T_g}{T}. \qquad (4.2.48)$$

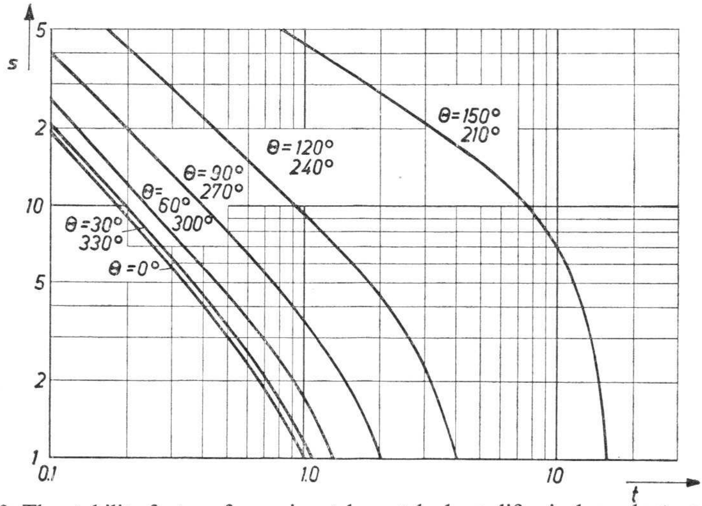


Fig. 4.9. The stability factor of a conjugately matched amplifier is dependent on the transistor properties t and Θ. For a certain combination of t and Θ the stability factor becomes $s = 1$ which means that for this value of Θ, the amplifier cannot be matched conjugately for larger values of t.

With Eqs. (4.2.34) and (4.2.44), the stability factor becomes:

$$s = \frac{2(1+\rho)^2}{t(1+\cos\Theta)}. \tag{4.2.49}$$

In Fig. 4.9 the stability factor s has been plotted as a function of t with Θ as parameter. For the point at which the curves for the various values of Θ intersect the (horizontal) line for $s = 1$, we have $t = t_g$.

Using the plots of s in Fig. 4.9, lines of constant stability factor have been drawn in Fig. 4.8 for $s = 1$, $s = 2$ and $s = 4$.

As follows from Fig. 4.8 a maximum gain in amplification of 6 dB above the maximum unilateralized gain of the transistor with $\Theta = 0°$ can be obtained by conjugate matching at input and output terminals. Then the amplifier is at the boundary of stability ($s = 1$). For stability factors of $s = 2$ and $s = 4$ the increase in transducer gain (for $\Theta = 0$) amounts to 2 dB and 1 dB respectively.

4.2.6 INCLUSION OF TUNED CIRCUITS IN THE CONJUGATELY MATCHED AMPLIFIER

In the preceding sub-sections the terminations of the conjugately matched amplifier have been referred to as the admittances Y_S and Y_L. Obviously, Y_S and Y_L may partly be formed by the admittances of input and output tuned circuits Y_1^* and Y_2^* respectively (in which $Y^* = G^* + jB^*$). Then the actual values of source and load conductances become:

$$G_S' = \rho g_{11} - G_1^*,$$
$$G_L' = \rho g_{22} - G_2^*.$$

By putting:

$$w_1' = \frac{G_1^*}{G_{1\ total}} = \frac{G_1^*}{G_S' + G_1^* + g_{in}} = \frac{G_1^*}{2g_{11} \cdot \rho}, \tag{4.2.50}$$

and

$$w_2' = \frac{G_2^*}{2g_{22} \cdot \rho}, \tag{4.2.51}$$

we obtain:

$$\left.\begin{aligned} G_S' &= g_{11}\rho(1 - 2w_1'), \\ G_L' &= g_{22}\rho(1 - 2w_2'). \end{aligned}\right\} \tag{4.2.52}$$

The susceptances $B_S + B_1^*$ and $B_L + B_2^*$ must be adjusted such that the correct values for conjugate matching are achieved. In most cases Y_S' and Y_L' are made real and equal to G_S' and G_L' given by Eq. (4.2.52) and the tuned circuits are detuned to such an extent that the required matching susceptances are obtained.

In Fig. 4.10 such an amplifier circuit is represented. The source and load dampings as well as the transistor input and output terminals are connected to tappings on the tuned circuits. In determining the proper values of G_S' and G_L' the tapping ratios must be taken into account.

Inclusion of the tuned circuits leads to a decrease in the actual transducer gain of the amplifier. The amount of power lost in the damping of the input tuned circuit equals $\left(\frac{2\rho g_{11} - G_1^*}{2\rho g_{11}}\right)^2 = (1 - w_1')^2$ times the total power supplied by the current source. Hence the transducer gain of the amplifier is decreased

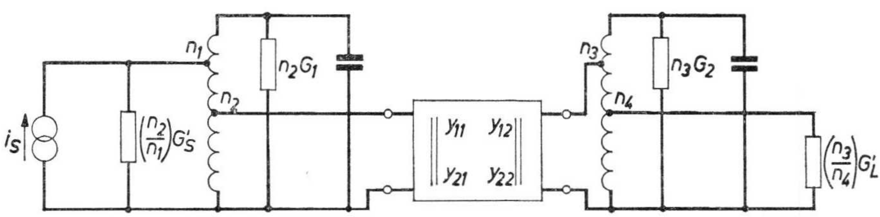


Fig. 4.10. Circuit diagram of a conjugately matched amplifier. The matching susceptances at input and output terminal pairs of the transistor are achieved by properly tuning the tuned circuits.

by this factor. Due to the damping of the output tuned circuit, the transducer gain is further reduced by a factor $(1 - w_2')^2$. The transducer gain of the amplifier including the tuned circuits then becomes with Eq. (4.2.38) [1]):

$$\Phi_{t\ \max} = \Phi_{uM} \cdot \frac{2}{1 - \frac{1}{2}t \cos \Theta + \rho} \cdot (1 - w_1')^2 (1 - w_2')^2 . \quad (4.2.53)$$

4.2.7 EXAMPLE

To illustrate the theory presented in the preceding sub-sections the gain and the terminations will be calculated for a conjugately matched amplifier with a transistor having the following parameters:

g_{11}	$=$ 10 m℧	b_{11}	$=$ 15 m℧
y_{12}	$=$ 0.25 m℧	φ_{12}	$=$ 260 ·
y_{21}	$=$ 100 m℧	φ_{21}	$=$ 310 ·
g_{22}	$=$ 0.5 m℧	b_{22}	$=$ 0.5 m℧

It follows from these parameters:

$$\Theta = 210°,\ t_g = 14.9,\ t = 5 \text{ and } \Phi_{uM} = 25 \text{ dB}.$$

Because $t < t_g$, the transistor is inherently stable (at the assumed frequency and biasing point) and hence conjugate matching of the amplifier is possible.

For $t = 5$, it follows from Fig. 4.9 that the stability factor s equals $s = 15$.

From Fig. 4.8 it follows that the maximum available gain of the transistor is 4.2 dB below the maximum unilateralized gain. For $\Phi_{uM} = 25$ dB, $\Phi_{aM} = 20.8$ dB.

Furthermore, from Fig. 4.7 it follows that $\rho = 2$, so that $G_S = 2g_{11} = 20$ m℧ and $G_L = 2g_{22} = 1$ m℧.

According to Eqs. (4.2.1) and (4.2.18) the susceptive part of the source admittances follows from:

$$B_S = -\, b_{11} + \tfrac{1}{2} g_{11} t \sin \Theta = -\, 15 + 12 = -\, 3 \text{ m℧}\,,$$

and from Eqs. (4.2.2) and (4.2.20):

$$B_L = -\, b_{22} + \tfrac{1}{2} g_{22} t \sin \Theta = -\, 0.5 + 0.125 = -\, 0.375 \text{ m℧}\,.$$

---

[1]) The factors $(1 - w')^2$ are not identical to the insertion losses $(1 - w)^2$ of a single-tuned bandpass filter as derived in Appendix II. The relation between w and w' follows from $w = w' \cdot \frac{2\rho}{1 + \rho}$. For a unilateral amplifier $w = w'$.

4.3 Single-Stage Amplifier with Defined Regeneration Coefficient

In amplifiers in which potentially unstable transistors are employed measures must be taken in order that stability of the amplifier is ensured. According to Chapter 2 this means that there is an upper limit for the regeneration coefficient T. In view of gain, as large a value of T as possible is desired. The upper limit for T mentioned can therefore be considered as being fixed when attempting to optimize the design of the amplifier with respect to power gain.

Similar conditions occur if the amplifier must operate between a source and load having given values of their damping. Then T is also constant.

In such an amplifier the parameters that may be varied to achieve maximum gain are the source and load susceptances only.

4.3.1 THE AMPLIFIER DETERMINANT

For an amplifier circuit as shown in Fig. 4.5 the main determinant (see subsection 2.1.2) can be written as:

$$\Delta = \begin{vmatrix} Y_1 & y_{12} \\ y_{21} & Y_2 \end{vmatrix}, \tag{4.3.1}$$

in which:

$$\begin{aligned} Y_1 &= G_1 + \mathrm{j}B_1, \\ &= G_S + g_{11} + \mathrm{j}(B_S + b_{11}), \end{aligned} \tag{4.3.2}$$

and

$$\begin{aligned} Y_2 &= G_2 + \mathrm{j}B_2, \\ &= G_L + g_{22} + \mathrm{j}(B_L + b_{22}). \end{aligned} \tag{4.3.3}$$

Furthermore we put:

$$Y_1 = G_1\left(1 + \mathrm{j}\,\frac{B_1}{G_1}\right) = G_1(1 + \mathrm{j}\tan\varphi_1)\,, \tag{4.3.4}$$

and

$$Y_2 = G_2\left(1 + \mathrm{j}\,\frac{B_2}{G_2}\right) = G_2(1 + \mathrm{j}\tan\varphi_2)\,. \tag{4.3.5}$$

Then Δ can be written as:

$$\Delta = G_1G_2 \cdot \delta, \tag{4.3.6}$$

in which:

$$\delta = \begin{vmatrix} 1 + \mathrm{j} \tan \varphi_1 & T \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j} \tan \varphi_2 \end{vmatrix} . \tag{4.3.7}$$

4.3.2 TRANSDUCER GAIN

The transducer gain of the amplifier is given by:

$$\Phi_t = 4 G_S G_L \frac{|y_{21}|^2}{G_1{}^2 G_2{}^2 |\delta|^2} . \tag{4.3.8}$$

For the type of amplifier under consideration T is constant, which implies that also the product $G_1 G_2 = A$ must be constant. To optimize Φ_t the quantities $|\delta|$, G_1 and G_2 may thus be varied taking into account constant values for T and A. Because δ only contains the constant T (the quantity Θ is a transistor parameter) whereas $G_S G_L$ only contains the constant A, the optimization procedures for Φ_t with respect to $|\delta|$ and G_1, G_2 may be carried out separately.

As stated in the introduction to Section 4.3, a certain class of amplifiers have to operate between a source and load with fixed values of G_S and G_L. For these amplifiers both G_1 and G_2 must remain constant and only $|\delta|$ may be varied to find the optimum value for Φ_t.

4.3.2.1 G_1 and G_2 are constant

As follows from Eq. (4.3.8) the maximum value of Φ_t is obtained when $|\delta|^2$ has the minimum value. According to Eq. (4.3.7) the variable quantities in $|\delta|^2$ are $\tan \varphi_1$ and $\tan \varphi_2$. In order to find the minimum value of $|\delta|^2$ we put:

$$\left.\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}(\tan \varphi_1)} |\delta|^2 = 0 , \\ \text{and} \qquad &\\ &\frac{\mathrm{d}}{\mathrm{d}(\tan \varphi_1)} |\delta|^2 = 0 . \end{aligned}\right\} \tag{4.3.9}$$

From Eq. (4.3.7):

$$\delta = (1 + \mathrm{j} \tan \varphi_1)(1 + \mathrm{j} \tan \varphi_2) - T \exp(\mathrm{j}\Theta), \tag{4.3.10}$$

or:

$$|\delta|^2 = (1 - \tan \varphi_1 \tan \varphi_2 - T \cos \Theta)^2 + (\tan \varphi_1 + \tan \varphi_2 - T \sin \Theta)^2. \tag{4.3.11}$$

With Eq. (4.3.9) we then obtain:

$$- \tan \varphi_2 (1 - T \cos \Theta) + \tan \varphi_1 \tan^2 \varphi_2 + \tan \varphi_1 + \tan \varphi_2 - T \sin \Theta = 0, \tag{4.3.12}$$

$$-\tan\varphi_1(1-T\cos\Theta)+\tan^2\varphi_1\tan\varphi_2+\tan\varphi_1+\tan\varphi_2-T\sin\Theta=0 \tag{4.3.13}$$

Subtracting these two equations gives:

$$\{\tan\varphi_1\tan\varphi_2-(1-T\cos\Theta)\}(\tan\varphi_2-\tan\varphi_1)=0 \tag{4.3.14}$$

Hence:

$$\tan\varphi_2-\tan\varphi_1=0 \tag{4.3.15}$$

and

$$\tan\varphi_1\tan\varphi_2-(1-T\cos\Theta)=0. \tag{4.3.16}$$

The solution given by Eq. (4.3.16) may be ignored because it leads to $\delta = 0$ as is shown by substitution in Eq. (4.3.11). Hence only the solution presented by Eq. (4.3.15) is useful. This indicates that the minimum value of $|\delta|$ occurs for:

$$\tan\varphi_1=\tan\varphi_2=\tan\varphi. \tag{4.3.17}$$

In order to evaluate δ_{min}, $\tan\varphi$ must first be calculated. Substitution of Eq. (4.3.17) into Eq. (4.3.12) leads to a third order polynomial in $\tan\varphi$ which cannot generally be solved.

However, because we are mainly interested in the optimum value of the transducer gain and hence in the minimum value of $|\delta|$, the value of $\tan\varphi$ need not necessarily be known. Then $|\delta|_{\min}$ may be obtained graphically as follows:

Substitution of Eq. (4.3.17) into Eq. (4.3.10) gives:

$$\delta=(1+\mathrm{j}\tan\varphi)^2-T\exp(\mathrm{j}\Theta). \tag{4.3.18}$$

The first term of the right hand side of this equation represents a parabola equivalent to the parabola $(1+\mathrm{j}x)^2$ considered in Chapter 2. Hence $|\delta|_{min}$ may be found by determining the distance between the extremity of the vector $T\exp(\mathrm{j}\Theta)$ and the parabola as shown in Fig. 4.11. The determination of $|\delta|_{min}$ is facilitated by Fig. 4.12 opposite page 112 in which a number of lines have been constructed which are equidistant to the parabola. For a given value of T and Θ the value of $|\delta|_{\min}$ may be read directly from the scale indicating the distance from the parabola to the line on which the extremity of T is situated.

In analogy with sub-section 2.4.3 the expression for the transducer gain can be written as:

$$\Phi_t=\frac{|y_{21}|^2}{4g_{11}g_{22}}\cdot\frac{4G_Sg_{11}}{(G_S+g_{11})^2}\cdot\frac{4G_Lg_{22}}{(G_L+g_{22})^2}\cdot\frac{1}{|\delta|^2_{\min}}, \tag{4.3.19}$$

or:

$$\Phi_t=\Phi_{uM}\cdot\Phi_{\mathrm{mm}_1}\cdot\Phi_{\mathrm{mm}_2}\cdot\frac{1}{|\delta|^2_{\min}}. \tag{4.3.20}$$

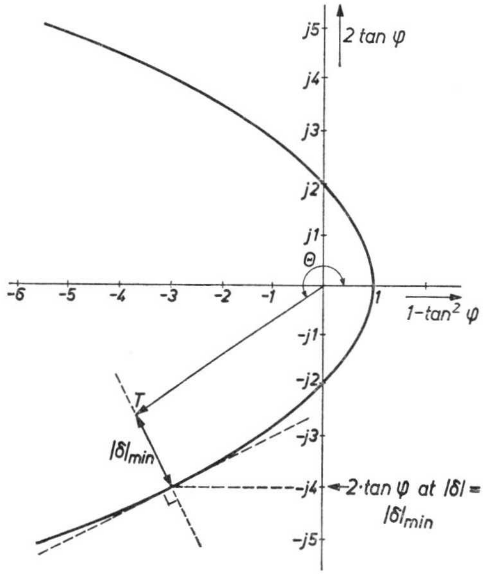


Fig. 4.11. To optimize the transducer gain of an amplifier of which the value of T is fixed for stability reasons or otherwise, the minimum value of $|\delta|$ must be found. This minimum is equal to the shortest distance between the extremity of the vector T and the parabola as indicated by $|\delta|_{min}$ in the figure. The shortest distance between the extremity of T and the parabola is found by constructing a line through this extremity perpendicular to the parabola. By drawing a line parallel to the real axis through the point of intersection of the perpendicular and the parabola the value 2 tan φ at which $|\delta| = |\delta|_{min}$ is found on the vertical axis.

In this expression Φ_{mm1} and Φ_{mm2} denote the mismatch losses at the input and output sides of the transistor if the transistor is taken to be unilateral. (See the footnote on page 51).

4.3.2.2 G_1 and G_2 are variable

In Eq. (4.3.8) the product $G_S G_L$ involves the variables G_1 and G_2 separately. In order to optimize Φ_t with respect to these variables we put (because the optimum value of δ has already been found):

$$\frac{\mathrm{d}}{\mathrm{d}\,G_1}(G_S G_L) = 0\,. \tag{4.3.21}$$

Now:

$$G_S G_L = (G_1 - g_{11})(G_2 - g_{22}),$$
$$= (G_1 - g_{11}) \cdot \left(\frac{A}{G_1} - g_{22}\right). \qquad (4.3.22)$$

Hence we obtain for the optimum value of G_1:

$$G_1 = \sqrt{\frac{g_{11}}{g_{22}} A}\,, \qquad (4.3.23)$$

and for the optimum value of G_2 (because $G_1 G_2 = A$):

$$G_2 = \sqrt{\frac{g_{22}}{g_{11}} A}\,. \qquad (4.3.24)$$

If:

$$N = \left|\frac{y_{21}}{y_{12}}\right|, \qquad (4.3.25)$$

Eq. (4.3.8) may be written:

$$\Phi_t = 4\,\frac{G_S}{G_1} \cdot \frac{G_L}{G_2} \cdot T \cdot N \cdot \frac{1}{|\delta|^2_{min}}, \qquad (4.3.26)$$

taking into account the optimum value of $|\delta|$ obtained in the preceding sub-section.
With:

$$M = |y_{12} y_{21}| \qquad (4.3.27)$$

and Eqs. (4.3.22) to (4.3.24), Eq. (4.3.26) may be written:

$$\Phi_t = 4\left(1 - \sqrt{\frac{T}{M} g_{11} g_{22}}\right)^2 \cdot T \cdot N \cdot \frac{1}{|\delta|^2_{min}}. \qquad (4.3.28)$$

4.3.3 INCLUSION OF TUNED CIRCUITS IN THE AMPLIFIER

In the same way as in sub-section 4.2.6 for the conjugately matched amplifier, tuned circuits may be incorporated in the amplifier with constant T. The admittances $Y_1{}^*$ and $Y_2{}^*$ of these tuned circuits form part of the load and source admittances. Again we assume that the susceptive parts of source and load admittances as required for maximum transducer gain are provided by the tuned circuits. Then the actual source and load admittances may be real and equal to $G_S{}'$ and $G_L{}'$. Now:

$$G_S{}' = G_1 - g_{11} - G_1{}^*,$$

and

$$G_L{}' = G_2 - g_{22} - G_1{}^*.$$

By putting:

$$\left.\begin{aligned} w_1 &= \frac{G_1{}^*}{G_1}, \\ \text{and} \qquad w_2 &= \frac{G_2{}^*}{G_2}, \end{aligned}\right\} \tag{4.3.29}$$

we obtain:

$$G_S' = G_1(1 - w_1) - g_{11}, \tag{4.3.30}$$

and

$$G_L' = G_2(1 - w_2) - g_{22}. \tag{4.3.31}$$

According to sub-section 2.4.3 the transducer gain of the amplifier is reduced by a factor $(1 - w_1)^2(1 - w_2)^2$ due to losses in these tuned circuits.

If the losses in the tuned circuits are denoted by Φ_{i1} and Φ_{i2} respectively, the transducer gain for an amplifier with constant T operating between a source and load with prescribed dampings becomes:

$$\Phi_{t,\,\max} = \Phi_{u\mathrm{M}} \cdot \Phi_{\mathrm{m}_1} \cdot \Phi_{\mathrm{m}_2} \cdot \Phi_{i_1} \cdot \Phi_{i_2} \cdot \frac{1}{|\delta|^2{}_{\min}}, \tag{4.3.32}$$

and for the amplifier in which G_S and G_L may be varied:

$$\Phi_{t,\,\max} = 4\left(1 - \sqrt{\frac{T}{M} g_{11} g_{22}}\right)^2 \cdot T \cdot N \cdot \Phi_{i_1} \cdot \Phi_{i_2} \cdot \frac{1}{|\delta|^2{}_{\min}}. \tag{4.3.33}$$

4.3.4 SOURCE AND LOAD SUSCEPTANCES FOR OPTIMUM TRANSDUCER GAIN

As follows from the preceding sub-sections the susceptive parts of source and load admittances must be given certain values in order to achieve the optimum transducer gain. These susceptances have, however, not been calculated because of the complexity involved (see sub-section 4.3.2.1). If necessary these susceptances can be calculated after a graphical evaluation of tan φ which relates:

$$\tan \varphi = \frac{B_S + b_{11}}{G_S + g_{11}} = \frac{B_L + b_{11}}{G_L + g_{11}}, \tag{4.3.34}$$

in which B_S, G_S, B_L and G_L include the admittances of the input and output tuned circuits.

The value of tan φ for optimum transducer gain for a given value of T and Θ can be determined as shown in Fig. 4.11. Fig. 4.13 (opposite page 113) presents a chart for determining tan φ for any value of T and Θ.

4.3.5 TUNING PROCEDURE FOR OPTIMUM TRANSDUCER GAIN

In single-stage amplifiers in which the source and load susceptances are provided by means of tuned circuits, these tuned circuits can easily be adjusted such that optimum transducer gain is obtained. This can be achieved by repeatedly tuning the output and input tuned circuits for maximum gain of the amplifier. The tuning of both circuits must be carried out repeatedly because the feedback of the transistor alters its output susceptance when tuning the input circuit and vice versa.

4.3.6 COMPARISON OF AMPLIFIER PERFORMANCE TUNED FOR OPTIMUM TRANSDUCER GAIN AND TUNED ACCORDING TO TUNING METHOD A

Comparison of Fig. 2.15 which is valid for a single-stage amplifier with given T tuned according to method A and Fig. 4.11 valid for the amplifier tuned for optimum transducer gain reveals that the only difference between the two amplifiers amounts to having the maximum transducer gain at different frequencies with respect to the tuning frequency. The tuning procedure for optimum transducer gain as described in the preceding sub-section produces the maximum gain at the tuning frequency whereas the (same) maximum in gain occurs at a frequency different from the tuning frequency when the amplifier is tuned according to method A. (see sub-section 2.5.2.1)

4.3.7 EXAMPLE

To illustrate the theory presented in this section we consider a single-stage amplifier which should be designed with a stability factor of $s = 4$ thereby delivering maximum gain. We assume that the transistor to be used has the following parameters:

$$g_{11} = 15\mho \quad \text{m} \qquad b_{11} = -3 \text{ m}\mho$$

$$|y_{12}| = 0.45 \text{ m}\mho \qquad \varphi_{12} = 250^\circ$$

$$|y_{21}| = 16 \text{ m}\mho \qquad \varphi_{21} = 95^\circ$$

$$g_{22} = 0.3 \text{ m}\mho \qquad b_{22} = 1.5 \text{ m}\mho.$$

Then it can be calculated:

$$\Phi_{uM} = 11.5 \text{ dB}, \qquad M = 7.2 \cdot 10^{-6} \mho^2, \qquad N = 36,$$

$$\Theta = 345^\circ \qquad T_g = 1.0 \qquad T = \frac{T_g}{4} = 0.25.$$

Then from the chart of Fig. 4.12, $|\delta|_{min} = 0{\cdot}75$, and from the chart of Fig. 4.13, $\tan \varphi = 0.125$. With Eq. (4.3.28) the transducer gain becomes 11.0 dB. The optimum values of G_1 and G_2 are obtained from Eqs. (4.2.23) and (4.3.24) which yield $G_1 = 38$ m$\mho$ and $G_2 = 0.76$ m$\mho$. Assuming that no tuned circuits are used the source and load dampings become

$$G_S = G_1 - g_{11} = 23 \text{ m℧},$$

$$G_L = G_2 - g_{22} = 0.46 \text{ m℧},$$

and with Eq. (4.3.34) we obtain for the source and load susceptances $B_S = - 1.75$ m℧ and $B_L = - 1.6$ m℧.

4.4 Single-Stage Amplifier with Prescribed Regeneration Coefficients and Prescribed Source Admittance

As already referred to in Section 4.1 for optimum noise performance of an amplifier certain values of real and imaginary parts of the source admittance are required. These values depend on the type of transistor used in the amplifier, its biasing point as well as on the frequency for which the amplifier has to be designed (see Bibliography [4.17]).

The design of the type of amplifier mentioned must thus be carried out taking into account a prescribed value of the source admittance. To optimize the gain of such an amplifier the real and imaginary parts of the load admittance are the only variables. In most amplifiers, however, potentially unstable transistors will be employed. This implies that the regeneration coefficient must also remain constant. This case will be considered in the following sections.

4.4.1 OPTIMIZATION OF TRANSDUCER GAIN

The transducer gain of the amplifier is given by Eqs. (4.3.8) and (4.3.7). Eq. (4.3.8) reads:

$$\Phi_t = 4G_SG_L \cdot \frac{|y_{21}|^2}{G_1{}^2G_2{}^2|\delta|^2}. \tag{4.4.1}$$

For constant value of T the quantities G_1 and hence G_2 are constant because G_S and hence G_L have prescribed values. The only variable is therefore:

$$\delta = \begin{vmatrix} 1 + \mathrm{j} \tan \varphi_1 & T \cdot \exp(\mathrm{j}\Theta) \\ 1 & 1 + \mathrm{j} \tan \varphi_2 \end{vmatrix}. \tag{4.4.2}$$

The transducer gain Φ_t can thus be optimized by finding the minimum value of $|\delta|$. Since $\tan \varphi_2$ is the only variable in δ we may put:

$$\frac{\mathrm{d}}{\mathrm{d}(\tan \varphi_2)} |\delta| = 0. \tag{4.4.3}$$

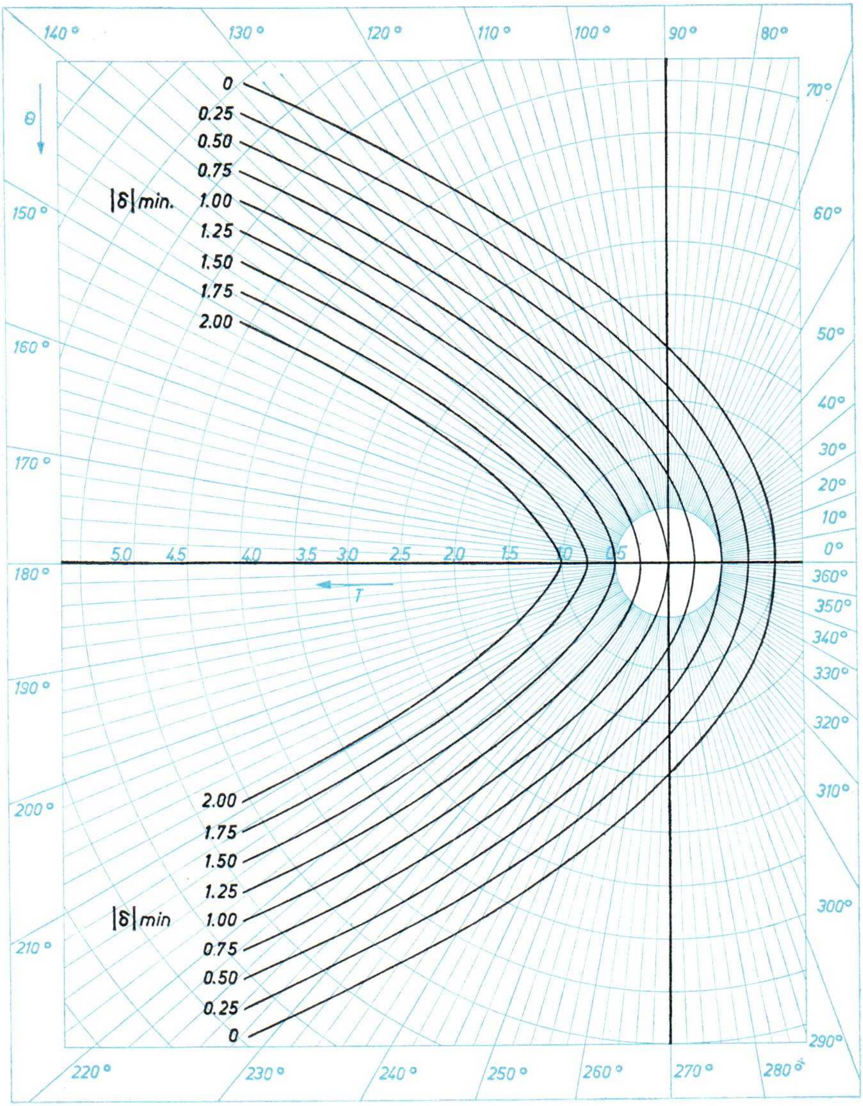


Fig. 4.12 Chart for determining $|\delta|_{min}$ for any value of T and Θ.

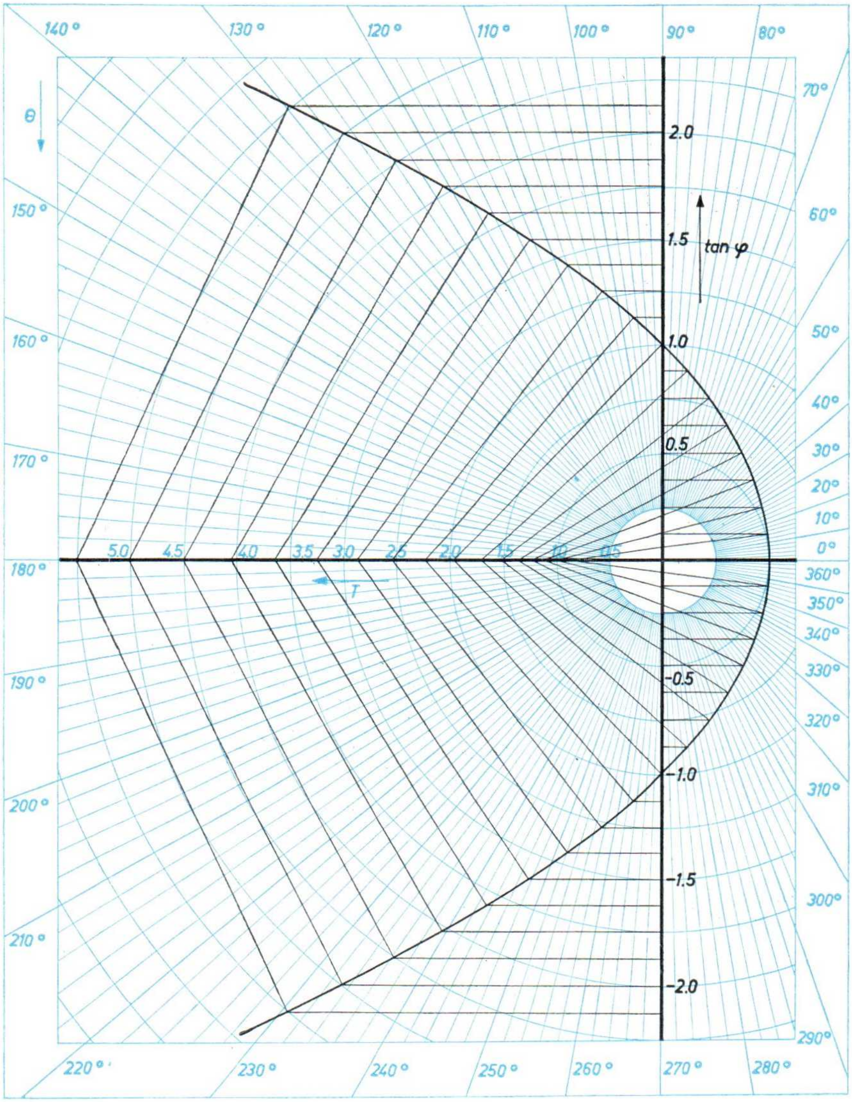


Fig. 4.13 Chart for determining tan φ at $|\delta| = |\delta|_{\min}$ for any value of T and Θ.

This yields for the optimum value of tan φ_2:

$$\tan \varphi_2 = \cos^2 \varphi_1 T \cos \Theta(\tan \Theta - \tan \varphi_1). \tag{4.4.4}$$

4.4.2 OPTIMUM VALUE OF LOAD ADMITTANCE

The value of G_2 follows from

$$G_2 = \frac{T}{G_1} = \frac{T}{G_S + g_{11}}.$$

Then the load damping G_L follows from:

$$G_L = G_2 - g_{22},$$

or:

$$G_L = \frac{T}{G_S + g_{11}} - g_{22}. \tag{4.4.5}$$

The value B_2 then becomes

$$B_2 = G_2 \cdot \tan \varphi_2.$$

This gives for the load susceptance B_L:

$$B_L = B_2 - b_{22}$$

or:

$$B_L = \tan \varphi_2 \cdot \frac{T}{G_S + g_{11}} - b_{22}. \tag{4.4.6}$$

When the amplifier is terminated by an admittance Y_L according to Eq. (4.4.5) and (4.4.6) the transducer gain becomes maximal taking into account the prescribed values of Y_S and T.

BIBLIOGRAPHY

[4.1] G. S. BAHRS, *Stable Amplifiers employing Potentially Unstable Elements*, 1957, I.R.E. Wescon Conv. Record, **1**, Pt. 2, p.185-189.

[4.2] E. F. BOLINDER, *Survey of Some Properties of Linear Networks*, Proc, I.R.E. **45**, p. 70-78, 1957.

[4.3] C. LE CAN, *High-Frequency Selective Fourpole with Complex Feedback*, parts I and II, Unpublished reports of Philips Semiconductor Electrical Development Laboratory, Nijmegen (Netherlands).

[4.4] L. G. CRIPPS and J. A. G. SLATTER, *Amplifier Gain and Stability*, J. Brit. I.R.E. **22**, p. 417, 1961.

[4.5] W. G. GÄRTNER, *Maximum Available Powergain of Linear Four-Poles*, Proc. I.R.E. **46**, p. 375-376, 1958.

[4.6] M. A. KARP, *Power Gain and Stability*, Proc. I.R.E. **45**, p. 339-340, 1957.

[4.7] YU. G. KRIUKOV, *An Analysis of Tuned Transistor Amplifiers with Non-Compensated Feedback*, Radiotekhnika, **16**, No. 8, p. 85-98, 1961.

[4.8] F. B. LLEWELLYN, *Some Fundamental Properties of Transmission Systems,* Proc. I.R.E. **40,** p. 271-283, 1952.

[4.9] J. G. LINVILL and J. F. GIBBONS, *Transistors and Active Circuits,* McGraw-Hill Book Comp., Inc., New York, 1961, Chapter 9.

[4.10] J. G. LINVILL and L. G. SCHIMPF, *The Design of Tetrode Transistor Amplifiers,* Bell Sys. Tech. **35,** p. 813-840, 1956.

[4.11] S. J. MASON, *Power Gain in Feedback Amplifiers,* I.R.E. Trans. Circuit Theory, **CT-1,** p. 20-25, 1954.

[4.12] J. M. ROLLETT, *Stability and Power Gain Invariants of Linear Twoports,* I.R.E. Trans. Circuit Theory, **CT-8**, p. 29-32, 1962.

[4.13] J. M. ROLLETT, *Invariant Stability Parameters,* Proc. I.R.E. **50,** p. 1854-55, 1962.

[4.14] J. O. SCANLAN and J. S. SINGLETON, *The Gain and Stability of Linear Two-Port Amplifiers,* I.R.E. Trans. Circuit Theory **CT-8** p. 240-246, 1962.

[4.15] J. O. SCANLAN and J. S. SINGLETON, *Two-Ports-Maximum Gain for a Given Stability Factor,* I.R.E. Trans. Circuit Theory **CT-9** p. 428-429, 1962.

[4.16] R. SPENCE, *On the Maximum Power Gain of a Two-Port Active Network,* Proc. I.R.E. **50,** p. 1709, 1962.

[4.17] W. SMULDERS, *Noise Properties of Transistors at High Frequencies,* Electronic Applications, Vol. **23**, no. 1 p. 1-25, 1962-63.

[4.18] S. VENKATESWARAN and A. R. BOOTHROYD, *Power Gain and Bandwidth of Tuned Transistor Amplifier Stages,* Trans. I.E.E., Part. C, p. 518-529, 1960.

[4.19] S. VENKATESWARAN, *An Invariant Stability Factor and its Physical Significance,* I.E.E. Monograph No. 468 E, September 1961.

CHAPTER 5

SINGLE-STAGE AMPLIFIER WITH TWO DOUBLE-TUNED BANDPASS FILTERS

5.1 General

In considering single-stage amplifiers comprising two double-tuned bandpass filters, the investigations will be based on the results previously obtained from the analysis of single-stage amplifiers with two single-tuned circuits. For the present analysis use will be made of the four-terminal network representation of the double-tuned bandpass filter as derived in Appendix III. The amplifier may then be considered to consist of three four-terminal networks in cascade, the first and last of which are passive (double-tuned bandpass filters), and the second active (transistor or electron tube). For this chain of four-terminal networks a matrix equation will be derived by means of which the transfer function of the complete amplifier can easily be evaluated. This transfer function then enables important amplifier properties such as stability, transducer gain, amplitude response and envelope delay to be determined.

The analysis of the single-stage amplifier with double-tuned bandpass filters is not only of practical importance in itself, but it also serves as an introduction to the analysis of multi-stage amplifiers comprising double-tuned bandpass filters, to be dealt with in Chapters 7 and 8.

5.2 Single-Stage Amplifier with Parallel-Parallel Tuned Double-Tuned Bandpass Filters

Fig. 5.1 shows a circuit of the single-stage amplifier comprising two double-tuned bandpass filters with parallel-tuned primary and secondary. In this circuit, a transistor in common emitter connection is shown, but any other transistor configuration or an electron tube might be used instead. The amplifier is driven by a current source having an admittance Y_S and loaded by an admittance Y_L.

It is assumed that inductive coupling is used for the double-tuned bandpass filters. However, both the method of analysis and the results obtained are the same if other types of coupling are employed.

In the multi-stage amplifiers to be investigated later, it will prove to be

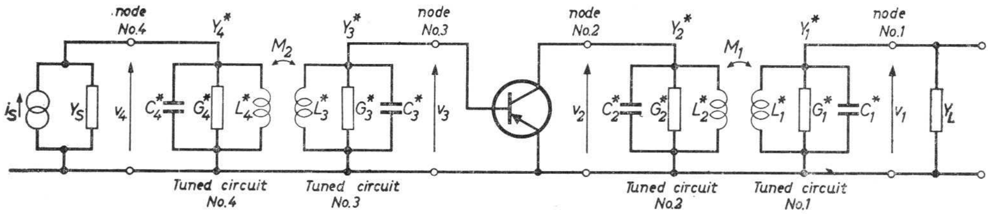


Fig. 5.1. Schematic circuit of a single-stage amplifier with two double-tuned bandpass filters with parallel-tuned primaries and secondaries.

convenient to start the analysis at the output side of the amplifier. For this reason the double-tuned bandpass filters and the resonant circuits forming these bandpass filters in the circuit of Fig. 5.1 are numbered consecutively, starting at the output side of the amplifier. The same procedure is used for numbering the voltages appearing at the terminals of the double-tuned bandpass filters.

By replacing the double-tuned bandpass filters by their equivalent four-terminal networks based on admittance parameters as derived in Appendix III, the equivalent circuit of Fig. 5.2 is obtained. In this circuit the active device is also represented as an admittance parameter equivalent circuit. To distinguish the admittance parameters of the passive and active four-terminal networks capital Y's are used to denote the former, and lower-case y's to denote the latter.

The indices which precede the admittance parameter symbols in Fig. 5.2 indicate the passive or active four-terminal network to which the parameters appertain. The symbol ${}_2Y_{11}$, for example, denotes the input admittance parameter of the penultimate bandpass filter of the amplifier.

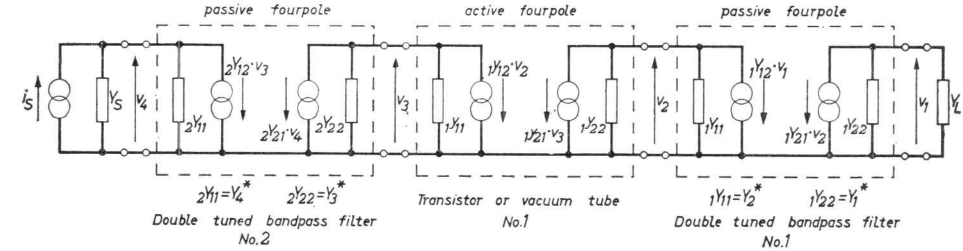


Fig. 5.2. Single-stage amplifier with two double-tuned bandpass filters. The bandpass filters and the active device (transistor or electron tube) are represented by four-terminal equivalent network based on admittance parameters.

Fig. 5.2 can be further simplified by combining the two admittances at the common points of the four-terminal networks into a single admittance. By so doing, the circuit of Fig. 5.3 is obtained, in which:

$$\left.\begin{aligned} Y_1 &= {}_1Y_{22} + Y_L, \\ Y_2 &= {}_1y_{22} + {}_1Y_{11}, \\ Y_3 &= {}_2Y_{22} + {}_1y_{11}, \\ Y_4 &= Y_S \;\; + {}_2Y_{11}. \end{aligned}\right\} \tag{5.2.1}$$

It is thus seen that the admittances Y_1 to Y_4 consist of an inductance, a capacitance and a conductance connected in parallel, forming a single-tuned circuit. According to Appendix II the admittances can then be expressed by:

$$Y = G(1 + \mathrm{j}x). \tag{5.2.2}$$

It is therefore possible to represent the complete single-stage amplifier by an equivalent circuit containing four single-tuned resonant circuits and a number of current sources, as shown in Fig. 5.3.

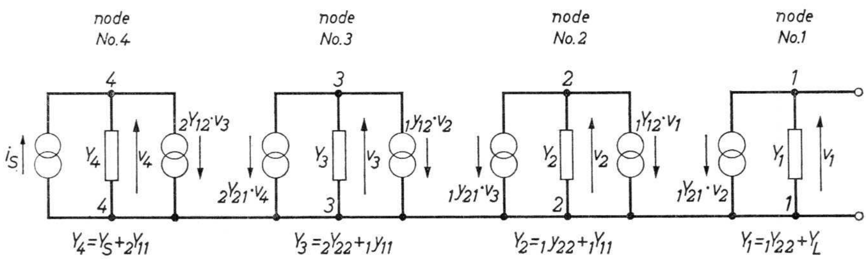


Fig. 5.3. Simplified equivalent circuit of the amplifier of Fig. 5.2.

According to Kirchhoff's first law, the following equations apply to the various nodes of the equivalent circuit of Fig. 5.3, viz,

to node 1: $\quad {}_1Y_{21} \cdot v_2 + Y_1 \cdot v_1 = 0, \qquad (5.2.3)$

to node 2:

$${}_1y_{21} \cdot v_3 + Y_2 \cdot v_2 + {}_1Y_{12} \cdot v_1 = 0, \tag{5.2.4}$$

to node 3:

$${}_2Y_{21} \cdot v_4 + Y_3 \cdot v_3 + {}_1y_{12} \cdot v_2 = 0, \tag{5.2.5}$$

and to node 4:

$$-\, i_S + Y_4 \cdot v_4 + {}_2Y_{12} \cdot v_3 = 0. \tag{5.2.6}$$

These four equations can be combined in a single matrix equation:

$$\begin{Vmatrix} i_S \\ 0 \\ 0 \\ 0 \end{Vmatrix} = \begin{Vmatrix} Y_4 & {}_2Y_{12} & 0 & 0 \\ {}_2Y_{21} & Y_3 & {}_1y_{12} & 0 \\ 0 & {}_1y_{21} & Y_2 & {}_1Y_{12} \\ 0 & 0 & {}_1Y_{21} & Y_1 \end{Vmatrix} \cdot \begin{Vmatrix} v_4 \\ v_3 \\ v_2 \\ v_1 \end{Vmatrix}. \qquad (5.2.7)$$

The first matrix of the right hand side of this equation is the definite admittance matrix of the amplifier circuit presented in Fig. 5.3. The method of deriving this matrix employed is in fact equivalent to that of Appendix I, Section 4.

As follows from the preceding chapters the determinant of the square matrix of Eq. (5.2.7) is important in analyzing the amplifier with respect to stability, gain and frequency response. This determinant, which will be denoted by Δ can be simplified by separating out the G's, making use of Eq. (5.2.2). Hence:

$$\Delta = G_1G_2G_3G_4 \begin{vmatrix} 1+jx_4 & \dfrac{{}_2Y_{12}}{G_4} & 0 & 0 \\ \dfrac{{}_2Y_{21}}{G_3} & 1+jx_3 & \dfrac{{}_1y_{12}}{G_3} & 0 \\ 0 & \dfrac{{}_1y_{21}}{G_2} & 1+jx_2 & \dfrac{{}_1Y_{12}}{G_2} \\ 0 & 0 & \dfrac{{}_1Y_{21}}{G_1} & 1+jx_1 \end{vmatrix}, \qquad (5.2.8)$$

Eq. (5.2.8) can be further simplified by dividing each column of the determinant by the Y_{21} (or y_{21}) term it contains, and multiplying the corresponding row (of equal index) by this same term, which gives:

$$\Delta = G_1G_2G_3G_4 \begin{vmatrix} 1+jx_4 & \dfrac{{}_2Y_{12}\cdot{}_2Y_{21}}{G_3G_4} & 0 & 0 \\ 1 & 1+jx_3 & \dfrac{{}_1y_{12}\cdot{}_1y_{21}}{G_2G_3} & 0 \\ 0 & 1 & 1+jx_2 & \dfrac{{}_1Y_{12}\cdot{}_1Y_{21}}{G_1G_2} \\ 0 & 0 & 1 & 1+jx_1 \end{vmatrix}. \qquad (5.2.9)$$

In Eq. (5.2.9) the index y in Δ_y indicates that Δ is obtained from the analysis using admittance parameters.

By putting the reduced determinant of Eq. (5.2.9) equal to δ_y we obtain:

$$\Delta_y = G_1 G_2 G_3 G_4 \cdot \delta_y, \tag{5.2.10}$$

and

$$\delta_y = \begin{vmatrix} 1 + jx_4 & \dfrac{{}_2Y_{12} \cdot {}_2Y_{21}}{G_3 G_4} & 0 & 0 \\ 1 & 1 + jx_3 & \dfrac{{}_1y_{12} \cdot {}_1y_{21}}{G_2 G_3} & 0 \\ 0 & 1 & 1 + jx_2 & \dfrac{{}_1Y_{12} \cdot {}_1Y_{21}}{G_1 G_2} \\ 0 & 0 & 1 & 1 + jx_1 \end{vmatrix}. \tag{5.2.11}$$

5.3 Single-Stage Amplifier with Two Parallel-Series Tuned Double-Tuned Bandpass Filters

Fig. 5.4 shows a circuit of a single-stage amplifier comprising two double tuned bandpass filters with parallel-tuned primaries and series-tuned secondaries. The amplifier is driven by a current source with admittance Y_S and loaded by an impedance Z_L.

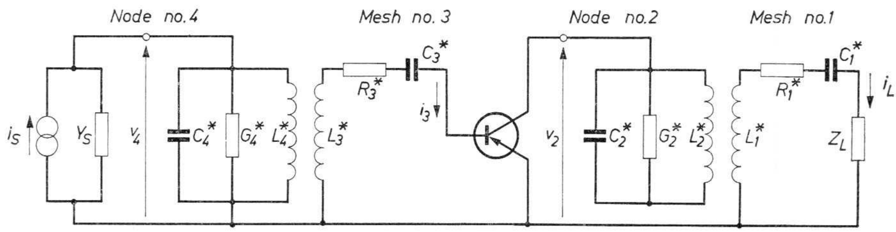


Fig. 5.4. Schematic circuit of a single-stage amplifier with two double-tuned bandpass filters with parallel-tuned primaries and series-tuned secondaries.

To analyze this amplifier the double-tuned bandpass filters are replaced by the equivalent four-terminal networks based on K-parameters as derived in Appendix III and the transistor is replaced by an H-parameter equivalent circuit. Then the output side of the K-network and the input side of the H-network form a series connection of two voltage sources and two impedances. To obtain a uniform direction of the current in this mesh it is necessary to assume a direction opposite to that of the adopted sign convention (see

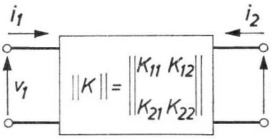


Fig. 5.5. *K*-four-terminal network with current directions according to the adopted sign convention. To ease the amplifier analysis the output current should have a reversed direction.

Chapter 1 and Fig. 5.5) for either the output current of the *K*-network or the input current of the *H*-network. Here an opposite direction will be assumed for the output current of the *K*-four-terminal network representing the double-tuned bandpass filters. The *K*-matrix of this network then becomes:

$$K = \begin{Vmatrix} K_{11} & -K_{12} \\ K_{21} & -K_{22} \end{Vmatrix} \tag{5.3.1}$$

whereas its equivalent circuit becomes as shown in Fig. 5.6.

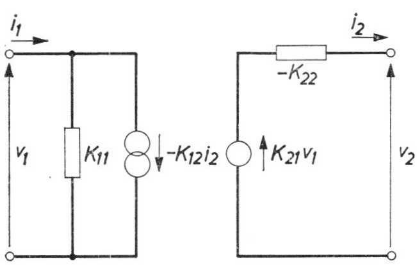


Fig. 5.6. Equivalent circuit for the *K*-four-terminal network with reversed direction of output current.

With these equivalent four-terminal networks the circuit of Fig. 6.4 becomes as shown in Fig. 5.7. By putting:

$$\left.\begin{aligned} Z_1 &= -\,{}_1K_{22} + Z_L, \\ Y_2 &= \ {}_1h_{22} + {}_1K_{11}, \\ Z_3 &= -\,{}_2K_{22} + {}_1h_{11}, \\ \text{and} \qquad Y_4 &= \ Y_S + {}_2K_{11}, \end{aligned}\right\} \tag{5.3.2}$$

the circuit of Fig. 5.7 may further be simplified to that presented in Fig. 5.8. Furthermore, according to Appendix II:

$$\left.\begin{aligned} Z &= R(1 + \mathrm{j}x), \\ \text{and} \qquad Y &= G(1 + \mathrm{j}x). \end{aligned}\right\} \tag{5.3.3}$$

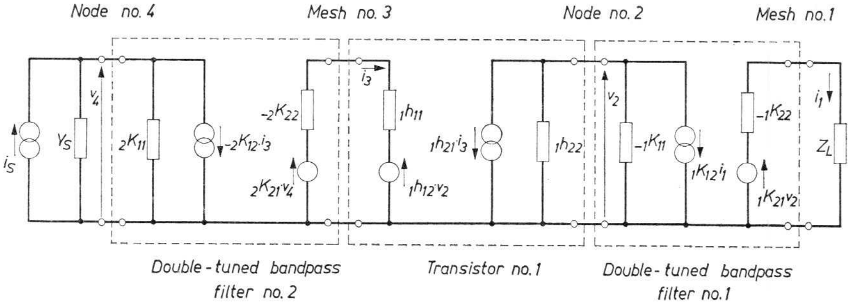


Fig. 5.7. Single-stage amplifier with two double-tuned bandpass filters according to Fig. 5.4.

For the equivalent amplifier circuit of Fig. 5.8 the following equations may be written down:

for mesh 1:

$$ {}_1K_{21}v_2 + Z_1 i_1 = 0, \tag{5.3.4}$$

for node 2:

$$ {}_1h_{21} \cdot i_3 + Y_2 v_2 - {}_1K_{12} \cdot i_1 = 0, \tag{5.3.5}$$

for mesh 3:

$$ -{}_2K_{21} \cdot v_4 + Z_3 \cdot i_3 + {}_1h_{12} \cdot v_2 = 0, \tag{5.3.6}$$

and for node 4:

$$ Y_4 \cdot v_4 - {}_2K_{12} \cdot i_3 = i_S. \tag{5.3.7}$$

Again these four equations may be combined in a single matrix equation:

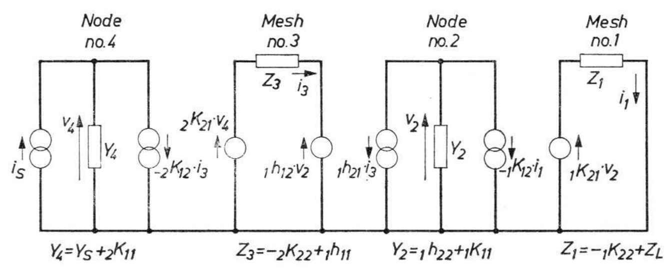


Fig. 5.8. Simplified equivalent diagram of the amplifier of Fig. 5.7.

$$\begin{Vmatrix} i_S \\ 0 \\ 0 \\ 0 \end{Vmatrix} = \begin{Vmatrix} Y_4 & -{}_2K_{12} & 0 & 0 \\ -{}_2K_{21} & Z_3 & {}_1h_{12} & 0 \\ 0 & {}_1h_{21} & Y_2 & -{}_1K_{12} \\ 0 & 0 & -{}_1K_{21} & Z_1 \end{Vmatrix} \cdot \begin{Vmatrix} v_4 \\ i_3 \\ v_2 \\ i_1 \end{Vmatrix}. \qquad (5.3.8)$$

The determinant of Eq. (5.3.8) which will be denoted by Δ_h can be simplified using Eq. (5.3.3) to:

$$\Delta_h = R_1G_2R_3G_4 \begin{vmatrix} 1+\mathrm{j}x_4 & \dfrac{-{}_2K_{12}}{G_4} & 0 & 0 \\ \dfrac{-{}_2K_{21}}{R_3} & 1+\mathrm{j}x_3 & \dfrac{{}_1h_{12}}{G_3} & 0 \\ 0 & \dfrac{{}_1h_{21}}{G_2} & 1+\mathrm{j}x_2 & \dfrac{-{}_1K_{12}}{G_2} \\ 0 & 0 & \dfrac{-{}_1K_{21}}{R_1} & 1+\mathrm{j}x_1 \end{vmatrix}. \qquad (5.3.9)$$

or, by putting the reduced determinant equal to δ_h:

$$\Delta_h = R_1G_2R_3G_4 \cdot \delta_h. \qquad (5.3.10)$$

By rearranging the elements of δ_h:

$$\delta_h = \begin{vmatrix} 1+\mathrm{j}x_4 & \dfrac{{}_2K_{12}\cdot{}_2K_{21}}{R_3G_4} & 0 & 0 \\ 1 & 1+\mathrm{j}x_3 & \dfrac{{}_1h_{12}\cdot{}_1h_{21}}{G_2R_3} & 0 \\ 0 & 1 & 1+\mathrm{j}x_2 & \dfrac{{}_1K_{12}\cdot{}_1K_{21}}{R_1G_2} \\ 0 & 0 & 1 & 1+\mathrm{j}x_1 \end{vmatrix}. \qquad (5.3.11)$$

5.4 The Reduced Determinant

The reduced determinants δ as defined by Eqs. (5.2.11) and (5.3.11) may be further simplified as follows: According to Appendix III, Eqs. (III.1.15) and (III.1.22) we have:

$$Y_{12} = Y_{21} = \mathrm{j}q\sqrt{G_pG_s}, \qquad (5.4.1)$$

and:

$$K_{12} = - K_{21} = - q \sqrt{G_p R_s}, \tag{5.4.2}$$

in which the indices p and s refer to primary and secondary of a double-tuned bandpass filter. With Eqs. (5.4.1) and (5.4.2):

$$\frac{Y_{12}Y_{21}}{G_p G_s} = \frac{K_{12}K_{21}}{G_p R_s} = - q^2 . \tag{5.4.3}$$

According to Chapter 2, sub-section 2.1.2, the term $\frac{y_{12}y_{21}}{G_2 G_3}$ equals the complex regeneration coefficient $T_y \cdot \exp(\mathrm{j}\Theta_y)$ in the Y-matrix environment. Also (see sub-section 2.1.3):

$$\frac{h_{12}h_{21}}{G_2 R_3} = T_h \cdot \exp(\mathrm{j}\Theta_h) .$$

Then the reduced determinant becomes:

$$\delta\gamma = \begin{vmatrix} 1 + \mathrm{j}x_4 & - q_2{}^2 & 0 & 0 \\ 1 & 1 + \mathrm{j}x_3 & T_\gamma \cdot \exp(\mathrm{j}\Theta_\gamma) & 0 \\ 0 & 1 & 1 + \mathrm{j}x_2 & - q_1{}^2 \\ 0 & 0 & 1 & 1 + \mathrm{j}x_1 \end{vmatrix} . \tag{5.4.4}$$

in which the index γ refers to either the Y or H-matrix environments [1]).

5.5 The Transfer Function of the Amplifier

The transfer function of an amplifier is defined as the ratio between a characteristic output parameter and a characteristic input parameter. For the amplifier circuit shown in Fig. 5.1 these characteristic parameters are v_1 and i_S respectively. Hence the transfer function equals the forward transfer impedance or transimpedance Z_t of the amplifier, i.e.:

$$Z_t = \frac{v_1}{i_S} . \tag{5.5.1}$$

For the circuit represented by Fig. 5.4 the characteristic quantities are i_1 and i_S. The transfer function therefore equals the forward transfer current ratio or current gain H_t [2]) of the amplifier. Hence:

---

[1]) It will be obvious that the index γ may also refer to the Z or K matrix environments provided the parameters of the transistor(s) and double-tuned bandpass filters are expressed in the appropriate matrix environments.

[2]) See note on page 24.

$$H_t = \frac{i_1}{i_S}. \tag{5.5.2}$$

To find Z_t for the amplifier with parallel-tuned double-tuned bandpass filters first v_1 is calculated using Eqs. (5.2.7) and (5.2.8). This gives:

$$v_1 = \frac{1}{\Delta_y} \begin{vmatrix} Y_4 & {}_2Y_{12} & 0 & i_S \\ {}_2Y_{21} & Y_3 & {}_1y_{12} & 0 \\ 0 & {}_1y_{21} & Y_2 & 0 \\ 0 & 0 & {}_1Y_{21} & 0 \end{vmatrix}. \tag{5.5.3}$$

It then follows for Z_t using Eqs. (5.2.10), (5.4.1), and (5.4.4):

$$Z_t = -\frac{{}_1y_{21} \cdot q_1 q_2}{\sqrt{G_1 G_2 G_3 G_4} \cdot \delta_y}. \tag{5.5.4}$$

In an analogous way it follows for H_t of the amplifier with parallel-series tuned double-tuned bandpass filters using Eqs. (5.3.10), (5.4.2), and (5.4.4):

$$H_t = \frac{{}_1h_{21} \cdot q_1 \cdot q_2}{\sqrt{R_1 G_2 R_3 G_4} \cdot \delta_h}. \tag{5.5.5}$$

Expressions (5.5.4) and (5.5.5) reveal that the factor δ_γ given by Eq. (5.4.4) is the only frequency-dependent part of the transimpedance function [1]. Furthermore, the factor δ_γ comprises the regeneration coefficient of the transistor; stability, transducer gain, amplitude response and envelope delay depend on the magnitude of this coefficient.

5.6 Stability

5.6.1 BOUNDARY OF STABILITY

In the single-stage amplifier with two double-tuned bandpass filters to be considered here, instability occurs as soon as the transfer function as given by Eq. (5.5.4) or Eq. (5.5.5) becomes infinite. This will be the case when δ_γ becomes zero. The amplifier is then said to be at the boundary of stability. Therefore, at the boundary of stability:

---

[1] The forward transfer immittance γ_{21} of the active four-pole is assumed to be frequency-independent with respect to modulus and argument (see Chapter 1).

$$\delta = \begin{vmatrix} 1+\mathrm{j}x_4 & -q_2{}^2 & 0 & 0 \\ 1 & 1+\mathrm{j}x_3 & T_{g1}\cdot\exp(\mathrm{j}\Theta) & 0 \\ 0 & 1 & 1+\mathrm{j}x_2 & -q_1{}^2 \\ 0 & 0 & 1 & 1+\mathrm{j}x_1 \end{vmatrix} = 0\,, \quad (5.6.1)$$

in which T_{g1} is the value of the regeneration coefficient on this boundary [1]). It can be calculated that:

$$T_{g1}\cdot\exp(\mathrm{j}\Theta) = \left(1+\mathrm{j}x_2+\frac{q_1{}^2}{1+\mathrm{j}x_1}\right)\left(1+\mathrm{j}x_3+\frac{q_2{}^2}{1+\mathrm{j}x_4}\right). \quad (5.6.2)$$

The right-hand side of this expression consists of the product of the reduced immittances presented to the transistor by the bandpass filters at its output and input terminals respectively (see Section 2.1 of Appendix III). This is analogous to the case of the single-stage amplifier with two single-tuned circuits. By putting $q^2 = 0$ in Eq. (5.6.2), Eq. (2.2.2) is obtained.

Working out the right-hand side of Eq. (5.6.2) gives:

$$T_{g1}\exp(\mathrm{j}\Theta) = 1 - x_2x_3 + q_1{}^2\frac{1+x_1x_3}{1+x_1{}^2} + q_2{}^2\frac{1+x_2x_4}{1+x_4{}^2} + q_1{}^2q_2{}^2\frac{1-x_1x_4}{(1+x_1{}^2)(1+x_4{}^2)} +$$
$$+\mathrm{j}\left\{x_2+x_3+q_1{}^2\frac{x_3-x_1}{1+x_1{}^2}+q_2{}^2\frac{x_2-x_4}{1+x_4{}^2}-q_1{}^2q_2{}^2\frac{x_1+x_4}{(1+x_1{}^2)(1+x_4{}^2)}\right\}. \quad (5.6.3)$$

If all circuits are assumed to be tuned synchronously, all values of x disappear at the tuning frequency, and the locus of $T_{g1}\cdot\exp(\mathrm{j}\Theta)$ plotted in the complex plane will by symmetrical with respect to the real axis.

In order to calculate the boundary of stability of the amplifier, it will be assumed that the geometrical means of the primary and secondary quality factors of both bandpass filters are identical. Since all values of β are identical, it is permissible to put:

$$\sqrt{x_1x_2} = \sqrt{x_3x_4} = x\,. \quad (5.6.4)$$

It is now convenient to introduce

$$\frac{Q_1}{Q_2} = \frac{x_1}{x_2} = r_1\,,$$

1) The suffix γ has been omitted here for reasons of simplicity in writing the various equations.

whence:

$$x_1 = x\sqrt{r_1}, \qquad \text{and} \qquad x_2 = x/\sqrt{r_1}, \tag{5.6.5}$$

and, similarly:

$$\frac{Q_3}{Q_4} = \frac{x_3}{x_4} = r_2,$$

whence:

$$x_3 = x\sqrt{r_2}, \qquad \text{and} \qquad x_4 = x/\sqrt{r_2}. \tag{5.6.6}$$

Substitution of these expressions in Eq. (5.6.3) gives:

$$\begin{aligned} T_{g1}\cdot\exp(\mathrm{j}\Theta) = {} & 1 - x^2\sqrt{r_2/r_1} + q_1^2\frac{1+x^2\sqrt{r_1r_2}}{1+x^2r_1} + q_2^2\cdot\frac{1+x^2/\sqrt{r_1r_2}}{1+x^2/r_2} + \\ & + q_1^2q_2^2\frac{1-x^2\sqrt{r_1/r_2}}{(1+x^2r_1)(1+x^2/r_2)} + \\ & + \mathrm{j}x\left\{\frac{1}{\sqrt{r_1}} + \sqrt{r_2} + q_1^2\frac{\sqrt{r_2}-\sqrt{r_1}}{1+x^2r_1} + q_2^2\frac{\frac{1}{\sqrt{r_1}}-\frac{1}{\sqrt{r_2}}}{1+x^2/r_2} - q_1^2q_2^2\frac{\sqrt{r_1}+\frac{1}{\sqrt{r_2}}}{(1+x^2r_1)(1+x^2/r_2)}\right\}. \end{aligned} \tag{5.6.7}$$

The latter expression enables $T_{g1}\cdot\exp(\mathrm{j}\Theta_1)$ to be calculated with x as the independent variable and to be plotted in the complex plane. This has been done in Figs. 5.9 and 5.10 for the various cases tabulated below.

graph	curve	r_1	r_2	$q_1^2 = q_2^2 = q^2$
Fig. 5.9	*A*	1	1	0.5
Fig. 5.9	*B*	1	1	1
Fig. 5.9	*C*	1	1	2
Fig. 5.10	*D*	2	2	1
Fig. 5.10	*E*	2	0.5	1
Fig. 5.10	*F*	0.5	2	1
Fig. 5.9, Fig. 5.10	*G*	(two single-tuned circuits)		

The coupling factors q^2 of both double-tuned bandpass filters are assumed to be equal. In Figs. 5.9 and 5.10 $T_{g1}\cdot\exp(\mathrm{j}\Theta_1)$ has been plotted only for positive values of x, because the corresponding curves for negative values of x are image-symmetrical to the former with respect to the real axis.

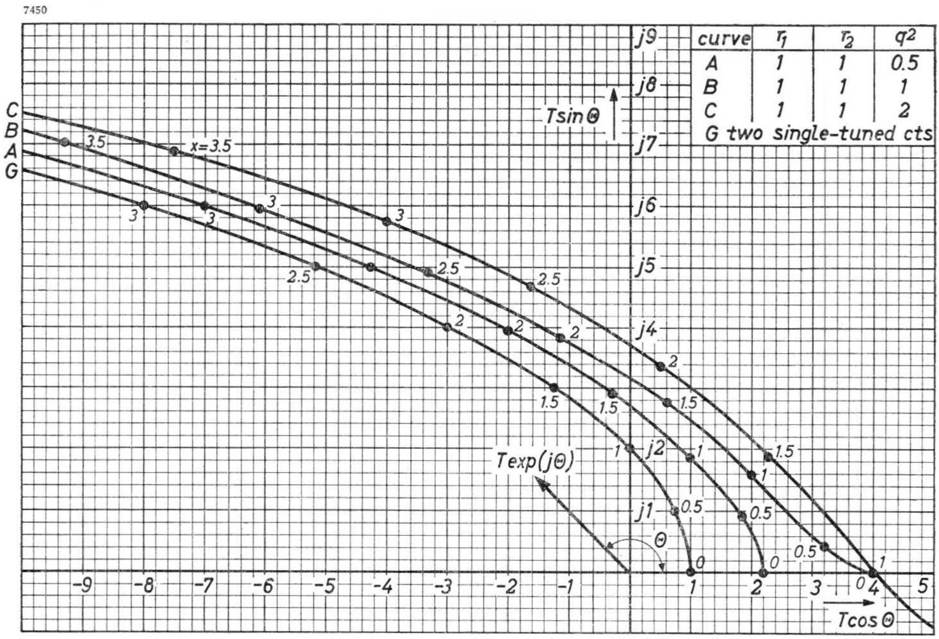


Fig. 5.9. Boundaries of stability of a single-stage amplifier with two double-tuned bandpass filters. The curves clearly show the influence of the value of q^2 on the boundary. For the sake of comparison curve G, representing the boundary of stability for a single-stage amplifier with two identical single-tuned circuits: ($q^2 = 0$), has also been plotted. Various values of x are indicated along the curves. Only the upper halves of the curves are drawn since the curves are symmetrical with respect to the real axis.

The curves in Figs. 5.9 and 5.10 thus represent the boundaries of stability for the single-stage amplifier with two double-tuned bandpass filters for several different cases. For the sake of comparison the boundary of stability for a single-stage amplifier with two identical single-tuned circuits has also been plotted in these graphs (curves G). All boundaries of stability for the stage with double-tuned bandpass filters are seen to lie outside the boundary for the stage with two single-tuned circuits. Fig. 5.9 moreover shows that T_g increases with the value of q^2 (cf. curves A, B and C). Fig. 5.10 further reveals that, when the quality factors of the primary and secondary are so chosen that circuits 1 and 4 have the highest quality factors (curve F), T_g assumes a larger value than when circuits 2 and 3 have the highest quality factors (curve E).

According to Eq. (5.6.7), the angle Θ corresponds to the argument of the right-hand side of this expression. Because Θ is a parameter which depends exclusively on the properties of the transistor with which the amplifier is equipped, it will be most useful to express T_g as a function of Θ. This has

7451

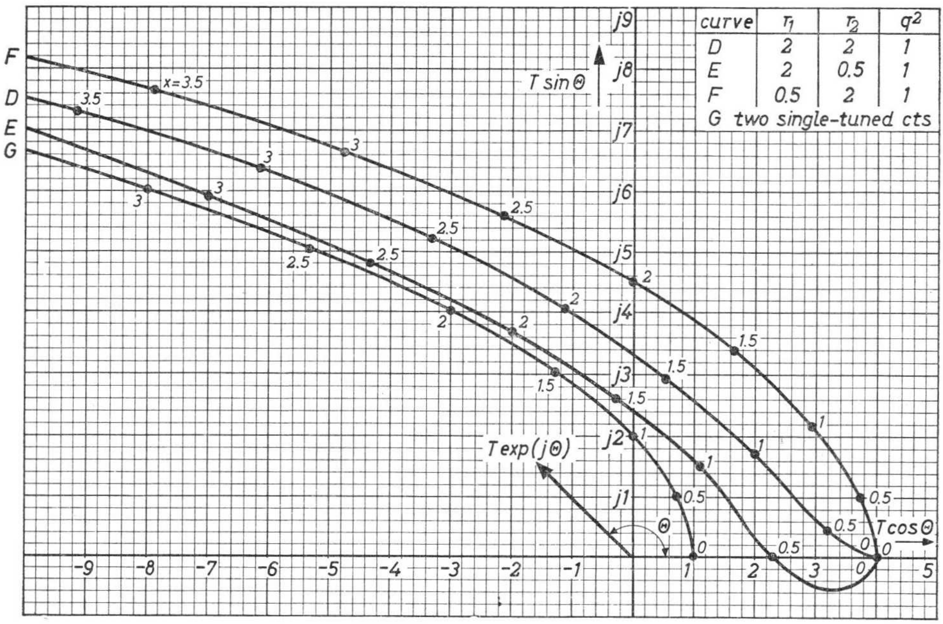


Fig. 5.10. As Fig. 5.9, but for different values of r_1 and r_2. These graphs clearly show the influence of these parameters.

been done in Figs. 5.11 and 5.12 for the amplifier under consideration for several values of q^2 and r. A logarithmic scale has been used for T_g in order to obtain the same relative accuracy for small and large values of T_g.

5.6.2 GRAPHICAL METHOD FOR DETERMINING THE BOUNDARY OF STABILITY

In the preceding sub-section the boundary of stability of the amplifier configuration in question has been considered using an analytical way of approach. There is, however, also a graphical method to determine this boundary. This method will prove to be very important in some specialized cases and, moreover, will be of help in understanding the stability problem in general.

Using Eqs. (III.2.8) and (III.2.9) of Appendix III, Eq. (5.6.2) can be written as:

$$T_{g1} \cdot \exp(\mathrm{j}\Theta) = y_{i1} \cdot y_{o2} = k_{i1} \cdot k_{i2}. \tag{5.6.8}$$

In the following considerations, which will lead to the graphical method for determining T_g, only the admittance matrix notation will be used. For the

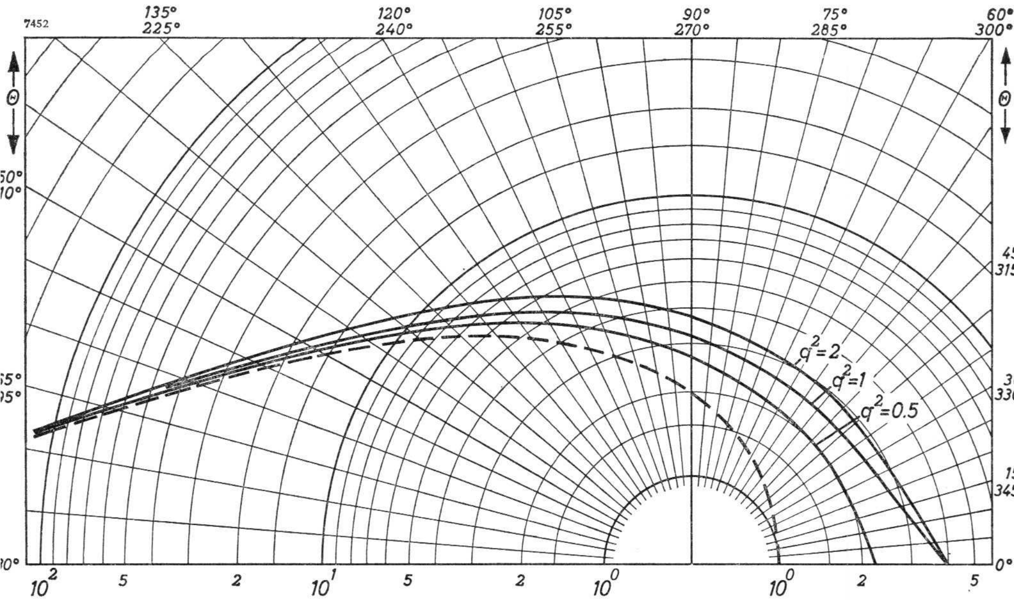


Fig. 5.11. Boundaries of stability of a single-stage amplifier with two identical double-tuned bandpass filters for $r = 1$ and several values of $q^2 = (kQ)^2$. For the sake of comparison the boundary of stability of a single-stage amplifier with two single-tuned circuits has also been plotted (curve in broken line).

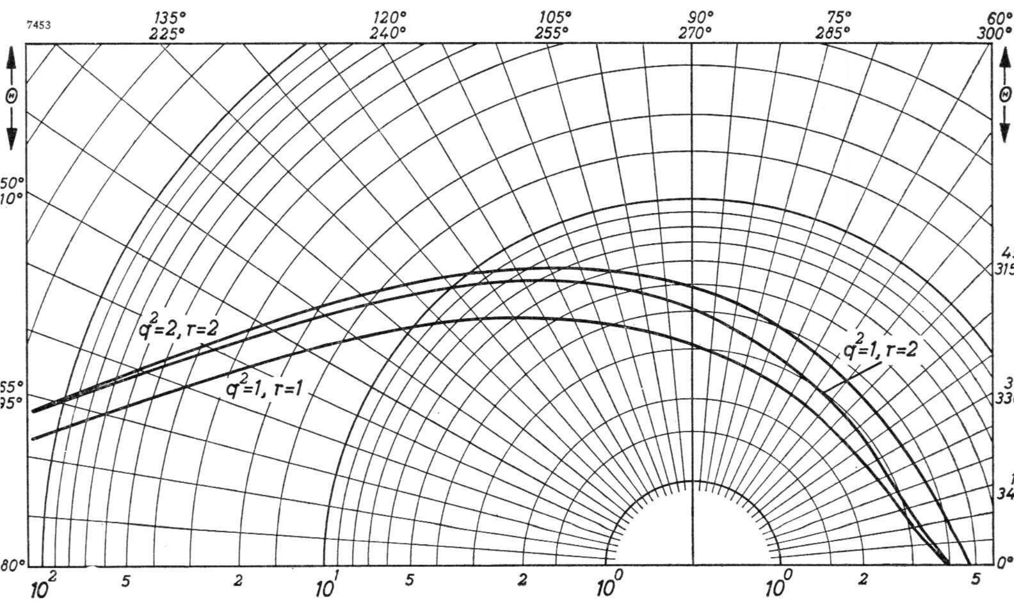


Fig. 5.12. As Fig. 5.11, but for different values of both q and r.

hybrid matrix notation corresponding results can be obtained by means of analogies. Eq. (5.6.8) may be written as:

$$T_{g1} = |y_{i1}| \cdot |y_{o2}| \tag{5.6.9}$$

and

$$\Theta - (\varphi_1 + \varphi_2) = 0 \pm k \cdot 2\pi, \quad k = 0, 1, 2, \ldots \tag{5.6.10}$$

in which φ_1 and φ_2 are the phase angles of y_{i1} and y_{o2} at the frequency at which instability will occur. If conditions (5.6.9) and (5.6.10) are satisfied, the internal loop gain of the amplifier stage is real and equal to unity. In Fig. 5.13 condition (5.6.10) is shown for two values of Θ. It follows that for values of Θ in the first or the second quadrant both φ_1 and φ_2 will be positive whereas for values of Θ in the third or the fourth quadrant φ_1 and φ_2 will be negative.

The graphical construction for T_{g1} is based on the fact that the phase shifts φ_1 and φ_2 necessary to fulfil condition (5.6.10) must be provided by y_{i1} and y_{o2} at the same frequency. In Fig. 5.14 such a construction is presented. The construction of the diagrams for y_{i1} and y_{o2} is carried out according to the method given in Appendix III, sub-section III.2.1.

It is assumed that $\Theta = 250°$; then $\varphi_1 + \varphi_2 = 110°$ and both phase shifts will be negative. This implies that instability will occur at a frequency below the resonant frequencies of the (synchronously tuned) double-tuned bandpass filters i.e. at negative values of the normalized detunings x_2 and x_3.

Furthermore, it will be assumed that the tuned circuits of which the two double-tuned bandpass filters are composed are identical; thus $x_1 = x_2 = x_3 = x_4 = x$, $r = 1$. The coupling factor of the bandpass filter at the output

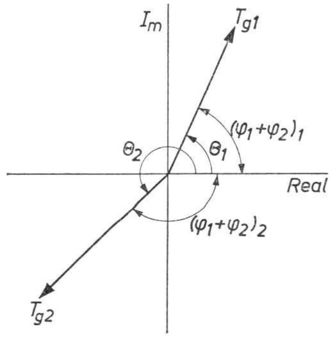


Fig. 5.13. At the boundary of stability of the amplifier the internal loopgain must be real. This is the case if $\Theta + \varphi_1 + \varphi_2 = 0 \pm k \, . \, 2\pi$ ($k = 0, 1, 2, \ldots$) which condition is shown for two values of Θ.

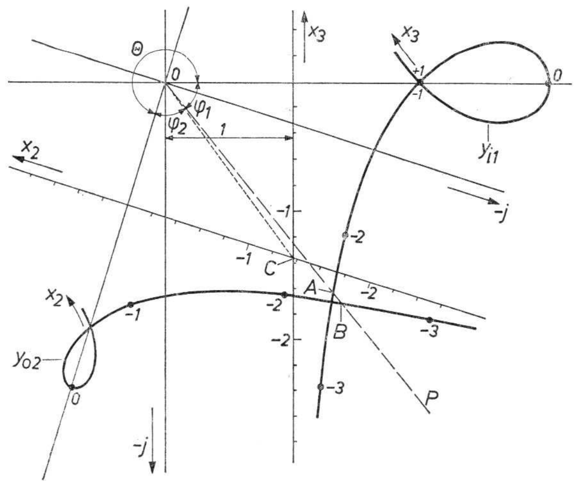


Fig. 5.14. Graphical construction for determining T_g of a single-stage amplifier with two double-tuned bandpass filters. Both bandpass filters are assumed to be identical except for the coupling factor q^2. For bandpass filter 1 (at the output terminals of the transistor), $q^2 = 2$ and for bandpass filter 2, $q^2 = 1.5$. For the angle Θ indicated, T_g equals the product of the sections OA and OB of the line OP. The line OP is drawn through the pole O such that it intersects the y_{i1} and y_{o2} diagrams at the same detuning ($x = -2.18$).

terminals of the transistor (bandpass filter No. 1) is assumed at $q_1{}^2 = 2.0$ whereas that of bandpass filter No. 2 equals $q_2{}^2 = 1.5$.

In Fig. 5.14 the diagram for y_{i1} has been drawn in a normal position. The diagram for y_{o2} has been constructed using the same pole O as for the diagram for y_{i1}. Furthermore the real axis of the y_{o2} diagram has been turned through an angle Θ and the y_{o2} diagram itself has been reflected with respect to this real axis. This means that $+\mathrm{j}$ and $-\mathrm{j}$ are interchanged. Thus the real axes of both digrams form an angle $360 - \Theta = \varphi_1 + \varphi_2$ and the parts of the respective diagrams for negative values of x intersect.

A line OP is drawn through the common pole O in such a way that it intersects the y_{i1} and y_{o2} curves at the same frequency. Then the line OA equals $|y_{i1}|$ and the line OB equals $|y_{o2}|$ at the frequency at which instability will occur. Because of the synchronous tuning of the double-tuned bandpass filters this happens at $x_2 = x_3 = x = 2.18$. With Eq. (5.6.9):

$$T_{g1} = OA \cdot OB. \tag{5.6.11}$$

Taking into account the proper scale factor we obtain from Fig. 5.14:

$$T_{g1} = 4.7.$$

In Fig. 5.14 the construction for T_{g1} for the case of a single-stage amplifier with two single-tuned bandpass filters is also carried out. This yields $T_{g1} = (OC)^2$, which equals $T_{g1} = 2.9$.

The construction for T_g as presented in Fig. 5.14 may be carried out for a single-stage amplifier with two double-tuned bandpass filters which need not be identical. In the case of non-synchronous tuning, the construction is also possible if the polar diagrams of y_{i1} or y_{o2} are provided with a frequency scale.

In fact the graphical determination of T_{g1} may be carried out for any network for which polar plots of y_{i1} and y_{o2} can be constructed. This renders it very useful, especially if in complicated cases the value of T_g is required for a limited number of values for Θ. In Chapter 12 we will demonstrate this when dealing with non-ideal transformers used to connect the bandpass filter terminals to the transistor terminals.

5.6.3 STABILITY FACTOR

By means of either the graphs of Figs. 5.9 to 5.12 or Eq. 5.6.3 as presented in sub-section 5.6.1 or the graphical method of sub-section 5.6.2 the value of T_{g1} can be ascertained for the single-stage amplifier.

As pointed out in sub-section 2.2.4, practical amplifiers should be designed with a certain margin of stability. For this purpose a stability factor s was introduced that relates the magnitude of the regeneration coefficient T on which the design of the amplifier is based, to the value of the boundary of stability T_g:

$$T = \frac{T_g}{s}. \tag{5.6.12}$$

The same argument holds for the single-stage amplifier with two double-tuned bandpass filters, the only difference being that the curve representing T_g as a function of Θ has a different shape from that given in Section 2.2. However, this is of importance only for the value of s, because for this type of amplifier also the value of T is chosen by considering the shape of the amplitude response and envelope delay characteristics for several values of T (cf. sub-section 2.5.2.2). It is therefore by no means certain that the exact value of T_{g1} as can be determined using the methods of sub-sections 5.6.1 or 5.6.2 is actually required for designing an amplifier.

It will often be sufficient to know the approximate value of the stability factor and hence, only an approximate value of T_g is required. A very rough approximation of the boundary of stability for this type of amplifier is obtained by using that for the single-stage amplifier considered in Chapter 2.

5.7 Tuning Procedures

In Section 2.3, dealing with single-stage amplifiers having single-tuned circuits, it has been pointed out that distinction should be made between three practical methods of aligning. The consequences of these tuning methods on the performance of the single-stage amplifier with double-tuned bandpass filters are dealt with in this section.

5.7.1 TUNING PROCEDURES FOR AN AMPLIFIER IN THE *Y*-MATRIX ENVIRONMENT

The type of amplifier analyzed in the chapter using the *Y*-matrix representation comprises double-tuned bandpass filters with parallel-tuned primary and secondary. In the following sub-sections this type of amplifier will be considered with regard to either of the tuning methods A, B and C.

5.7.2 TUNING METHOD A

As described in sub-section 2.3.2, tuning an amplifier according to what is referred to as method A amounts to each resonant circuit of the amplifier being tuned to the desired frequency whilst the resonant circuits immediately preceding and following it are so heavily damped that the remaining part of the amplifier has no influence on the circuit to be tuned. Its admittance can then be expressed by:

$$Y = G(1 + \mathrm{j}x), \tag{5.7.1}$$

as shown in sub-section 2.3.1. In the preceding calculations on the single-stage amplifier with double-tuned bandpass filters it has been tacitly assumed that the amplifier was tuned according to this method A. (Hence, in deriving the matrix equation (5.4.4); Eq. (5.7.1) was assumed to be applicable.

5.7.3 TUNING METHOD B

The tuning procedure referred to as method B consists in aligning the various tuned circuits of the amplifier successively, starting at the output side. During alignment of a particular resonant circuit, the circuit immediately preceding it must then be heavily damped or detuned. In so doing, the preceding part of the amplifier has no influence on the admittance of the circuit to be tuned, whereas the part of the amplifier which follows this circuit does influence its admittance.

The admittances of the various tuned circuits of the single-stage amplifier will now be calculated in succession for the case in which tuning method B is applied. At the same time it will be shown how this tuning procedure may be carried out in practice, reference being made to Figs. 5.1, 5.2 and 5.3.

Tuned Circuit 1

To align circuit 1, circuit 2 must be made inoperative. This can be achieved most easily by connecting a low-impedance signal generator across its terminals. The signal generator is adjusted to the desired frequency and circuit 1 is tuned for maximum deflection of a detector voltmeter connected to the output terminals of the amplifier. This voltmeter must not load the tuned output circuit (circuit 1) to any appreciable extent.

The output voltage v_o depends, except for a constant, exclusively on the frequency-dependent part of the circuit, so that:

$$v_o = C_1(1 + jx), \tag{5.7.2}$$

in which the constant C_1 is inversely proportional to the damping G_1 of circuit 1 and to the amplitude of the signal supplied by the generator.

Putting $P_1 = 1 + jx_1$ and denoting the value of P_1 at which v_o is at a maximum for the chosen tuning frequency by P_{1M}, gives:

$$P_{1M} = 1. \tag{5.7.3}$$

Tuned Circuit 2

In order to align circuit 2, circuit 3 must be made inoperative, for example by connecting the low-impedance signal generator which supplies the signal for aligning the amplifier, across it, Circuit 1 remains operative. Now circuit 2 is tuned in such a way that the deflection of the output meter is at a maximum.

The output voltage v_o depends, except for a different constant, C_2, on the frequency-dependent part of the admittance of circuit 2, and on the coupling coefficient of the double-tuned bandpass filter. Hence:

$$v_o = \frac{C_2}{\begin{vmatrix} 1 + jx_2 & -q_1^2 \\ 1 & 1 + jx_1 \end{vmatrix}},$$

but, since circuit 1 has been tuned previously, $x_1 = 0$, whence:

$$v_o = \frac{C_2}{P_2} = \frac{C_2}{\begin{vmatrix} 1 + jx_2 & -q_1^2 \\ 1 & 1 \end{vmatrix}}. \tag{5.7.4}$$

It can be shown that in this case the constant C_2 depends on the amplitude of the signal supplied by the generator, on the dampings G_1 and G_2 and on the forward transconductance ${}_1Y_{21}$ of the double-tuned bandpass filter.

The output voltage v_o is at a maximum when $x_2 = 0$ and

$$P_{2M} = 1 + q_1^2. \tag{5.7.5}$$

Tuned Circuit 3

To align circuit 3, circuit 4 is made inoperative, whereas circuits 2 and 1 remain operative. In this case too, correct tuning is achieved when the deviation of the output meter is at a maximum, which gives:

$$v_o = \frac{C_3}{P_3} = \frac{C_3}{\begin{vmatrix} 1 + \mathrm{j}x_3 & T_1 \cdot \exp(\mathrm{j}\Theta_1) & 0 \\ 1 & 1 & -q_1^2 \\ 0 & 1 & 1 \end{vmatrix}}. \tag{5.7.6}$$

The output voltage v_o is at a maximum when the imaginary part of the first term of the determinant is equal to:

$$T_1 \sin \Theta_1 \cdot \frac{1}{1 + q_1^2} = x_3'. \tag{5.7.7}$$

It is seen that v_o is now at a maximum when $x_3 \neq 0$, and this conflicts with the requirement that x_3 should disappear at the tuning frequency. The relative admittance [1]) of this resonant circuit will therefore be defined by (cf.: Appendix II and sub-sections (2.1.2) and (2.3.3):

$$(1 + \mathrm{j}(x_3 + x_3'). \tag{5.7.8}$$

Substitution of this relative admittance for the first term of the determinant in Eq. (5.7.6), using x_3' as defined by Eq. (5.7.7), gives:

$$P_{3M} = 1 + q_1^2 - T_1 \cos \Theta_1. \tag{5.7.9}$$

Tuned Circuit 4

The tuned input circuit 4 of the single-stage amplifier is aligned with all other circuits operative, whence:

$$v_o = \frac{C_4}{P_4} = \frac{C_4}{\begin{vmatrix} 1 + \mathrm{j}x_4 & -q_2^2 & 0 & 0 \\ 1 & 1 + \mathrm{j}x_3' & T_1 \cos \Theta_1 & 0 \\ 0 & 1 & 1 & -q_1^2 \\ 0 & 0 & 1 & 1 \end{vmatrix}}. \tag{5.7.10}$$

---

[1]) The term "relative admittance" employed here denotes the tuned circuit admittance with respect to the admittance at resonance ($x = 0$).

The output voltage v_o is therefore at a maximum when $x_4 = 0$, which gives:

$$P_{4M} = (1 + q_1^2)(1 + q_2^2) - T_1 \cos \Theta_1. \tag{5.7.11}$$

Summarizing the tuning procedures for circuits 1 to 4, it is seen that the numerator of the expressions for v_o consists of those terms of δ_{ij} (given by Eq. (5.4.4)) which are operative during the particular alignment, the results of the previous alignments being taken into account. The numerator of Ee. (5.2.6), for example, applicable to the alignment of circuit 3, consists of the 3×3 minor determinant derived from the determinant δ_{ij} with $x_1 = x_2 = 0$ because the tuning of circuits 1 and 2 has been carried out previously. The following chapter, dealing with multi-stage amplifiers, will show the usefulness of this conclusion.

If the relative admittance of each tuned circuit is represented in the form

$$1 + \mathrm{j}(x + x'),$$

it follows from the above comments that the various tuning corrections term are as follows:

$$\left.\begin{aligned} x_1' &= 0, \\ x_2' &= 0, \\ x_3' &= T_1 \sin \Theta_1 \cdot \frac{1}{1 + q_1^2} = T_1 \sin \Theta_1 \cdot \frac{P_{1M}}{P_{2M}}. \\ x_4' &= 0 \end{aligned}\right\} \tag{7.5.12}$$

Now the determinant δ according to Eq. (5.4.4), can be rewritten as follows, taking the influence of tuning method B into account:

$$\delta_y = \begin{vmatrix} 1 + \mathrm{j}(x_4 + x_4') & -q_2^2 & 0 & 0 \\ 1 & 1 + \mathrm{j}(x_3 + x_3') & T_1 \exp(\mathrm{j}\Theta_1) & 0 \\ 0 & 1 & 1 + \mathrm{j}(x_2 + x_2') & -q_1^2 \\ 0 & 0 & 1 & 1 + \mathrm{j}(x_1 + x_{21}') \end{vmatrix}. \tag{5.7.13}$$

Furthermore, the quantities P_M are seen to be the magnitudes of the minor determinants of δ_{ij} at the tuning frequency:

$$\left.\begin{aligned} P_{1M} &= 1, \\ P_{2M} &= P_{1M} + q_1^2 = 1 + q_1^2, \\ P_{3M} &= P_{2M} - T_1 \cos \Theta_1 \cdot P_{1M} = 1 + q_1^2 - T_1 \cos \Theta_1, \\ P_{4M} &= P_{3M} + q_2^2 P_{2M} = (1 + q_1^2)(1 + q_2^2) - T_1 \cos \Theta_1. \end{aligned}\right\} \tag{5.7.14}$$

5.7.4 TUNING METHOD C

As described in sub-section 2.3.4, the tuning procedure referred to as method C consists in aligning the various tuned circuits in succession, starting at the input side and rendering inoperative the tuned circuit which immediately follows the circuit to be aligned. In practice this tuning method can be carried out by connecting a signal generator, which is adjusted to the desired frequency, to the input circuit of the amplifier that is to be tuned. The admittance of this signal generator must be sufficiently low so that the circuit is not loaded to an appreciable extent. The exact tuning point can be ascertained by means of a detector voltmeter connected across the circuit following that which is to be tuned. The input admittance of this voltmeter should be increased to such an extent that the resulting quality factor of the circuit across which the voltmeter is connected becomes very low, thus fulfilling the condition that the circuit is made inoperative. The sensitivity of the voltmeter should remain sufficiently high to give an indication of the correct tuning point.

To tune circuits 4, 3 and 2, the detector voltmeter (with increased input admittance) is similarly connected across circuits 3, 2 and 1 respectively. It should be recognized that, since for tuning the output circuit the detector voltmeter must be connected directly across this circuit, it should not appreciably load the circuit in this particular case.

The influences of this tuning procedure can be calculated on the same lines as explained for method B. The minor determinants derived from δ_y will now be denoted by Q_4, Q_3, Q_2 and Q_1, and their maximum values at the tuning by Q_{4M}, Q_{3M}, Q_{2M} and Q_{1M} respectively[1]). The tuning correction factors applicable to tuning method C will be denoted by x'', in analogy with section 2.3. Therefore:

$$\left.\begin{aligned} x_4'' &= 0, \\ x_3'' &= 0, \\ x_2'' &= T_1 \sin \Theta_1 \cdot \frac{1}{1+q_2^2} = T_1 \sin \Theta_1 \cdot \frac{Q_{4M}}{Q_{3M}}, \\ x_1'' &= 0, \end{aligned}\right\} \qquad (5.7.15)$$

and

$$\left.\begin{aligned} Q_{4M} &= 1, \\ Q_{3M} &= 1 + q_2^2, \\ Q_{2M} &= 1 + q_2^2 - T_1 \cos \Theta_1, \\ Q_{1M} &= (1 + q_1^2)(1 + q_2^2) - T_1 \cos \Theta_1. \end{aligned}\right\} \qquad (5.7.16)$$

---

[1]) The symbols Q and Q_M used here to denote the minor determinants obtained with tuning method C should not be confused with the symbols Q and Q_0 used to denote the quality factors of the tuned circuits af the amplifier. In all cases it will be obvious from the context which quantity is actually meant.

TABLE. 5.1. TUNING PROCEDURES FOR AMPLIFIERS IN THE *Y*-MATRIX ENVIRONMENT

1	2	3	4	5
Method of tuning	Generator connected to:	Detector voltmeter connected to:	Large damping or detuning required for:	Operation required for tuning each circuit of the amplifier
A (sections 2.3.2 and 5.7.2)	tuned circuit preceding the circuit to be aligned: must have a low impedance, except for tuning the input circuit	tuned circuit following the circuit to be aligned: must have a low impedance, except for tuning the output circuit	tuned circuits immediately preceding and following the circuit to be aligned: is automatically provided for by the generator and detector voltmeter	(1) Connect generator to the circuit preceding the circuit to be tuned. (2) Connect detector voltmeter to the circuit following the circuit to be tuned. (3) Tune the circuit.
B (sections 2.3.3 and 5.7.3)	tuned circuit preceding the circuit to be aligned: must have a low impedance, except for tuning the input circuit	output terminals of the amplifier: must have a high impedance	tuned circuit preceding the circuit to be aligned: is automatically provided for by the generator	Connect the detector voltmeter to the output terminals of the amplifier. Subsequently: (1) Connect the generator. (2) Tune the circuit.
C (sections 2.3.4 and 5.7.4)	input terminals of amplifier: must present a high impedance to the input terminals of the amplifier	tuned circuit following the circuit to be aligned: must have a low input impedance, except for tuning the output circuit	tuned circuit following the circuit to be aligned: is automatically provided for by the detector voltmeter	Connect the signal generator to the input terminals of the amplifier. Subsequently: (1) Connect the detector voltmeter. (2) Tune the circuit.

Taking the influence of tuning method C into account, the determinant δ_y can be rewritten:

$$\delta_y = \begin{vmatrix} 1 + \mathrm{j}(x_4 + x_4'') & -q_2^2 & 0 & 0 \\ 1 & 1 + \mathrm{j}(x_3 + x_3'') & T_1 \exp(\mathrm{j}\Theta) & 0 \\ 0 & 1 & 1 + \mathrm{j}(x_2 + x_2'') & -q_1^2) \\ 0 & 0 & 1 & 1 + \mathrm{j}(x_1 + x_1'') \end{vmatrix} . \tag{5.7.17}$$

5.7.5 SUMMARY OF TUNING PROCEDURES FOR AMPLIFIERS IN THE Y-MATRIX ENVIRONMENT

In Table 5.1 all practical aspects of tuning methods A, B and C for amplifiers in the Y-matrix environment as considered in the preceding sub-sections have been set out. Columns 2 to 5 indicate the method in which the tuning procedure is carried out. Inspection of the table learns that tuning an amplifier, especially a complicated one, according to methods B or C requires approximately two thirds of the number of operations as are required to tune the same amplifier according to method A.

5.7.6 TUNING PROCEDURES FOR AN AMPLIFIER IN THE H-MATRIX ENVIRONMENT

In sub-section 2.3.6 it is shown that for a single-stage amplifier with single-tuned bandpass filters, identical mathematical expressions which describe the influences of the tuning methods A, B and C for amplifiers in either the Y- or the H-matrix environment can be derived. Also for single-stage amplifiers with double-tuned bandpass filters the expressions obtained for the various tuning methods for amplifiers in both matrix environments are identical. This may be shown by deriving the expression for the amplifier in the H-matrix environment. This can easily be done by means of analogies to the preceding sub-sections.

When considering the various methods of tuning for amplifiers in either the Y- or H-matrix environments it is essential to take into account the basic definitions for tuning methods A, B and C as presented in sub-section 2.3.5.

The practical methods of carrying out these tuning procedures for amplifiers in the H-matrix environment are set out in Table 5.2.

5.7.7 REDUCED DETERMINANT FOR THE VARIOUS METHODS OF TUNING

The three different methods of tuning can be combined in a single mathematical expression by using co-factors p_1 and p_2, as given in the table below:

TABLE 5.2. TUNING PROCEDURES FOR AMPLIFIERS IN THE *H*-MATRIX ENVIRONMENT

1	2	3	4	5
Method of tuning	Generator connected to:	Detector voltmeter connected to:	If the circuit to be tuned forms a parallel circuit:	If the circuit to be tuned forms a series circuit:
A (sections 2.3.6.1 and 5.7.6)	tuned circuit preceding the circuit to be aligned	tuned circuit following the circuit to be aligned.	both generator and detector voltmeter must present a high impedance. All other parallel-tuned circuits of the amplifier must be heavily damped.	both generator and detector voltmeter must present a low impedance.
B (sections 2.3.6.2 and 5.7.6)	tuned circuit preceding the circuit to be aligned.	output terminals of the amplifier: must have a high impedance.	generator must present a high impedance. The parallel-tuned circuit preceding the one to be aligned must be heavily damped.	generator must present a low impedance.
C (sections 2.3.6.3 and 5.7.6)	input terminals of the amplifier: must present a high impedance to the input terminals of the amplifier.	tuned circuit following the one to be aligned.	detector voltmeter must present a high impedance. The parallel-tuned circuit following the one to be aligned must be heavily damped.	detector voltmeter must present a low impedance

	Tuning method A	Tuning method B	Tuning method C
p_1	0	1	0
p_2	0	0	1

From Eqs. (5.7.13) and (5.7.17) the amplifier determinant then becomes:

$$\delta = \begin{vmatrix} 1+j(x+p_1x'+p_2x'')_4 & -q_2^2 & 0 & 0 \\ 1 & 1+j(x+p_1x'+p_2x'')_3 & T_1\exp(j\Theta_1) & 0 \\ 0 & 1 & 1+j(x+p_1x'+p_2x'')_2 & -q_1^2 \\ 0 & 0 & 1 & 1+j(x+p_1x'+p_2x'')_1 \end{vmatrix}. \tag{5.7.18}$$

The quantities x' and x'' occurring in this expression are given by Eqs. (5.7.12) and (5.7.15) respectively.

To evaluate the transfer function of the amplifier from Eqs. (5.5.4) and (5.5.5), the determinant δ as given by (5.7.18) must be used because the influences of the different tuning methods are then incorporated.

5.7.8 INFLUENCE OF THE VARIOUS METHODS OF TUNING ON THE STABILITY FACTOR

As a matter of fact, the boundary of stability should actually also be determined from δ as given by (5.7.18). However, since the tuning correction terms depend on $T \sin \Theta$ it would then be necessary also to take into account the parameter s which relates T to $T_g = sT$. Considering the large variety in parameters already used in section 5.6.1 to calculate the boundary of stability of the single-stage amplifier tuned according to method A, the general calculation of the boundary of stability for other methods of tuning would become very complex.

Using the graphical method of determining the boundary of stability as considered in sub-section 5.6.2, however, the *change of stability factor* due to tuning methods B and C can easily be determined. This can probably best be illustrated by means of an example which extends the case considered in sub-section 5.6.2 ($\Theta = 250°$, $r_1 = r_2 = 1$, $q_1^2 = 1.5$, $q_2^2 = 2.0$). For tuning method A it was found that $T_g = 4.7$. For $s = 4$, the regeneration coefficient becomes $T = T_g/s = 1.18$. With Eq. (5.7.12) the tuning correction term for tuning method B becomes $x_3' = -0.34$. The construction for T_g is carried out in Fig. 5.15. The line OP' joins points of the same frequency on the y_{i1} curve and the (shifted) y_{o2} curve. The boundary of stability equals $T_g' = OA'.OB' = 4.9$. For $T = 1.18$ (the value for which x_3' and, hence, the

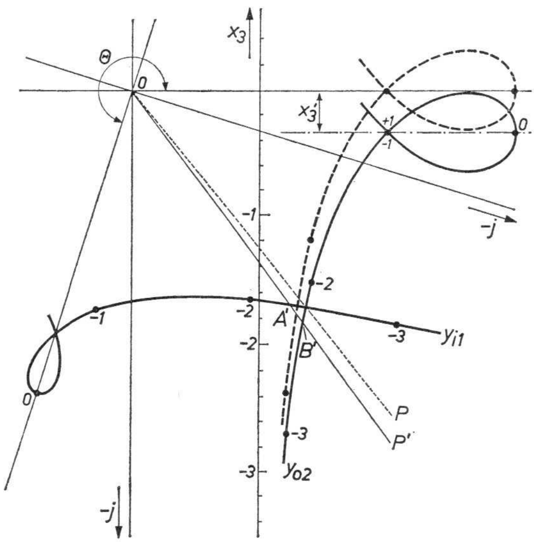


Fig. 5.15. Graphical construction for determining the stability factor of an amplifier designed with a certain value of s for tuning method A when it is tuned according to method B.

shift of the y_{o2} curve was determined), the stability factor now becomes $s' = 4.2$

From the above considerations it may be concluded that when an amplifier is designed for, say, $s = 4$ for tuning method A, this stability factor slightly increases (in this particular case to $s = 4.2$) when the amplifier is tuned according to methods B or C. This is in accordance with the effects found for the single-stage amplifier with single-tuned bandpass filters, see sub-section 2.3.8.

However, it is in most cases not essential to know the exact value of T_g, the more so because it is judged from the amplitude and envelope delay characteristics whether an amplifier design is acceptable or not. It is only necessary to know the value of T_g approximately for determining the stability factor s with a view to interchangeability requirements (cf. sub-sections 2.2.4. 2.5.2.3 and 5.6.3 and Chapter 11). For these reasons the exact calculation of T_g is omitted here.

5.8 Transducer Gain

The transducer gain Φ_t of the single-stage amplifier with two double-tuned bandpass filters, defined at the tuning frequency ($x = 0$), is given by:

$$\Phi_t = 4G_S G_L |Z_{to}|^2, \tag{5.8.1}$$

or:

$$\Phi_t = 4G_S R_L |H_{to}|^2, \qquad (5.8.2)$$

in which (see Eq. (5.5.4)):

$$Z_{to} = \frac{-\,{}_1y_{21} \cdot q_1 \cdot q_2}{\sqrt{G_1 G_2 G_3 G_4} \cdot \delta_0}, \qquad (5.8.3)$$

and (see Eq. (5.5.5)):

$$H_{to} = \frac{{}_1h_{21} \cdot q_1 \cdot q_2}{\sqrt{R_1 G_2 R_3 G_4} \cdot \delta_0}. \qquad (5.8.4)$$

Furthermore (see Eq. (5.7.18)):

$$\delta_0 = \begin{vmatrix} 1 + \mathrm{j}(p_1 x' + p_2 x'')_4 & -q_2^2 & 0 & 0 \\ 1 & 1 + \mathrm{j}(p_1 x' + p_2 x'')_3 & T_1 \exp(\mathrm{j}\Theta) & 0 \\ 0 & 1 & 1 + \mathrm{j}(p_1 x' + p_2 x'')_2 & -q_1^2 \\ 0 & 0 & 1 & 1 + \mathrm{j}(p_1 x' + p_2 x'')_1 \end{vmatrix}. \qquad (5.8.5)$$

The values of x' and x'' in this expression are given by Eqs. (5.7.12) and (5.7.15) respectively, whilst the values of p_1 and p_2 again follow from the table on page 145.

For tuning method A ($p_1 = p_2 = 0$):

$$\delta_0 = (1 + q_1^2)(1 + q_2^2) - T \exp(\mathrm{j}\Theta), \qquad (5.8.6)$$

and for tuning methods B ($p_1 = 1, p_2 = 0$) and C ($p_1 = 0, p_2 = 1$):

$$\delta_0 = (1 + q_1^2)(1 + q_2^2) - T \cos \Theta. \qquad (5.8.7)$$

Eqs. (5.8.6) and (5.8.7) may be combined as:

$$\delta_0 = (1 + q_1^2)(1 + q_2^2) - T \cos \Theta - \mathrm{j}(1 - p_1 - p_2) T \sin \Theta. \qquad (5.8.8)$$

This expression shows that, in general, with tuning methods B and C the value of δ_0 will be smaller, in other words: Φ_t will be larger than with tuning method A.

From Eqs. (5.8.3) and (5.8.1):

$$\Phi_t = 4G_S G_L \cdot \frac{|y_{21}|^2 q_1^2 q_2^2}{G_1 G_2 G_3 G_4 \cdot |\delta_0|^2}, \qquad (5.8.9)$$

or

$$\Phi_t = \frac{|y_{21}|^2}{4g_{11}g_{22}} \cdot \frac{G_L}{G_1} \cdot \frac{g_{22}}{G_2} \cdot \frac{g_{11}}{G_3} \cdot \frac{G_S}{G_4} \cdot \left(\frac{2q_1}{1+q_1^2}\right)^2 \cdot \left(\frac{2q_2}{1+q_2^2}\right)^2 \cdot \frac{(1+q_1^2)(1+q_2^2)^2}{|\delta_0|^2} . \qquad (5.8.10)$$

According to Appendix I:

$$\frac{|y_{21}|^2}{4g_{11}g_{22}} = \Phi_{u\mathrm{M}} .$$

By putting:

$$\frac{G^*}{G} = w , \qquad (5.8.11)$$

it follows that (c.f. Fig. 5.2):

$$\left.\begin{aligned} G_1 &= G_L + G_1^* \quad &\text{or} \quad G_L &= (1-w_1)G_1, \\ G_2 &= {}_1g_{22} + G_2^* \quad &\text{or} \quad {}_1g_{22} &= (1-w_2)G_2, \\ G_3 &= {}_1g_{11} + G_3^* \quad &\text{or} \quad {}_1g_{11} &= (1-w_3)G_3, \\ G_4 &= G_S + G_4^* \quad &\text{or} \quad G_S &= (1-w_4)G_4. \end{aligned}\right\} \qquad (5.8.12)$$

According to Appendix III, Section III.6, the transducer losses of a double-tuned bandpass filter Φ_{tb} are equal to

$$\Phi_{tb} = \left(\frac{2q}{1+q^2}\right)^2 \cdot (1-w_p)(1-w_s) . \qquad (5.8.13)$$

From Eqs. (5.8.10) to (5.8.13) the transducer gain becomes:

$$\Phi_t = \Phi_{u\mathrm{M}} \cdot \Phi_{tb_1} \cdot \Phi_{tb_2} \cdot \frac{(1+q_1^2)(1+q_2^2)^2}{|\delta_0|^2} . \qquad (5.8.14)$$

It follows from Eq. (5.8.8) that if $T = 0$, that is to say if the amplifier has no feedback, Φ_t becomes:

$$\Phi_t = \Phi_{u\mathrm{M}} \cdot \Phi_{tb1} \cdot \Phi_{tb2} .$$

The last factor of Eq. (5.8.14) thus represents the losses due to the feedback. These losses will be denoted by Φ_f [1]), which gives:

$$\Phi_f = \frac{(1+q_1^2)^2(1+q_2^2)^2}{|\delta_0|^2} , \qquad (5.8.15)$$

and:

$$\Phi_t = \Phi_{u\mathrm{M}} \cdot \Phi_{tb_1} \cdot \Phi_{tb_2} \cdot \Phi_f . \qquad (5.8.16)$$

---

1) In analogy with sub-section 2.4.3 the quantity Φ_f represents the losses due to the extra admittance present at the input terminals of the transistor because of its non-unilateral character. If tuning methods B or C are applied, Φ_f represents the losses due to the real part of this extra admittance.

In some cases it will be more convenient to write the expression for Φ in a somewhat different form. For this purpose the various factors of Eq. (5.8.9) are rearranged as follows:

$$\Phi_t = 4\frac{G_S}{G_4} \cdot \frac{G_L}{G_1} \cdot \frac{|y_{21}| \cdot |y_{12}|}{G_2 G_3} \cdot \frac{|y_{21}|}{|y_{12}|} \cdot \frac{q_1^2 q_2^2}{|\delta_0|^2}. \tag{5.8.17}$$

By putting

$$\frac{|y_{21}|}{|y_{12}|} = N, \tag{5.8.18}$$

Eq. (5.8.17) becomes:

$$\Phi_t = 4\frac{G_S}{G_4} \cdot \frac{G_L}{G_1} \cdot T_y \cdot N \cdot \frac{q_1^2 q_2^2}{|\delta_0|^2}, \tag{5.8.19}$$

in which T_y is the regeneration coefficient in the Y-matrix environment (see Section 5.4).

The above expressions for the transducer gain are derived for an amplifier in the Y-matrix environment. Starting with Eqs. (5.8.2) and (5.8.4) corresponding expressions can be derived in an analogous way for an amplifier in the H-matrix environment. This results in:

$$\Phi_t = 4\frac{G_S}{G_4} \cdot \frac{R_L}{R_1} \cdot T_h \cdot N \cdot \frac{q_1^2 q_2^2}{|\delta_0|^2}, \tag{5.8.20}$$

in which T_h is defined in Section 5.4, and

$$N = \left|\frac{h_{21}}{h_{12}}\right| = \left|\frac{y_{21}}{y_{12}}\right|. \tag{5.8.21}$$

If Eq. (5.8.16) is used to determine Φ_t, $\Phi_{u\mathrm{M}}$ is given by (see Appendix V):

$$\Phi_{u\mathrm{M}} = \frac{|h_{21}|^2}{4Re(h_{11})Re(h_{22})}, \tag{5.8.22}$$

and Φ_{tb} and Φ_f are given by Eqs. (5.8.13) and (5.8.15) respectively.

Since, for a given transistor, the value of T is determined by the required stability, Eqs. (5.8.19) and (5.8.20) clearly show the influence of the ratio N on the transducer gain of the amplifier.

5.8.1 THE OPTIMUM VALUE OF q^2

The expressions (5.8.19) and (5.8.20) for the transducer gain of the amplifier contain a factor:

$$\frac{q_1^2 q_2^2}{|\delta_0|^2} = \Phi_F, \tag{5.8.23}$$

which accounts for the influence of the coupling factors of the double-tuned bandpass filters and the feedback of the transistor. Hence, Φ_t may be optimized by optimizing Φ_F. Therefore we put:

$$\frac{\partial \Phi_F}{\partial q_1} = 0 \quad \text{and} \quad \frac{\partial \Phi_F}{\partial q_2} = 0\,. \tag{5.8.24}$$

After some calculations for either of the tuning methods A, B or C:

$$\left.\begin{aligned} (1 - q_1^2)(1 + q_2^2) - T\cos\Theta = 0, \\ (1 + q_1^2)(1 - q_2^2) - T\cos\Theta = 0. \end{aligned}\right\} \tag{5.8.25}$$

From Eq. (5.8.25) it follows that:

$$q_1^2{}_{\mathrm{opt}} = q_2^2{}_{\mathrm{opt}} = q^2{}_{\mathrm{opt}}, \tag{5.8.26}$$

and

$$q^2{}_{\mathrm{opt}} = \sqrt{1 - T\cos\Theta}\,. \tag{5.8.27}$$

In Fig. 5.16 the optimum value of q^2 according to Eq. (5.8.27) has been plotted as a function of $T\cos\Theta$. At $T = 0$, $q^2 = 1$ which is the value of critical coupling of a double-tuned bandpass filter. This critical coupling gives maximum transfer of energy from primary to secondary.

Substituting $q^2{}_{\mathrm{opt}}$ from Eq. (5.8.27) into Eq. (5.8.23) gives:

$$\Phi_{F\mathrm{opt}} = \frac{1}{4(1 + \sqrt{1 - T\cos\Theta})^2}\,. \tag{5.8.28}$$

In Fig. 5.16, the factor $\Phi_{F\mathrm{opt}}$ expressed in dB's has been plotted as a function of $T\cos\Theta$. The value of Φ_F at $T = 0$ is taken as 0 dB. It should be noted that Eq. (5.8.29) is only valid when T has such a value that stability of the amplifier is ensured over the entire passband.

5.9 Amplitude Response Curve

In analogy with sub-section 2.5.2, the normalized amplitude response curve, that is to say the amplitude response curve having unity magnitude at $x = 0$, is given by the relation $|\delta_0/\delta|$, which is the reduced determinant of the entire

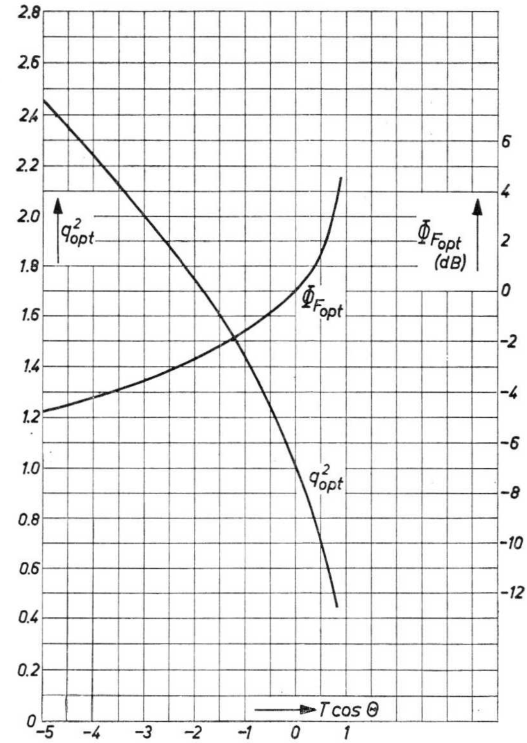


Fig. 5.16. At a certain value $q^2{}_{opt}$ of the coefficient of coupling of the double-tuned bandpass filters of the single stage amplifier the transducer gain becomes maximal. This optimum value is plotted as a function of $T \cos \Theta$. At this optimum value of $q_{opt}{}^2$ the losses due to feedback of the transistor and the coefficients of coupling of the double-tuned bandpass filters have a minimum value as appears from the plot of $\Phi_{Fopt} = q_1{}^2 q_2{}^2/|\delta_0|^2$.

amplifier, as given by Eq. (5.7.18). The quantity δ_0 which equals δ at $x = 0$, is given by Eq. (5.8.5) or Eq. (5.8.8).

The amplitude response curve for a single-stage amplifier with two identical double-tuned bandpass filters is plotted in Fig. 5.16. It is assumed that the bandpass filters have equal primary and secondary quality factors (whence $r = 1$), that $q^2 = (KQ)^2 = 1$ and, moreover, that $T = 2$ and $\Theta = 225°$. For the sake of comparison the amplitude response curve for the unilateral amplifier ($T = 0$) has also been plotted.

This figure shows that the curve for this amplifier with $T = 2$ differs only slightly from that for the amplifier with $T = 0$. This means that in this amplifier with two double-tuned bandpass filters the feedback of the transistor distorts the symmetry of the response curve to a lesser extent than in the

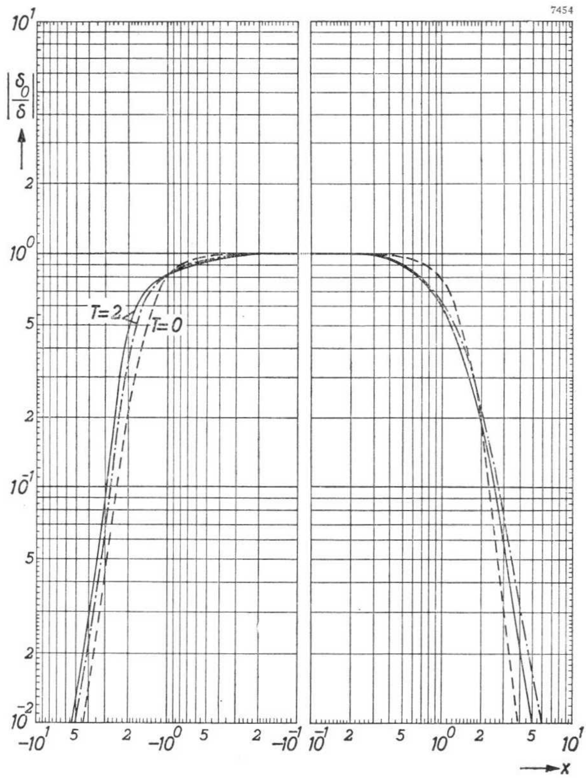


Fig. 5.17. Amplitude response curves of a single-stage amplifier with two identical double-tuned bandpass filters having the following data: $q_1{}^2 = q_2{}^2 = 1$, $r = 1$, $T = 2$ and $\Theta = 225°$. The fully drawn curve is applicable to tuning method A and the dash-dot curve to tuning methods B and C. For the sake of comparison the response curve of the unilateral amplifier ($T = 0$) has also been plotted (curve in broken line).

amplifier with two single-tuned bandpass filters (cf. the curves given in Fig. 2.17). Considering that the stability factor $s = T_g/T$ is roughly equal to 4 in both cases, this emphasizes once again that the performance of the amplifier cannot be judged from the stability factors.

Because all resonant circuits of this amplifier are identical, it makes no difference to its performance whether tuning method B or C is used. However, tuning method A will lead to different results. This is also illustrated by Fig. 5.17 which shows that better symmetry is obtained with tuning method B or C than with method A. Since methods B or C also yield a higher gain than method A, these methods are preferable for tuning the amplifier.

Fig. 5.18 illustrates another important aspect of the design of bandpass amplifiers equipped with transistors. In this graph the amplitude response curves have been plotted for an amplifier (also with $\Theta = 225°$) with two identical double-tuned bandpass filters of which $q^2 = (KQ)^2 = 2$. The curve for $T = 0$ is double humped (because of the overtransitional coupling). The curve for $T = 2$, however, has a flat top, whereas that for $T = 3$ is even slightly rounded off. This can be explained as follows:

Assume that tuning method B is employed. Then the output bandpass filter of the amplifier is tuned as if the amplifier had no feedback. Furthermore, assume that the admittance parameter notation is used. When the secondary of the first bandpass filter is tuned, the input admittance of the

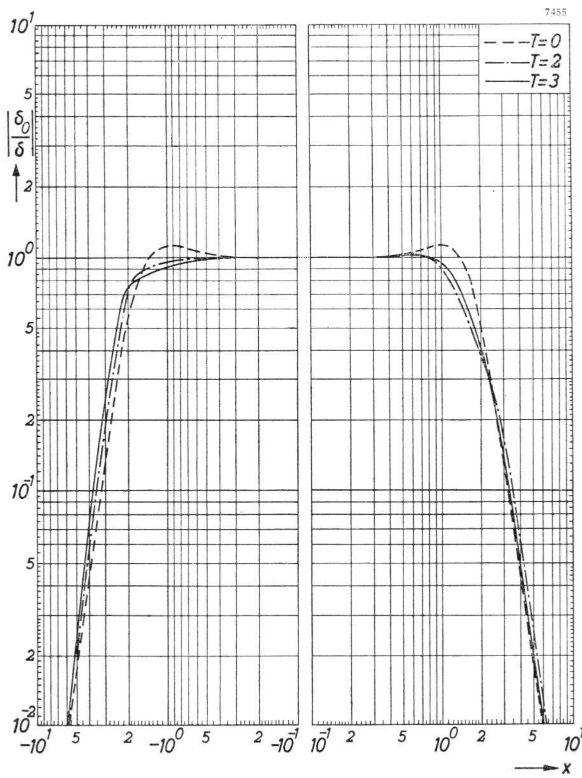


Fig. 5.18. Curves similar to those plotted in Fig. 5.17 but for $q_1{}^2 = q_2{}^2 = 2$ and $r = 1$, the dash-dot curve being valid for $T = 2$, the fully drawn curve for $T = 3$ and the curve in broken line for $T = 0$ (unilateral amplifier). These curves illustrate the disappearance of the humps due to the presence of negative real feedback.

transistor is [1]):

$$y_{in} = y_{11} - \frac{y_{12}y_{21}}{G_2(1 + q^2)}. \tag{5.9.1}$$

This means that, in addition to g_{11}, an extra damping:

$$g_{in\,fb} = \frac{Re(y_{12}y_{21})}{G_2(1 + q^2)}, \tag{5.9.2}$$

appears at the input terminals of the transistor.

With Eqs. (2.1.11) and (2.1.12) this extra damping can be expressed by:

$$g_{in\,fb} = - G_1 \cdot T \cos \Theta \cdot \frac{1}{1 + q^2}. \tag{5.9.3}$$

For $\Theta = 225°$, $T \cos \Theta$ is negative, as a result of which, according to Eq. (5.9.3), the damping on the secondary of the first double-tuned bandpass filter is increased. The "working" quality factor Q of the secondary is therefore decreased, and since the coupling coefficient k between the primary and secondary is constant, the "working" KQ is also decreased. This explains the disappearance of the humps at $T = 2$ and $T = 3$ in Fig. 5.18. If $T \cos \Theta$ had been positive, this would obviously have resulted in an increase of the "working" KQ and hence in an increase of the humps.

From the above argument it can be concluded that when choosing $q = KQ$ it is necessary also to take the quantities T and Θ into consideration. (In fact, this is one of the points which render the syntheses of amplifiers in which T differs from zero extremely complex.)

5.10 Envelope Delay Curve

According to section 2.5.3, the envelope delay of the amplifier is given by:

$$t_e \simeq \tau_e \cdot \frac{2Q}{\omega_0}, \tag{5.10.1}$$

in which (cg. Eq. (2.5.38):

$$\tau_e = \frac{\Delta\varphi}{\Delta x}. \tag{5.10.2}$$

The angle φ must be derived from the transfer function of the amplifiers:

$$Z_t = - \frac{{}_1y_{21} \cdot q_1 \cdot q_2}{\sqrt{G_1 G_2 G_3 G_4} \cdot \delta}, \tag{5.10.3}$$

---

[1]) Eq. (5.9.1) is in accordance with Eq. (2.3.2)in which Y_2 has been replaced by the input damping at $x = 0$ of the second bandpass filter of the amplifier (cf. Appendix III).

or
$$H_t = \frac{{}_1h_{21} \cdot q_1 \cdot q_2}{\sqrt{R_1 G_2 R_3 G_4} \cdot \delta}, \tag{5.10.4}$$

in which δ denotes the general amplifier determinant, including the influences of the tuning procedure as given by Eq. (5.7.18).

Since the relative bandwidth of the amplifier under consideration is fairly small, the phase angle of y_{21} may be assumed constant. (In fact, this is one of the assumptions on which this theory is based; cf. Chapter 1.) In order to evaluate φ, only δ need therefore be considered.

$$\varphi = \tan^{-1} \frac{I_m(\delta)}{Re(\delta)}. \tag{5.10.5}$$

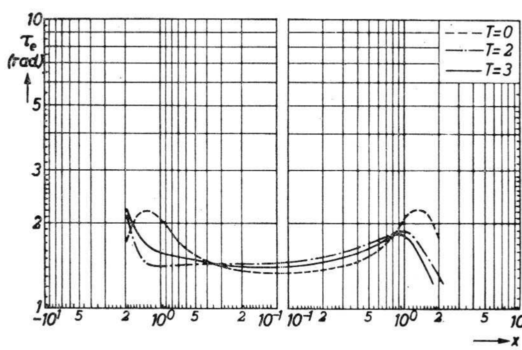


Fig. 5.19. Reduced envelope delay τ_e of a single-stage amplifier with two identical double-tuned bandpass filters having the following characteristics: $q_1^2 = q_2^2 = 2$, $r = 1$, $\Theta = 225°$, it being assumed that tuning method B or C is applied. The dash-dot curve applies to $T = 2$, the fully drawn curve to $T = 3$, whilst the curve in broken line is applicable to the unilateral amplifier ($T = 0$).

The reduced envelope delay τ_e has been determined by way of example for a single-stage amplifier with two identical double-tuned bandpass filters, it being assumed that $q^2 = (kQ)^2 = 2$, $r = 1$, and $\Theta = 225°$. The values of τ_e thus obtained for $T = 0$, $T = 2$ and $T = 3$ have been plotted in Fig. 5.19.

It should be recognized that the curves for $T = 2$ and $T = 3$ in the graph of Fig. 5.19 are slightly flatter than the corresponding curve for $T = 0$. This means that the feedback which is present in the amplifier has a flattening effect on the envelope delay characteristic, provided the feedback is not excessive.

BIBLIOGRAPHY

[5.1] J. CARSTAEDT, H. SCHOEN and F. WEITZSCH, *Nichtneutralisierte und teilneutralisierte Z.F. Verstärker in AM/FM Empfängern mit Transistoren*, Valvo Ber. **VI**, p. 81-105, 1960 (No. 3).

[5.2] S. H. CHANG, *Parabolic Loci for Two Tuned Coupled Circuits*, Proc. I.R.E. **36**, p. 1384-1388, 1948.

[5.3] A. H. J. NIEVEEN VAN DIJKUM, *The Design of Transistor I.F. Amplifiers with Double-tuned Bandpass Filters*, Unpublished report of Philips' Semiconductor Electrical Development Laboratory, Nijmegen, Netherlands.

[5.4] G. E. VALLEY and H. WALLMAN, *Vacuum Tube Amplifiers*, Chapter IV, McGraw Hill New-York 1948.

[5.5] F. WEITZSCH, *Einige theoretische Untersuchungen zur Leistungsübertragung und Stabilität in transistorbestückten Z.F. Verstärkern bei Verwendung von Bandfiltern*, Valvo Ber. **V**, p. 98-112, 1959 (No. 3).

[5.6] Simple Method of Designing Transistorised I.F. Amplifiers with Double-tuned Bandpass Filters, Advanced Information **108**, Philips Electr. Markets Dev., 1960.

CHAPTER 6

MULTI-STAGE AMPLIFIERS WITH SYNCHRONOUS SINGLE-TUNED BANDPASS FILTERS

The design of multi-stage transistor amplifiers is complicated by the fact that transistors are non-unilateral devices. A transistor considered as an active four-terminal network will have a mutual interaction between its input and output terminals. If then a number of stages containing such transistors are cascaded, the interaction between the input and output terminals of any transistor influences to a certain degree the operating conditions of all other transistors. The degree of interaction between successive stages obviously depends on the degree of coupling of the stages due to the interstage network employed. The larger this coupling, the larger the interaction.

It is, therefore, not possible to design a multi-stage amplifier on a stage-by-stage basis. This means that the conventional theory of linear amplifier circuits employing unilateral amplifying elements is not applicable to transistor amplifiers and recourse must be made to an analysis of the multi-stage amplifier as a whole.

In amplifiers employing single-tuned bandpass filters as interstage coupling networks, as considered in this chapter, there is a very light coupling between the successive stages. In this type of amplifier, problems due to the interaction between the stages present themselves very seriously.

These problems, relating to stability, power gain amplitude response and envelope delay, will be considered in detail in the following sections.

6.1. General Amplifier Circuit

In Fig. 6.1 a circuit diagram of an amplifier containing n transistors and

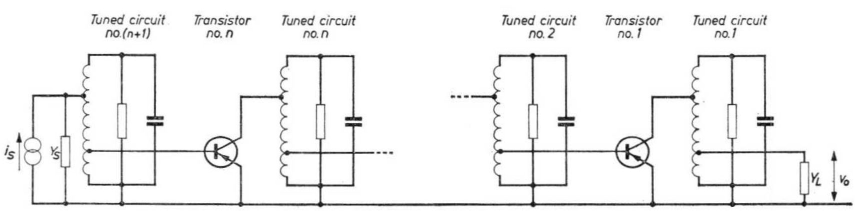


Fig. 6.1. Circuit diagram of an n-stage amplifier with $n + 1$ single-tuned bandpass filters.

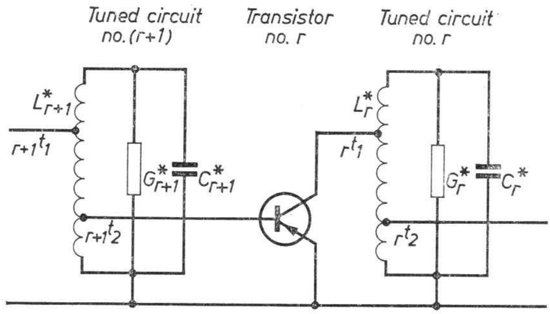


Fig. 6.2. Stage No.r of the amplifier showing the taps on the single-tuned bandpass filters and the notation thereof.

$(n+1)$ single-tuned bandpass filters is presented. The input and output terminals of the transistors, which are, by way of example, shown in the common emitter configuration, are connected to taps on the single-tuned circuits. To arrive at a more uniform and more schematic circuit diagram, we consider the rth stage of this n-stage amplifier, see Fig. 6.2. This rth stage comprises transistor No. r and the rth and $(r+1)$th tuned circuits. The notations used for the elements of these single-tuned circuits as used in the diagram are analogous to those of Chapter 2.

The output terminals of the transistor are connected to a tap $_rt_1$ on the rth tuned circuit and the input terminals of the transistor to a tap $_{(r+1)}t_2$ on the $(r+1)$th tuned circuit. We may replace the tuned circuits having taps by an admittance Y^* and two ideal transformers with transformer ratios $t_1 : 1$ and $1 : t_2$ as shown in Fig. 6.3. Furthermore, let the transistor be replaced by an equivalent four-terminal network based on admittance parameters and let the input and output currents and voltages of the transistor network be related as:

$$\left.\begin{aligned} i_1' &= y_{11}'v_1' + y_{12}'v_2' , \\ i_2' &= y_{21}'v_1' + y_{22}'v_2' . \end{aligned}\right\} \tag{6.1.1}$$

The transistor equivalent network can now be transformed to the top of the tuned circuits taking into account the proper transformer ratios. Then a normalized equivalent four-terminal network is obtained as indicated in Fig. 6.3. Now:

$$\left.\begin{aligned} i_1 &= {}_ry_{11}v_1 + {}_ry_{12}v_2, \\ i_2 &= {}_ry_{21}v_1 + {}_ry_{22}v_2, \end{aligned}\right\} \tag{6.1.2}$$

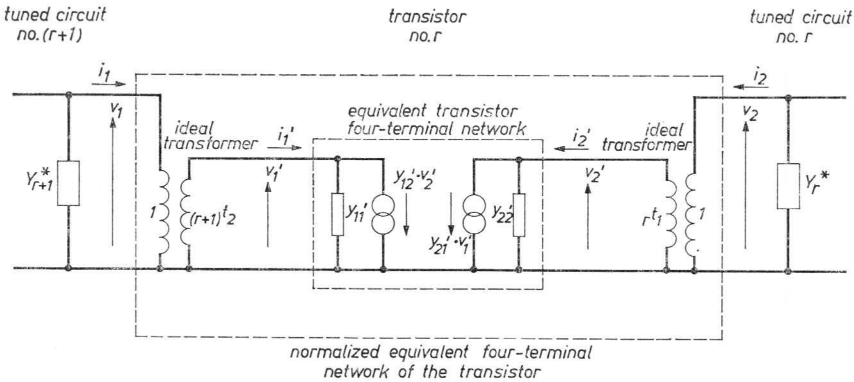


Fig. 6.3. Equivalent circuit diagram of the rth stage of an amplifier. The taps on the single-tuned bandpass filters are represented by ideal transformers. These transformers are included in a normalized equivalent four-terminal network representing the transistor.

in which:

$$\left.\begin{aligned} {}_r y_{11} &= {}_{(r+1)}t_2^2 \cdot {}_r y_{11}', & {}_r y_{12} &= {}_r t_1 \cdot {}_{(r+1)}t_2 \cdot {}_r y_{12}', \\ {}_r y_{21} &= {}_r t_1 \cdot {}_{(r+1)}t_2 \cdot {}_r y_{21}', & {}_r y_{22} &= {}_r t_1^2 \cdot {}_r y_{22}'. \end{aligned}\right\} \quad (6.1.3)$$

Using this normalized equivalent four-terminal network, the amplifier circuit around the rth transistor can be represented as shown in Fig. 6.4

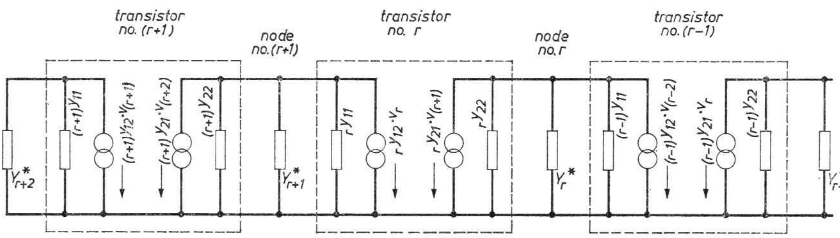


Fig. 6.4. Equivalent circuit diagram of a part of the n-stage amplifier.

which may further be simplified to Fig. 6.5 by combining the output self-admittance of the rth transistor, the admittance of the rth tuned circuit and the input admittance of the $(r-1)$th transistor into a single admittance Y_r.

6.2 General Amplifier Determinant

An amplifier comprising n transistors and $(n+1)$ single-tuned bandpass

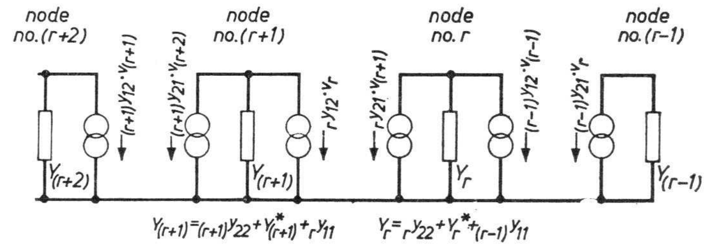


Fig. 6.5. Simplified equivalent circuit diagram of a part of the n-stage amplifier.

filters driven by a current source with admittance Y_S and loaded by an admittance Y_L is represented in Fig. 6.6. When:

$$Y_{n+1} = Y_S + Y_n{}^*{}_{+1} + {}_n y_{11}, \tag{6.2.1}$$

$$Y_1 \;\; = {}_1 y_{22} + Y_1{}^* \;\; + Y_L, \tag{6.2.2}$$

and the further notations are as indicated in the preceding section we may write the nodal equations for the n-stage amplifier in the form of a matrix equation:

$$\left\| \begin{array}{c} i_S \\ 0 \\ - \\ - \\ 0 \\ 0 \end{array} \right\| = \left\| \begin{array}{cccccc} Y_{n+1} & {}_n y_{12} & - & - & 0 & 0 \\ {}_n y_{21} & Y_n & - & - & 0 & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & Y_2 & {}_1 y_{12} \\ 0 & 0 & - & - & {}_1 y_{21} & Y_1 \end{array} \right\| \cdot \left\| \begin{array}{c} v_{n+1} \\ v_n \\ - \\ - \\ v_2 \\ v_1 \end{array} \right\| \tag{6.2.3}$$

The determinant Δ of this equation may be simplified by firstly: dividing all rows by the Y-term it contains, and secondly: by dividing all columns by the y_{21}/Y term it now contains and multiplying the corresponding rows by this same term. Then Δ may be written as:

$$\Delta = \prod_{m=1}^{m=n+1} Y_m \cdot \delta, \tag{6.2.4}$$

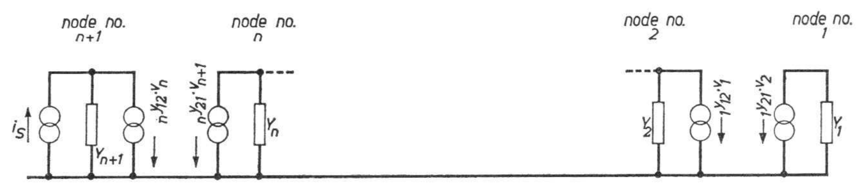


Fig. 6.6. Simplified equivalent circuit diagram of the n-stage amplifier showing source and load terminations.

$$\delta = \begin{vmatrix} 1 & \dfrac{{}_n y_{12} \cdot {}_n y_{21}}{Y_{n+1} \cdot Y_n} & — & — & 0 & 0 \\ 1 & 1 & — & — & 0 & 0 \\ — & — & — & — & — & — \\ — & — & — & — & — & — \\ 0 & 0 & — & — & 1 & \dfrac{{}_1 y_{12} \cdot {}_1 y_{21}}{Y_2 \cdot Y_1} \\ 0 & 0 & — & — & 1 & 1 \end{vmatrix} \qquad (6.2.5)$$

6.3 Loopgains and Stability

Use will be made of the loopgain concept to analyze the multi-stage amplifier with single-tuned bandpass filters with respect to stability. Using this concept enables us to illustrate the various stability and instability phenomena in a multi-stage amplifier in a plausible manner.

6.3.1 THE LOOPGAIN OF AN ISOLATED AMPLIFIER STAGE

In order to illustrate the loopgain concept we will first consider an isolated amplifier stage. Let such an isolated amplifier stage be represented by Fig.6.7. The amplifier determinant for this stage equals:

$$\Delta = \begin{vmatrix} Y_1 & y_{12} \\ y_{21} & Y_2 \end{vmatrix}, \qquad (6.3.1)$$

which may also be written as:

$$\Delta = Y_1 Y_2 \begin{vmatrix} 1 & \dfrac{y_{12} \cdot y_{21}}{Y_1 Y_2} \\ 1 & 1 \end{vmatrix}. \qquad (6.3.2)$$

Furthermore, the forward voltage gain of the stage equals:

$$-\frac{y_{21}}{Y_2},$$

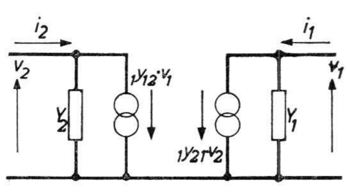


Fig. 6.7. Isolated amplifier stage.

whereas the reverse voltage gain is given by:

$$-\frac{y_{12}}{Y_1}.$$

Hence, the loopgain of the stage amounts to

$$\frac{y_{12}\,y_{21}}{Y_1 Y_2}.$$

If we denote the loopgain of the isolated amplifier stage by

$$u = \frac{y_{12}\,y_{21}}{Y_1\,Y_2}, \tag{6.3.3}$$

we may write for the general amplifier determinant:

$$\Delta = Y_1 Y_2 \begin{vmatrix} 1 & u \\ 1 & 1 \end{vmatrix}. \tag{6.3.4}$$

If the isolated amplifier stage forms a single-stage amplifier and does not constitute a part of a chain of amplifier stages (see next section), Eq. (6.3.3) represents the complete amplifier determinant, and, apparently, when $u_1 = 1$ ($= {}_1u_g$) the amplifier is at the boundary of stability (because then $\Delta_1 = 0$).

6.3.2 ISOLATED STAGE LOOPGAINS IN THE REDUCED AMPLIFIER DETERMINANT

The reduced determinant for the n-stage amplifier given by Eq. (6.2.5) may, taking into account Eq. (6.3.3.), be written as [1]):

$$\delta_u = \begin{vmatrix} 1 & u_n & 0 & - & - & 0 & 0 & 0 \\ 1 & 1 & u_{n-1} & - & - & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & - & - & 1 & 1 & u_1 \\ 0 & 0 & 0 & - & - & 0 & 1 & 1 \end{vmatrix}, \tag{6.3.5}$$

in which u_r denotes the loopgain of the rth stage of the n-stage amplifier when considered as isolated. Expressed in a formula:

$$u_r = \frac{{}_r y_{12} \cdot {}_r y_{21}}{Y_r \cdot Y_{r+1}}. \tag{6.3.6}$$

[1]) The index u in δ_u indicates that δ_u refers to the reduced amplifier determinant expressed in terms of isolated loopgains. Remember that δ (without index u) denotes the reduced determinant in terms of regeneration coefficient.

6.3.3 BOUNDARY OF STABILITY OF AN n-STAGE AMPLIFIER

The boundary of stability of an n-stage amplifier comprising $(n+1)$ single tuned bandpass filters is obtained by equating δ_u given by Eq. (6.3.5) to zero (provided all Y's are different from zero). In the case where all transistors and all single-tuned bandpass filters are identical, all loopgains at the boundary of stability are equal. These loopgains, which will be denoted by $_nu_g$, are calculated in Appendix VI. The results are compiled in Table 6.1 for amplifiers comprising up to 10 stages. For a single-stage amplifier $_1u_g = 1$, whereas for an infinite number of stages $_nu_g = 0.25$.

When the various stages of the amplifier are not identical, instability will occur for values of u_r other than $_nu_g$. Substituting values for u_r in Eq. (6.3.5) it can be checked whether the amplifier is stable ($\delta < 0$) or not by evaluating this determinant (or by checking the cascaded loopgains of the various stages, see sub-section 6.3.4).

Using the notations of Chapter 2, Eqs. (2.1.4), (2.1.11), (2.1.12) and (2.2.6) we may write for Eq. (6.3.6):

$$T_r \exp(\mathrm{j}\Theta_r) = u_r (1 + \mathrm{j}x_r)(1 + \mathrm{j}x_{r+1}). \tag{6.3.7}$$

At the boundary of stability we obtain for the amplifier with identical stages by eliminating x from the last expression (all x'es are assumed to be identical):

$$_nT_g = \frac{2\,_nu_g}{1 + \cos\Theta}, \tag{6.3.8}$$

or:

$$_nT_g = {_nu_g} \cdot {_1T_g}. \tag{6.3.9}$$

TABLE 6.1

n	$_nu_g$
1	1.0000
2	0.5000
3	0.3820
4	0.3333
5	0.3081
6	0.2928
7	0.2831
8	0.2764
9	0.2715
10	0.2680
∞	0.2500

6.3.4 CASCADED LOOPGAINS IN A MULTI-STAGE AMPLIFIER

In a multi-stage amplifier the loopgain[1]) of any stage depends on the transfer parameters of the transistor in the stage in question and on the total admittances seen by the forward and reverse current generators of the transistor.

Let ${}_nU_r$ denote the loopgain of the rth stage of the cascade of n stages. Then, for a single-stage amplifier, obviously,

$$ {}_1U_1 = u_1 \tag{6.3.10} $$

(see sub-section 6.3.1).

In a two-stage amplifier the forward transfer current generator of the output transistor sees an admittance Y_1, see Fig. 6.8, whereas the reverse transfer current generator sees an admittance

$$ Y_2 - \frac{{}_2y_{12} \cdot {}_2y_{21}}{Y_3} = Y_2\,(1 - u_2)\,. $$

The loopgain ${}_2U_1$ of this output stage therefore becomes:

$$ {}_2U_1 = \frac{{}_1y_{12} \cdot {}_1y_{21}}{Y_1 \cdot Y_2\,(1 - u_2)}\,, $$

or:

$$ {}_2U_1 = \frac{u_1}{1 - u_2}\,. \tag{6.3.11} $$

In the same way the loopgain of the input stage is found to be:

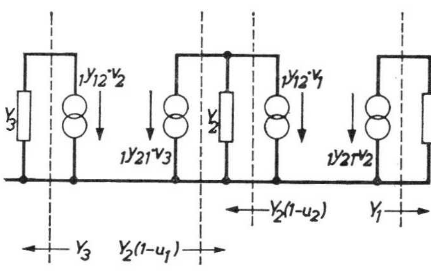


Fig. 6.8. Simplified equivalent circuit diagram of a two-stage amplifier illustrates the definitions of cascaded loopgains.

[1]) The method of considering the loopgain of a stage out of a cascade of stages in terms of the loopgains of these stages considered as isolated, has been indicated firstly by C. J. McCluskey in a private communication to the author.

$$ {}_2U_2 = \frac{u_2}{1 - u_1}. \tag{6.3.12} $$

When the loopgain of any of the stages of the amplifier becomes unity, the amplifier is at the boundary of stability. For the two-stage amplifier this boundary is therefore reached if ${}_2U_1 = 1$ or ${}_2U_2 = 1$, or:

$$ 1 - u_1 - u_2 = 0. \tag{6.3.13} $$

For identical stages $u = 0.5$, which is also obtained in the preceding section.

In an analogous way we can derive for the various stages of a three-stage amplifier:

$$ {}_3U_1 = \frac{u_1}{1 - \dfrac{u_2}{1 - u_3}}, \tag{6.3.14} $$

$$ {}_3U_2 = \frac{u_2}{(1 - u_1)\,(1 - u_3)}, \tag{6.3.15} $$

$$ {}_3U_3 = \frac{u_3}{1 - \dfrac{u_2}{1 - u_1}}. \tag{6.3.16} $$

The boundary of stability is reached for

$$ {}_3U_1 = 1,\ {}_3U_2 = 1 \text{ or } {}_3U_3 = 1\,, $$

or:

$$ u_1 + u_2 + u_3 - u_1u_3 = 1\,, \tag{6.3.17} $$

which leads to ${}_3u_g = 0.38$ if $u_1 = u_2 = u_3$.

For a four-stage amplifier we may derive:

$$ {}_4U_1 = \frac{u_1}{1 - \dfrac{u_2}{1 - \dfrac{u_3}{1 - u_4}}}, \tag{6.3.18} $$

$$ {}_4U_2 = \frac{u_2}{(1 - u_1)\left(1 - \dfrac{u_3}{1 - u_4}\right)}, \tag{6.3.19} $$

$$_4U_3 = \frac{u_3}{\left(1 - \dfrac{u_2}{1 - u_1}\right)(1 - u_4)}, \tag{6.3.20}$$

$$_4U_4 = \frac{u_4}{1 - \dfrac{u_3}{1 - \dfrac{u_2}{1 - u_1}}}, \tag{6.3.21}$$

whereas at the boundary of stability:

$$u_1 + u_2 + u_3 + y_4 - u_1u_3 - u_1u_4 - u_2u_3 = 1. \tag{6.3.22}$$

In an analogous way we obtain for an n-stage amplifier:

$$_nU_1 = \frac{u_1}{1 - \dfrac{u_2}{1 - \dfrac{u_3}{\text{etc.}}}}, \tag{6.3.23}$$

$$_nU_2 = \frac{u_2}{(1 - u_1)\left(1 - \dfrac{u_3}{1 - \dfrac{u_4}{\text{etc.}}}\right)}, \tag{6.3.24}$$

$$_nU_3 = \frac{u_3}{\left(1 - \dfrac{u_2}{1 - u_1}\right)\left(1 - \left(\dfrac{u_4}{1 - \dfrac{u_5}{\text{etc.}}}\right)\right)}. \tag{6.3.25}$$

For the rth stage we may write (see also Fig. 6.9):

$$_nU_r = \frac{{}_ry_{12} \cdot {}_ry_{21}}{Y_r \cdot \dfrac{P_r}{P_{r-1}} \cdot Y_{r+1} \cdot \dfrac{Q_{r+1}}{Q_{r+2}}}, \tag{6.3.26}$$

or:

$$_nU_r = \frac{u_r}{\dfrac{P_r}{P_{r-1}} \cdot \dfrac{Q_{r+1}}{Q_{r+2}}}, \tag{6.3.27}$$

in which P and Q are minor determinants of δ defined as indicated in Eq. (6.3.28):

$$\delta_u = \begin{vmatrix} 1 & u_n & 0 & & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\ 1 & 1 & u_{n-1} & & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 1 & 1 & & 0 & 0 & 0 & 0 & & 0 & 0 & 0 \\ - & - & - & \mathbf{Q}_{r+1} & & & & & & - & - & - \\ - & - & - & & & & & & & - & - & - \\ 0 & 0 & 0 & & 1 & u_{r+1} & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & & 1 & 1 & u_r & 0 & & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 1 & 1 & u_{r-1} & & 0 & 0 & 0 \\ 0 & 0 & 0 & & 0 & 0 & 1 & 1 & & 0 & 0 & 0 \\ - & - & - & & & & & & \mathbf{P}_r & - & - & - \\ - & - & - & & & & & & & - & - & - \\ 0 & 0 & 0 & & 0 & 0 & 0 & 0 & & 1 & u_2 & 0 \\ 0 & 0 & 0 & & 0 & 0 & 0 & 0 & & 1 & 1 & u_1 \\ 0 & 0 & 0 & & 0 & 0 & 0 & 0 & & 0 & 1 & 1 \end{vmatrix} \qquad (6.3.28)$$

The minor P_r is thus obtained by starting at the lower right hand side of δ_u and taking so many rows and columns from δ_u that the loop gains u_1 up to and including u_{r-1} are contained in the minor (r columns and rows). The minor Q_{r+1} is obtained by starting at the upper left hand side of δ_u; the loop gains u_{r+1} up to and including u_n are to be contained in this minor ($n + 1 - r$ columns and rows).

The quantity $Y_r \dfrac{P_r}{P_{r-1}}$ in Eq. (6.3.26) denotes the total admittance seen by the forward transfer current generator of the rth transistor. Analogously,

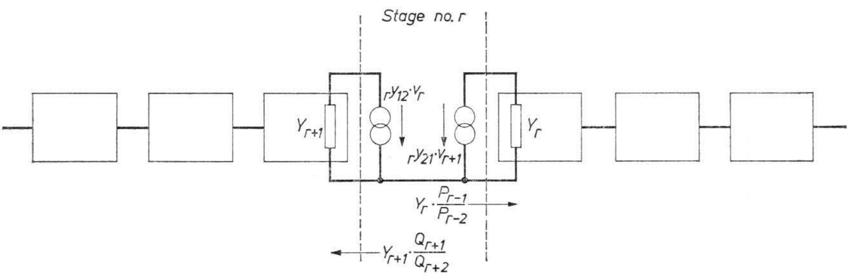


Fig. 6.9. Part of an n-stage amplifier illustrating the cascaded loopgain of the rth stage.

the quantity $Y_{r+1} \dfrac{Q_{r+1}}{Q_{r+2}}$ denotes the total admittance seen by the reverse transfer current generator. These total admittances can easily be written as continued fractions. This might also be concluded from the expressions obtained for the simpler cases first considered. If, therefore, all u's of the n-stage amplifier are known, all ${}_nU$'s can be calculated. Interesting questions which might be put forward in view of amplifier stability are:

a) what are the values found for the various ${}_nU$'s of an n-stage amplifier if all u's are made equal?
b) if equal values for all ${}_nU$'s of the n-stage amplifier are required, what values are to be given to the various u's?

These questions will be dealt with in the next section.

6.3.5 STABILITY FACTORS IN AN n-STAGE AMPLIFIER

The stability factor of an amplifier is generally defined as the reciprocal of its maximum real loopgain. In an n-stage amplifier with single-tuned band-pass filters this loopgain may either be an isolated loopgain (u) or a cascaded loopgain (${}_nU$). This implies that stability factors associated with both kinds of loopgain should be considered.

The *"isolated" stability factor* s_r for the rth stage is therefore defined as:

$$s_r = \frac{1}{u_r}, \tag{6.3.29}$$

whereas the *"cascaded" stability factor* ${}_nS_r$ is defined as:

$${}_nS_r = \frac{1}{{}_nU_r}, \tag{6.3.30}$$

The two questions put forward in the preceding section thus refer respectively to:

a) equal isolated stability factors,
b) equal cascaded stability factors.

Both cases will be considered in the following sub-sections (see also Bibliography (6.1)).

6.3.5.1 *Equal Isolated Stability Factors*

If in an amplifier the isolated stability factors of the various stages are (made) identical, we may calculate the cascaded stability factors of these stages by using the expressions derived in sub-section 6.3.4. In Figs. 6.10 and 6.11 the cascaded stability factors ${}_nS_r$ are plotted as a function of the isolated

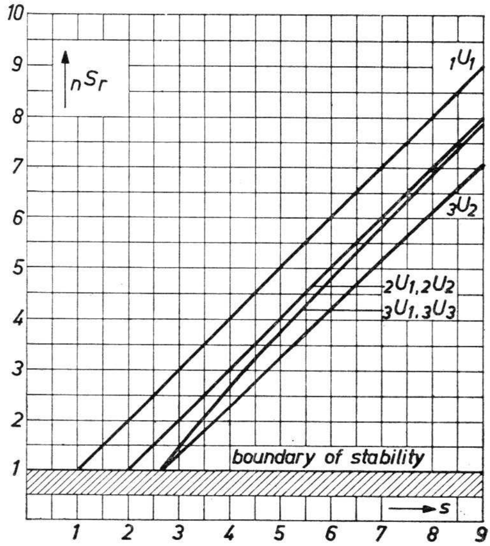


Fig. 6.10. Relation between the cascaded stability factors and the isolated stability factors of a two and a three-stage amplifier. The isolated stability factors of all stages are taken to be identical.

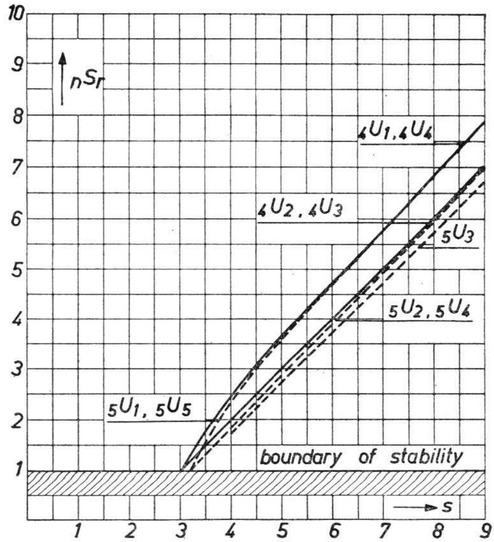


Fig. 6.11. As Fig. 6.10, but for four and five-stage amplifiers.

stability factor s for the various stages of amplifiers comprising up to five stages. It follows from these graphs as well as from the expressions presented in sub-section 6.3.4 that, for a three-stage amplifier or for a multi-stage amplifier having an odd number of stages, the centre stage has the lowest cascaded stability factor. In a multi-stage amplifier with an even number of stages this applies to the two centre stages. Furthermore, the cascaded stability factors of the various stages have values which are symmetrical with respect to the centre stage(s) of the amplifier and which increase gradually going from the centre stage(s) to the outer stages. This has also been illustrated in Table 6.2 below, which presents the various stability factors of a five-stage amplifier of which each stage has an isolated stability factor of $s = 6$.

TABLE 6.2 STABILITY FACTORS IN A FIVE-STAGE AMPLIFIER

Stage no	s	${}_5S_r$
1	6	5.25
2	6	3.95
3	6	3.75
4	6	3.95
5	6	5.25

6.3.5.2 Equal Cascaded Stability Factors

If it is required that the cascaded stability factors of the various stages of the amplifier are identical, it can be calculated by means of the expression derived in sub-section 6.3.4, which values of isolated stability factor should be given to the various stages in order to fulfil this condition.

Considering a three-stage amplifier we find that for equal cascaded stability factors S of the various stages:

$$S = \frac{1}{{}_3U_1} = \frac{1}{{}_3U_2} = \frac{1}{{}_3U_3}\,. \tag{6.3.31}$$

From Eqs. (6.3.14), (6.3.15) and (6.3.16) it then follows for the isolated stability factors $\left(s = \dfrac{1}{u}\right)$:

$$\left.\begin{aligned} s_1 = s_2 &= 1 + S\,, \\ s_2 &= \frac{(1+S)^2}{S}\,. \end{aligned}\right\} \tag{6.3.32}$$

In an analogous way it follows for a four-stage amplifier using Eqs. (6.3.18) to (6.3.21):

$$\left.\begin{aligned} s_1 = s_4 &= 1 + S \quad , \\ s_2 = s_3 &= \frac{(1+S)^2}{S} \, . \end{aligned}\right\} \tag{6.3.33}$$

Using Eqs. (6.3.23) to (6.3.28) it can be calculated for an n-stage amplifier with equal cascaded stability factors:

$$s_1 = s_n = 1 + S, \tag{6.3.34}$$

and

$$s_m = \frac{(1+S)^2}{S}, \tag{6.3.35}$$

in which $m = 2,3, \ldots n - 1.$

It thus follows that for equal cascaded stability factors of the amplifiers the first and the last stages have isolated loopgains different from those of the inner stages. In Fig. 6.12 the isolated loopgains as given by Eqs. (6.3.34) and (6.3.35) have been plotted as a function of the cascaded loopgain S.

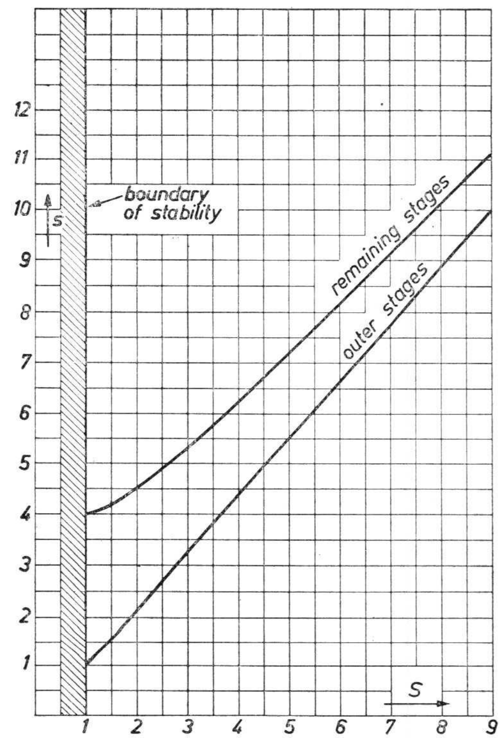


Fig. 6.12. Relation between the isolated stability factors and the cascaded stability factors for an n-stage amplifier. The cascaded stability factors are identical for all stages.

6.3.5.3 Choice of Stability Factor System for Practical Amplifier Constructions

Whether, in a practical amplifier, the cascaded stability factors or the isolated stability factors should be made equal depends on the relative merits of the two systems in amplifier design and construction.

When the isolated stability factors are made equal, all single tuned bandpass filters of the amplifier become identical which production problems. When equal cascaded stability factors are required the input and output bandpass filters are different from the others. Equal isolated stability factors are therefore preferable in the light of production techniques. This however, implies a certain sacrifice in gain of the amplifier compared with the case of equal cascaded stability factors. This may become apparent from the following considerations: In an amplifier the most important stability factors are those which apply to the various stages under actual operating conditions: the cascaded stability factors. These cascaded stability factors should have a certain minimum value. This minimum value, defined for a nominal amplifier, depends on spreads in transistor parameters and tolerance of components as well as on the allowable distortion of the response curve. If the isolated stability factors are made equal, the worst cascaded stability factor (centre stage of the amplifier) should have this minimum value. The cascaded stability factors of the remaining stages are then higher than required. This means that more gain than necessary has been sacrificed in achieving stability. For amplifiers with practical values of stability factors this extra sacrifice in gain, compared with the case of equal cascaded stability factors, is generally very small, see sub-section 6.6.11 and Bibliography (6.1).

From these considerations it may be concluded that there are no distinct advantages in either of the two systems, except that the system with equal isolated stability factors is generally preferable from the point of view of amplifier construction. For this reason we will confine ourselves in the further analysis of this type of amplifier mainly to the equal isolated stability factor system.

6.4 Regeneration Coefficients

6.4.1 REGENERATION COEFFICIENTS IN THE GENERAL AMPLIFIER DETERMINANT

Considering that:

$$Y_r = G_r\,(1 + \mathrm{j}x_r), \tag{6.4.1}$$

and

$$\frac{{}_r y_{12} \cdot {}_r y_{21}}{G_r \cdot G_{r+1}} = T_r \cdot \exp\,(\mathrm{j}\Theta_r)\,, \tag{6.4.2}$$

the determinant of Eq. (6.2.3) can be written as:

$$\Delta = \prod_{m=1}^{m=n+1} G_m \cdot \delta\,, \tag{6.4.3}$$

and

$$\delta = \begin{vmatrix} 1+jx_{n+1} & T_n \exp(j\Theta_n) & - & - & 0 & 0 \\ 1 & 1+jx_n & - & - & 0 & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & 1+jx_2 & T_1 \exp(j\Theta_1 \\ 0 & 0 & - & - & - & 1+jx_1 \end{vmatrix}. \tag{6.4.4}$$

With Eq. (6.4.4) we have obtained the normal form of reduced amplifier determinant which will be used for investigating gain and frequency response of the amplifier.

6.4.2 REGENERATION COEFFICIENTS AND STABILITY FACTORS

According to Chapter 2 the boundary of stability, in terms of regeneration coefficients, of an isolated amplifier stage is given by:

$$T_g = \frac{2}{1+\cos\Theta}\,. \tag{6.4.5}$$

The isolated stability factor s relates this boundary to the actual regeneration coefficient as:

$$T = \frac{T_g}{s}\,. \tag{6.4.6}$$

To obtain values for the regeneration coefficients T of the various stages the isolated stability factors are thus required. The design on stability of an amplifier should, however, be based on cascaded stability factors. When these factors are known, the isolated stability factors can be ascertained by means of the graphs of Figs. 6.10, 6.11 and 6.12.

If the amplifier is equipped with identical transistors in the various stages, the equal isolated stability factor system leads to equal values of T for all stages. When the equal cascaded stability factor system is employed the values of T_1 and T_n are larger than from those of $T_2 \ldots T_{n-1}$.

6.5 Tuning Procedures

In the multi-stage amplifier with synchronous single-tuned bandpass filters tuning may also be carried out according to either of the methods A, B or C as described for the single-stage amplifiers in Chapters 2 and 5. We therefore introduce tuning correction terms x' for tuning method B and x'' for tuning method C. With p_1 and p_2 having values given in Table 2.1 on page 46 the relative admittance of the rth tuned circuit of the amplifier becomes:

$$y_r = 1 + \mathrm{j}\,(x_r + p_1 x_r' + p_2 x_r''). \tag{6.5.1}$$

Then the reduced determinant of the amplifier may be written as:

$$\delta = \begin{vmatrix} y_{n+1} & T_n \exp(j\Theta_n) & — & — & 0 & 0 \\ 1 & y_n & — & — & 0 & 0 \\ — & — & — & — & — & — \\ — & — & — & — & — & — \\ 0 & 0 & — & — & y_2 & T_1 \exp(\mathrm{j}\Theta_1) \\ 0 & 0 & — & — & 1 & y_1 \end{vmatrix}. \tag{6.5.2}$$

The tuning methods A, B and C as well as the calculation of the tuning correction terms have been considered in detail in Chapter 2 for a single-stage amplifier with two single-tuned bandpass filters, and in Chapter 5 for a single-stage amplifier with two double-tuned bandpass filters (comprising four single-tuned circuits).

Calculation of the tuning correction terms for this particular amplifier, which may be carried out analogously, yields for tuning method B:

$$\left.\begin{aligned}
x_1' &= 0,\\
x_2' &= T_1 \sin\Theta_1 \cdot \frac{1}{P_{1\mathrm{M}}} = T_1 \sin\Theta_1\,,\\
x_3' &= T_2 \sin\Theta_2 \cdot \frac{P_{1\mathrm{M}}}{P_{2\mathrm{M}}} = T_2 \sin\Theta_2 \cdot \frac{1}{1 - T_1\cos\Theta_1},\\
x_4' &= T_3 \sin\Theta_3 \cdot \frac{P_{2\mathrm{M}}}{P_{3\mathrm{M}}} = T_3 \sin\Theta_3 \cdot \frac{1 - T_1\cos\Theta_1}{1 - T_1\cos\Theta_1 - T_2\cos\Theta_2},\\
\text{and}\quad x_r' &= T_r \sin\Theta_r \cdot \frac{P_{(r-2)\mathrm{M}}}{P_{(r-1)\mathrm{M}}}.
\end{aligned}\right\} \tag{6.5.3}$$

For tuning method C we obtain:

$$\left.\begin{aligned}
x''_{n+1} &= 0, \\
x_n'' &= T_n \sin \Theta_n \cdot \frac{1}{Q_{(n+1)\mathrm{M}}} = T_n \sin \Theta_n , \\
x''_{n-1} &= T_{n-1} \sin \Theta_{n-1} \cdot \frac{Q_{(n+1)\mathrm{M}}}{Q_{n\mathrm{M}}} = T_{n-1} \sin \Theta_{n-1} \cdot \frac{1}{1 - T_n \cos \Theta_n}, \\
x_r'' &= T_r \sin \Theta_r \cdot \frac{Q_{(r+2)\mathrm{M}}}{Q_{(r+1)\mathrm{M}}} .
\end{aligned}\right\} \quad (6.5.4)$$

In these expressions P and Q are minor determinants obtained from Eq. (6.3.28) when starting at the lower right hand side corner and the upper left hand side corner respectively. The first index of P and Q denotes the order of the minor which is, obviously, equal to the number of tuned circuits included. The index M in $P_{r\mathrm{M}}$ and $Q_{r\mathrm{M}}$ denotes that these minors apply to parts of an amplifier tuned according to method B or method C respectively in such a way that these parts give maximum response at the tuning frequency.

In Chapter 5 an analogous notation has been used for the minors of the determinant for a single-stage amplifier with two double-tuned bandpass filters.

6.6 Gain

6.6.1 TRANSDUCER GAIN

The transducer gain of an amplifier is given by:

$$\Phi_t = 4\, G_S G_L \cdot |Z_t|^2, \tag{6.6.1}$$

in which Z_t represents the transimpedance of the amplifier. For the n-stage amplifier this transimpedance can be obtained from Eq. (6.2.3) as:

$$Z_t = \frac{\prod_{m=1}^{m=n} {}_m y_{21}}{\Delta}, \tag{6.6.2}$$

or, with Eq. (6.4.3):

$$Z_t = \frac{\prod_{m=1}^{m=n} {}_m y_{21}}{\prod_{m=1}^{m=n+1} G_m \;\delta}. \tag{6.6.3}$$

The reduced determinant δ including the effects of the tuning procedures is given by Eq. (6.5.2). At the tuning frequency all x'es in δ vanish and the transducer gain becomes:

$$\Phi_{t,n} = 4\, G_S G_L \frac{\prod\limits_{m=1}^{m=n} |{}_m y_{21}|^2}{\prod\limits_{m=1}^{m=n+1} G_m{}^2 \cdot |\delta_0|^2}, \tag{6.6.4}$$

in which

$$\delta_0 = \begin{vmatrix} y_{(n+1),0} & T_n \exp(\mathrm{j}\Theta_n) & - & - & 0 & 0 \\ 1 & y_{n,0} & - & - & 0 & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & y_{2,0} & T_1 \exp(\mathrm{j}\Theta_1) \\ 0 & 0 & - & - & 1 & y_{1,0} \end{vmatrix}, \tag{6.6.5}$$

and

$$y_{r,o} = 1 + \mathrm{j}\,(p_1 x_r' + p_2 x_r''). \tag{6.6.6}$$

In analogy with sub-section 2.4.3 we may express the transducer gain of the multi-stage amplifier given by Eq. (6.6.4) as the product of the maximum unilateral gains of the transistors [1]), Φ_{uM}, the insertion losses of the tuned circuits [2]), Θ_i, the mismatch losses across the tuned circuits [3]), Θ_{mm} and a factor $1/|{}_n\delta_0|^2$, ${}_n\Theta_f$, accounting for the non-unilateral properties of the transistors. In a similar way as we obtained Eq. (2.4.15), we find:

$$\Phi_{t,n} = \prod_{m=1}^{m=n} {}_m\Phi_{uM} \cdot \prod_{m=1}^{m=n+1} ({}_m\Phi_i \cdot {}_m\Phi_{mm}) \cdot {}_n\Phi_f\,. \tag{6.6.7}$$

For an amplifier consisting of identical stages, this expression reduces to:

$$\Phi_{t,n} = \Phi_{uM}{}^n \cdot \Phi_i{}^n \cdot \Phi_{mm}{}^n \cdot {}_n\Phi_f\,. \tag{6.6.8}$$

Expressions (6.6.7) and (6.6.8) are of special importance for amplifier designs in which the contribution of the various parts to the total transducer gain is known (as a design requirement) or is required afterwards for other purposes. If, however, the transducer gain of the amplifier obtainable with a certain type of transistor is of prime interest, Eq. (6.6.4) can better be expressed in an alternative form. If $N = \left|\dfrac{y_{21}}{y_{12}}\right|$, Eq. (6.6.4) can be written as:

---

1) See Appendix V.
2) See Appendix II.
3) See Appendix II and footnote on page 51.

$$\Phi_{t,n} = 4 \frac{G_S}{G_{n+1}} \cdot \frac{G_L}{G_1} \cdot \prod_{m=1}^{m=n} (T \cdot N)_m \cdot {}_n\Phi_f , \tag{6.6.9}$$

in which T is generally determined by stability requirements.

For the purpose of comparing various transistors, it may be assumed that the ratios G_S/G_{n+1} and G_L/G_1 are kept constant (which may be achieved by a change of tapping ratio) and, furthermore, that all stages of the amplifier are identical. Evaluation of $T^n N^n / |\delta_0|^2$ then gives the transducer gain of the amplifier (except for a constant) obtainable with the different types of transistors.

The factor $\frac{1}{|\delta_0|^2} = {}_n\Phi_f$ appearing in the various transducer gain expressions may be obtained by evaluating the determinant given by Eq. (6.6.5). When tuning methods B or C are applied and the various stages of the amplifier are identical with respect to T and Θ, $|\delta_0|$ is a function of $T \cos \Theta$ only and can therefore easily be represented graphically. Fig. 6.13 shows such a graph for amplifiers comprising up to five stages. For tuning method A, representation of $|\delta_0|$ in a single graph is not possible because the quantities

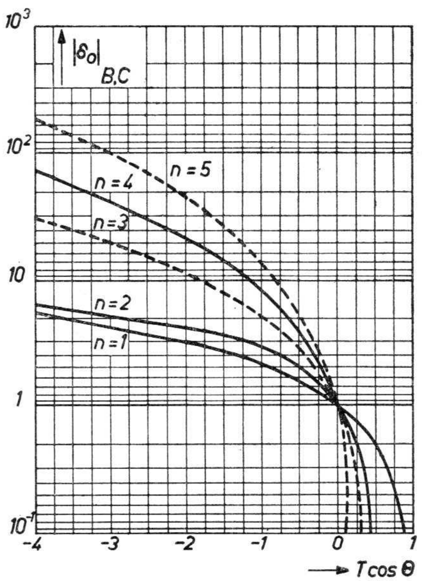


Fig. 6.13. The value of the reduced determinant $|\delta|$ at $x = 0$ for an amplifier tuned according to methods **B** or C as a function of T cos Θ.

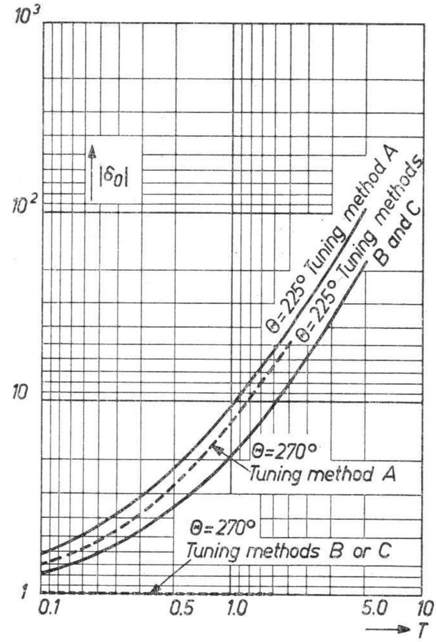


Fig. 6.14. Comparison of values of $|\delta_0|$ for an amplifier tuned according to either method A or methods B or C for $\Theta = 225°$ and $\Theta = 270°$. It follows that the differences in $|\delta_o|$ are larger for larger values of $|\sin \Theta|$.

$T \cos \Theta$ as well as $T \sin \Theta$ are contained in $|\delta_0|$. To show the differences in gain obtained with tuning methods A and B or C, $|\delta_0|$ has been plotted in Fig. 6.14 as a function of T for $\Theta = 270°$ and $\Theta = 225°$.

6.6.1.1 Comparison of Equal Isolated Stability Factor and Equal Cascaded Stability Factor Systems

As pointed out in sub-section 6.3.5.3 some gain is sacrificed when the equal isolated stability factor system is applied in an amplifier which must have a given minimum cascaded stability factor. The difference in gain obtained with the two systems will be calculated for a four-stage amplifier in which $S \geqslant 4$. For the case of equal isolated stability factors it then follows from Fig. 6.11 that $s = 6$. For the case of equal cascaded stability factors it follows from Fig. 6.12 that $s = 6.25$ for stages 1 and 4 and $s = 4.4$ for stages 2 and 3. The various regeneration coefficients T can now be ascertained from $T = T_g/s$.

With Eq. (6.6.9), $\Phi_{t,n}$ can be calculated for the two systems taking into account the different values of T and $|\delta_0|$. The factors G_S/G_5 and G_L/G_1 do not change noticeably because for the outer stages the isolated stability factors equal either $s = 6$ or $s = 6.25$. Then the ratio of the transducer gain obtained from the amplifier with equal cascaded stability factors and from the amplifier with equal isolated factors becomes:

$$\frac{\Phi_{t,4} \text{ equal } S}{\Phi_{t,4} \text{ equal } s} =$$

$$= \frac{6^4}{(4.4)^2 \cdot (6.25)^2} \cdot \frac{1 - 4\frac{T_g}{6}\cos\Theta + 3\left(\frac{T_g}{6}\cos\Theta\right)^2}{1 - 2\left(\frac{T_g}{4.4} + \frac{T_g}{6.25}\right)\cos\Theta + \left\{\left(\frac{T_g}{4.4}\right)^2 + 2\left(\frac{T_g}{6.25}\right)^2\right\}\cos^2\Theta}, \quad \ldots \quad (6.6.10)$$

in which T_g is the boundary of stability of an isolated amplifier stage:

$$T_g = \frac{2}{1 + \cos\Theta}. \qquad (6.6.11)$$

In Fig. 6.15, Eq. (6.6.10) has been plotted as a function of the regeneration phase angle Θ. It follows from this curve that for a four-stage amplifier with a cascaded stability factor of $S \geqslant 4$, an increase in transducer gain can be obtained of 3.3 dB at $\Theta = 0°$ by making all cascaded stability factors equal instead of making all isolated stability factors equal. At $\Theta = 270°$ (or 90°) this increase amounts to 2.4 dB whereas at $\Theta = 210°$ (or 150°), 1.5 dB is gained.

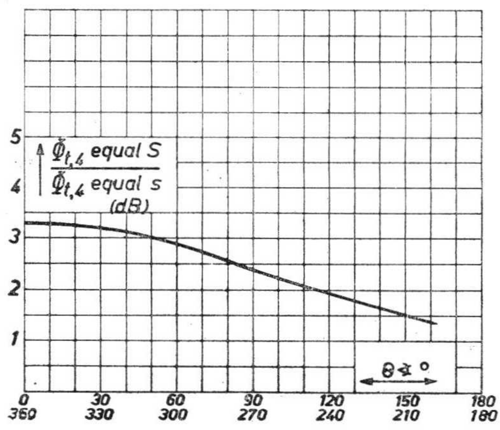


Fig. 6.15. Difference in transducer gain as a function of T of a four-stage amplifier designed with equal cascaded stability factors and with equal isolated stability factors for $\Theta = 225°$ and $S = 4$.

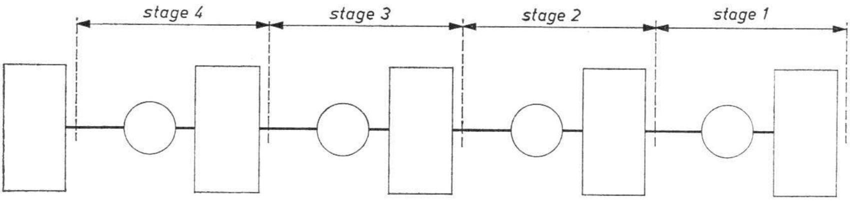


Fig. 6.16. Diagram illustrating the numbering of stages of the amplifier.

From these considerations it may be concluded that the increase in gain per stage obtained in this way is generally less than 1 dB (see also sub-section 6.3.5.3).

6.6.2 POWER GAIN PER STAGE

The power gain of the rth stage of an amplifier comprising the rth transistor and the rth single-tuned bandpass filter (see Fig. 6.16) is composed of the factors ${}_r\Phi_{uM}$, the maximum unilateralized power gain of the transistor; ${}_r\Phi_i$ and ${}_r\Phi_{mm}$, the insertion losses of the rth single-tuned bandpass filter and the mismatch losses across this bandpass filter and Φ_{fr}, the losses in power gain due to the feedback of the transistor. Therefore:

$$\Phi_r = {}_r\Phi_{uM} \cdot {}_r\Phi_i \cdot {}_r\Phi_{mm} \cdot \Phi_{fr}. \tag{6.6.12}$$

As we have found in Chapter 2, the losses (or gain) due to the feedback of the transistor, defined at the tuning frequency, equal the squared ratio of the total admittance at the transistor input terminals *without* feedback to that *with* feedback. The total admittance including the influences of the feedback will be calculated in the following sub-section.

6.6.2.1 The Input Admittance of a Particular Stage of the Amplifier

In order to calculate the input admittance of the rth transistor of the amplifier, we consider the circuit of Fig. 6.17. For this part of the amplifier we can write down the following matrix equation:

$$\left\| \begin{matrix} (i_m)_r \\ 0 \\ - \\ - \\ 0 \\ 0 \end{matrix} \right\| = \left\| \begin{matrix} {}_ry_{11} & {}_ry_{12} & - & - & 0 & 0 \\ {}_ry_{21} & Y_r & - & - & 0 & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & Y_2 & {}_1y_{12} \\ 0 & 0 & - & - & {}_1y_{21} & Y_1 \end{matrix} \right\| \cdot \left\| \begin{matrix} (v_{in})_r \\ v_r \\ - \\ - \\ v_2 \\ v_1 \end{matrix} \right\| . \tag{6.6.13}$$

The first term of the determinant of this matrix equation equals the input self-admittance of the rth transistor. Now:

$$_r y_{11} = {}_r g_{11} + {}_r b_{11}. \tag{6.6.14}$$

In order to manipulate with the determinant in such a way that for each transistor its regeneration coefficient T appears, we must relate $_r g_{11}$ to G_{r+1}, which is a factor of the denominator of

$$T_r = \frac{_r|y_{12}y_{21}|}{G_r \cdot G_{r+1}} .$$

Therefore we put:

$$\zeta_{r+1} = \frac{G_{r+1} - {}_r g_{11}}{G_{r+1}}$$

or:

$$_r g_{11} = (1 - \zeta_{r+1})\, G_{r+1} . \tag{6.6.15}$$

Eq. (6.6.14) then becomes:

$$_r y_{11} = G_{r+1} \left(1 - \zeta_{r+1} + \mathrm{j}\, \frac{b_{11}}{G_{r+1}}\right) . \tag{6.6.16}$$

After substituting

$$Y = G\,\{1 + \mathrm{j}(x + p_1 x')\} ,$$

and $_r y_{11}$ given by Eq. (6.6.16) in the determinant of Eq. (6.6.13), the G's may be separated out. Let the reduced determinant be denoted by P_{r+1}. Then Eq. (6.6.13) may be written:

$$\begin{Vmatrix} (i_m)_r \\ - \\ - \\ 0 \end{Vmatrix} = \prod_{m+1}^{m=r+1} G_m \cdot \begin{Vmatrix} P_{r+1} \end{Vmatrix} \cdot \begin{Vmatrix} (v_{in})_r \\ - \\ - \\ v_1 \end{Vmatrix} . \tag{6.6.17}$$

Furthermore, the determinant P_{r+1} becomes:

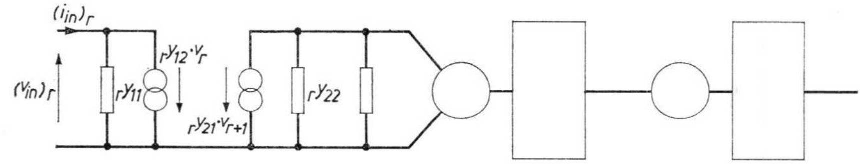


Fig. 6.17. The input admittance of the rth stage of an n-stage amplifier is defined with this transistor loaded by the amplifier stages Nos. 1 to ($r - 1$).

$$P_{r+1} = \left(1 - \zeta_{r+1} + \mathrm{j}\,\frac{{}_rb_{11}}{G_{r+1}}\right) P_r - T_r \exp(\mathrm{j}\Theta_r) \cdot P_{r-1}\,. \qquad (6.6.18)$$

From Eqs. (6.6.17) and (6.6.18) it then follows for ${}_ry_{in}$:

$${}_ry_{in} = G_{r+1} \cdot \frac{P_{r+1}}{P_r}\,,$$

or:

$${}_ry_{in} = G_{r+1} \left\{ 1 - \zeta_{r+1} + \mathrm{j}\,\frac{{}_rb_{11}}{G_{r+1}} - T_r \exp(\mathrm{j}\Theta_r) \frac{P_{r-1}}{P_r} \right\}. \qquad (6.6.19)$$

For tuning method B the minors P are real at the tuning frequency. If these values of the minors are denoted by P_{M}, we may write:

$${}_rg_{in} = G_{r+1} \left\{ 1 - \zeta_{r+1} - T_r \cos\Theta_r \cdot \frac{P_{(r-1)\mathrm{M}}}{P_{r\mathrm{M}}} \right\}, \qquad (6.6.20)$$

and

$${}_rb_{in} = {}_rb_{11} - T_r \sin\Theta_r \cdot \frac{P_{(r-1)\mathrm{M}}}{P_{r\mathrm{M}}} \cdot G_{r+1}\,. \qquad (6.6.21)$$

Eqs. (6.6.19) to (6.6.21) give the input admittance of any of the transistors in the amplifier for tuning methods B and C at the tuning frequency. These expressions may be used to calculate the power gain of a particular stage of the amplifier.

6.6.2.2 The Feedback Losses of the rth Stage of the Amplifier

Consider an amplifier with single-tuned bandpass filters tuned according to either tuning methods A or B. The total admittance at the input terminals of the rth transistor of this amplifier, disregarding the feedback of the $(r+1)$th transistor (this feedback is accounted for in stage $(r+1)$), equals:

$${}_{(r+1)}y_{22} + Y^*_{r+1} + {}_ry_{in}. \qquad (6.6.22)$$

At the frequency of tuning and for tuning method A this total admittance becomes with Eq. (6.6.19):

$$G_{r+1} \left\{ 1 - T_r \exp(\mathrm{j}\Theta_r) \cdot \frac{P_{r-1}}{P_r} \right\}, \qquad (6.6.23)$$

and for tuning method B with Eq. (6.6.20):

$$G_{r+1}\left\{1 - T_r \cos \Theta_r \cdot \frac{P_{(r-1)\mathrm{M}}}{P_{r\mathrm{M}}}\right\}.$$

If the rth transistor has no feedback, $T_r = 0$ and the admittance amounts to G_{r+1} in both cases.

The losses due to feedback of the rth stage then become for tuning method A:

$$\Phi_{fr} = \frac{G_{r+1}{}^2}{\left|G_{r+1}\left(1 - T_r \exp(\mathrm{j}\Theta_r)\frac{P_{r-1}}{P_r}\right)\right|^2},$$

$$= \frac{1}{\left|1 - T_r \exp(\mathrm{j}\Theta_r)\frac{P_{r-1}}{P_r}\right|^2}, \tag{6.6.24}$$

and for tuning method B:

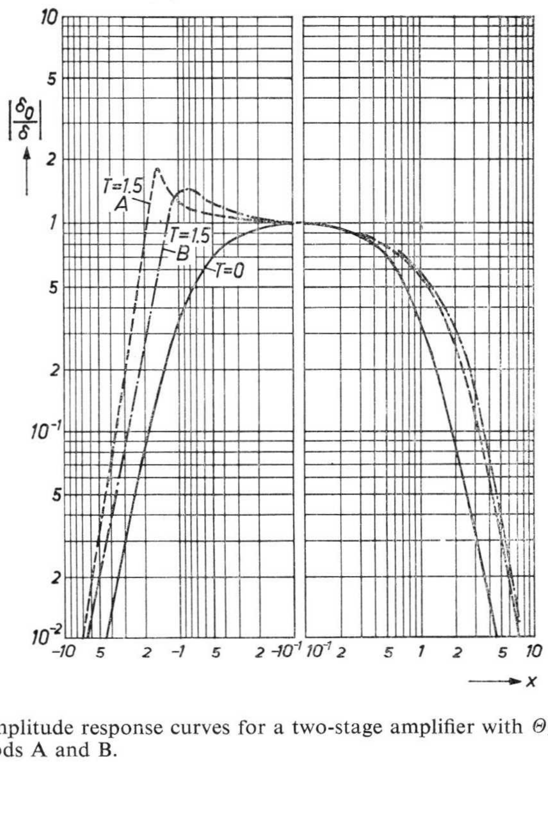


Fig. 6.18. Amplitude response curves for a two-stage amplifier with $\Theta_1 = \Theta_2 = 225°$ for tuning methods A and B.

$$\Phi_{fr} = \frac{1}{\left(1 - T_r \cos \Theta_r \cdot \frac{P_{(r-1)\mathrm{M}}}{P_{r\mathrm{M}}}\right)^2}. \tag{6.6.25}$$

The power gain Φ of any stage of the amplifier can now be calculated from Eq. (6.6.12) together with Eqs. (6.6.24) and (6.6.25).

6.7 Response curve

6.7.1 AMPLITUDE RESPONSE CURVE

The amplitude response curve of the amplifier, which is defined as $a = |\delta_0/\delta|$, can be determined by evaluating δ as a function of the normalized detuning x.

In Fig. 6.18 the amplitude response curve of a two-stage amplifier with three single-tuned bandpass filters has been plotted for tuning methods A and B. For this amplifier $\Theta = 225°$ and $x_1 = x_2 = x_3 = x$. It follows from these curves that they become more asymmetrical for increasing values of T and that the response curves obtained with tuning method B are less asymmetrical than those obtained with tuning method A taking the same values for T.

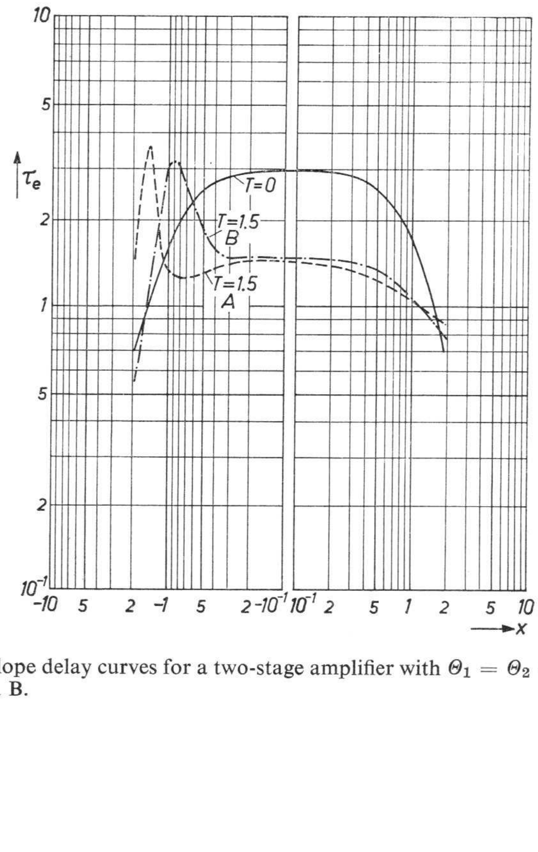


Fig. 6.19. Envelope delay curves for a two-stage amplifier with $\Theta_1 = \Theta_2 = 225°$ for tuning methods A and B.

6.7.2 ENVELOPE DELAY CURVE

According to sub-section 2.5.3 the envelope delay curve of an amplifier can be determined by evaluating $\Delta\varphi/\Delta x$, in which $\varphi = \tan^{-1} \{I_m(\delta)/R_e(\delta)\}$, as a function of x for suitably small values of the interval Δx.

In Fig. 6.19 envelope delay curves have been plotted for the two-stage amplifier considered in the preceding sub-section.

BIBLIOGRAPHY

[6.1] L. G. CRIPPS, *Multi-stage Amplifier Stability*, Electr. and Radio Eng., Vol. **36,** p. 454-458, 1959.

[6.2] Z. JENOLEK and R. S. SIDOROWICZ, *Bandpass Amplifiers*, Wireless Eng., Vol. **31,** p. 84-99, 1954.

[6.3] M. LIM, *Power Gain and Stability of Multi-stage, Narrow Band Amplifiers Employing Non-unilateral Electron Devices*, I.R.E. Trans. on Circuit Theory, **CT-8,** p. 158-166, 1960.

[6.4] C. J. McCLUSKEY, *Stability Criteria for Single-Tuned Transistor Amplifiers*, Unpublished report of the Radio and Television Engineering Department, Philips Electronic Industries Ltd., Ontario, Canada, 1959.

[6.5] C. J. McCLUSKEY, *A General Theory for the Analysis of Tuned Transistor Amplifiers*, Unpublished report of the Radio and Television Engineering Department, Philips Electrical Industries, Ltd, Ontario, Canada, 1960.

[6.6] C. J. McCLUSKEY, *Capacitive Feedback in Synchronously Tuned Transistor Amplifiers*, Unpublished report of the Radio and Television Engineering Department, Philips Electronic Industries Ltd., Ontario, Canada, 1960.

[6.7] C. J. McCLUSKEY, *Bandpass Transistor Amplifiers*, Electronic Techn. Vol. **38,** p. 183-186, 1961.

[6.8] IU. L. SIMONOV, *Analysis of the Stability of Tuned Transistor Amplifiers*,. Radiotekhika i elektronika, Vol. 5, p. 105-116, 1959.

CHAPTER 7

MULTI-STAGE AMPLIFIERS WITH DOUBLE-TUNED BANDPASS FILTERS

In the preceding chapters the analyses of single-stage amplifiers with two single-tuned bandpass filters and with two double-tuned bandpass filters as well as the analysis of multi-stage amplifiers with single-tuned bandpass filters were given. In practice, however, most bandpass amplifiers, for example those used in radio and television receivers, contain more than one stage, the inter-stage coupling usually being formed by double-tuned bandpass filters.

This chapter deals with such multi-stage amplifiers, use being made of the results of the analyses of the preceding chapters. Again use will be made of a determinant method to represent the amplifier performance following the method indicated by McCluskey (See Bibliography [7.4]). The higher order determinants encountered in this analysis will prove to be simple extensions of the determinants used for the single-stage amplifier of Chapter 5.

7.1 Equivalent Circuit of an n-Stage Amplifier with $(n+1)$ Double-Tuned Bandpass Filters

7.1.1 AMPLIFIERS IN THE ADMITTANCE MATRIX ENVIRONMENT

In Section 5.2 it was shown how the equivalent circuit of a complete single-stage amplifier with two double-tuned bandpass filters in the admittance matrix environment is obtained. The transistor(s) and the parallel-parallel-tuned double-tuned bandpass filters are both represented by equivalent admittance parameter four-terminal networks and placed in the correct sequence.

The output terminals of the first four-terminal network are now connected to the input terminals of the second network, and the output terminals of the latter to the input terminals of the third network. Next, the self-admittances of the networks at the points where they are interconnected are combined into one admittance. In this way the equivalent circuit of the single-stage amplifier shown in Fig. 5.3 was obtained.

The same procedure can be followed to combine n transistors (or electron tubes) and $(n + 1)$ double-tuned bandpass filters into an equivalent circuit. Fig. 7.1 shows a block diagram of such an amplifier and Fig. 7.2 its equivalent circuit.

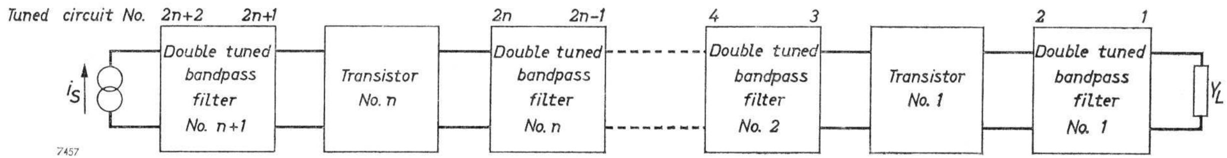


Fig. 7.1. Block diagram of an n-stage amplifier with $(n + 1)$ double-tuned bandpass filters.

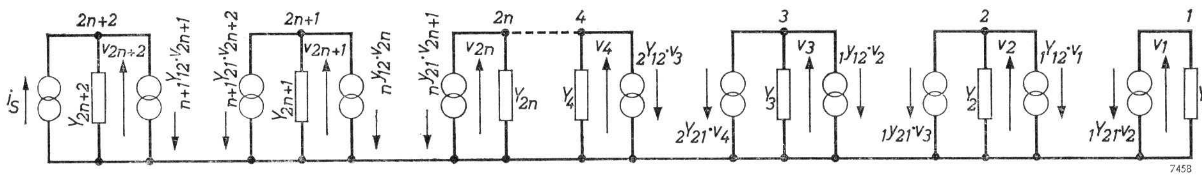


Fig. 7.2. Schematic diagram of an amplifier consisting of n active four-terminal networks (transistors or electron tubes) and $(n + 1)$ double-tuned bandpass filters with parallel-tuned primaries and secondaries. In this diagram

$$Y_{2n+2} = Y_S + {}_{n+1}Y_{11} \qquad Y_3 = {}_2Y_{22} + {}_1y_{11}$$
$$Y_{2n+1} = {}_{n+1}Y_{22} + {}_ny_{11} \qquad Y_2 = {}_1y_{22} + {}_1Y_{11}$$
$$Y_{2n} = {}_ny_{22} + {}_nY_{11} \qquad Y_1 = {}_1Y_{22} + Y_L$$

in which Y_S and Y_L denote the source and load admittance respectively.

The numbering of the transistors, the double-tuned bandpass filters and the single-tuned circuits of which these bandpass filters are composed is again consecutive, beginning at the output side of the amplifier. Suffixes which precede the admittance parameters indicate to which four-terminal network the parameter belongs. As in Chapter 5, capitals are used for the admittance parameters of the double-tuned bandpass filters, and lower case letters for the admittance parameters of the transistors.

The symbol ${}_2Y_{12}$ for example, denotes the Y_{12} parameter of the last but one double-tuned bandpass filter, and the symbol ${}_ny_{21}$ denotes the y_{21} parameter of the n^{th} transistor, numbered from the output side of the amplifier.

7.1.2 AMPLIFIERS IN THE H-MATRIX ENVIRONMENT

In Section 5.3 the equivalent circuit of a single-stage amplifier with two parallel-series tuned double-tuned bandpass filters is derived. It was shown that this equivalent circuit could easily be derived if the properties of the transistor were expressed in the hybrid H-matrix environment and those of the double-tuned bandpass filters were expressed in the K-matrix environment. Using the same method, the equivalent circuit of the n-stage amplifier with $(n+1)$ double-tuned bandpass filters as represented in Fig. 7.3 can be derived.

7.2 The Reduced Amplifier Determinant

Considering the amplifier in the admittance matrix environment it follows that there are $2n+2$ nodal points, see Fig. 7.2, at which, according to Kirchhoff's first law, the sum of the currents equals zero. These current equations for all nodes can be combined into the following matrix equation:

$$\left\| \begin{matrix} i_S \\ 0 \\ 0 \\ - \\ - \\ 0 \\ 0 \\ 0 \end{matrix} \right\| = \left\| \begin{matrix} Y_{2n+2} & {}_{n+1}Y_{12} & 0 & - & - & 0 & 0 & 0 \\ {}_{n+1}Y_{21} & Y_{2n+1} & {}_ny_{12} & - & - & 0 & 0 & 0 \\ 0 & {}_ny_{21} & Y_{2n} & - & - & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ 0 & 0 & 0 & - & - & Y_3 & {}_1y_{12} & 0 \\ 0 & 0 & 0 & - & - & {}_1y_{21} & Y_2 & {}_1Y_{12} \\ 0 & 0 & 0 & - & - & 0 & {}_1Y_{21} & Y_1 \end{matrix} \right\| \cdot \left\| \begin{matrix} v_{2n+2} \\ v_{2n+1} \\ v_{2n} \\ - \\ - \\ v_3 \\ v_2 \\ v_1 \end{matrix} \right\| . \tag{7.2.1}$$

By proceeding in similar manner to that in Section 5.4 it can be derived that the main determinant of Eq. (7.2.1) becomes:

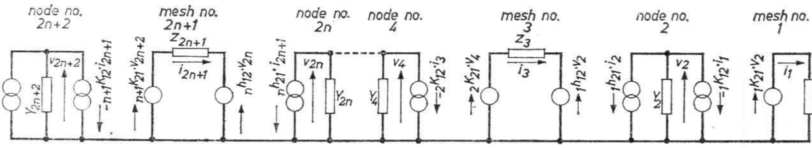


Fig. 7.3. Schematic diagram of an amplifier consisting of n active four-terminal networks and $n + 1$ double-tuned bandpass filters with parallel-tuned primaries and series-tuned secondaries. In this diagram

$$Y_{2n+2} = Y_S + {}_{n+1}K_{11} \qquad Y_4 = {}_2h_{22} + {}_2K_{11}$$
$$Z_{2n+2} = -{}_{n+1}K_{22} + {}_nh_{11} \qquad Z_3 = -{}_2K_{22} + {}_1h_{11}$$
$$Y_2 = {}_1h_{22} + {}_1K_{11}$$
$$Z_1 = -{}_1K_{22} + Z_L$$

$$\Delta_y = \prod_{m=1}^{m=2n+2} G_m \cdot \delta_y, \tag{7.2.2}$$

in which the reduced determinant δ_y is given by:

$$\delta = \begin{vmatrix} 1+jx_{2n+2} & \dfrac{{}_{n+1}Y_{12} \cdot {}_{n+1}Y_{21}}{G_{2n+2} \cdot G_{2n+1}} & 0 & -- & 0 & 0 & 0 \\ 1 & 1+jx_{2n+1} & \dfrac{{}_ny_{12} \cdot {}_ny_{21}}{G_{2n+1} \cdot G_{2n}} & -- & 0 & 0 & 0 \\ 0 & 1 & 1+jx_{2n} & -- & 0 & 0 & 0 \\ - & - & - & -- & - & - & - \\ - & - & - & -- & - & - & - \\ 0 & 0 & 0 & -- & 1+jx_3 & \dfrac{{}_1y_{12} \cdot {}_1y_{21}}{G_3 \cdot G_2} & 0 \\ 0 & 0 & 0 & -- & 1 & 1+jx_2 & \dfrac{{}_1Y_{12} \cdot {}_1Y_{21}}{G_2 \cdot G_1} \\ 0 & 0 & 0 & -- & 0 & 1 & 1+jx_1 \end{vmatrix} \cdot \tag{7.2.3}$$

By introducing transistor regeneration coefficients T_y and regeneration phase angles Θ_y, see Eq. (2.1.11) and (2.1.12), and also the coupling factors q^2 for the double-tuned bandpass filters, see Eq. (5.4.3), the reduced determinant becomes [1]):

[1]) The suffixes y in T_y and Θ_y have been omitted for reasons of simplicity in writing Eq. (7.2.4).

$$\delta = \begin{vmatrix} 1+jx_{2n+2} & -q^2{}_{n+1} & 0 & -- & 0 & 0 & 0 \\ 1 & 1+jx_{2n+1} & T_n \exp(j\Theta_n) & - & 0 & 0 & 0 \\ 0 & 1 & 1+jx_{2n} & -- & 0 & 0 & 0 \\ - & - & - & -- & - & - & - \\ - & - & - & -- & - & - & - \\ 0 & 0 & 0 & -- & 1+jx_3 & T_1 \exp(j\Theta_1) & - \\ 0 & 0 & 0 & -- & 1 & 1+jx_2 & -q_1{}^2 \\ 0 & 0 & 0 & -- & 0 & 1 & 1+jx_1 \end{vmatrix} \cdot \quad (7.2.4)$$

The amplifier in the hybrid-matrix environment can be considered in an analogous way. The various mesh and nodal equations of the equivalent circuit represented in Fig. 7.3 may also be combined in a single matrix equation. By manipulating with the determinant of this equation in the same manner as above using the method of Section 5.4, the reduced determinant of the amplifier becomes as given by Eq. (7.2.4). Values for the quantities T and Θ related to transistor parameters expressed in the H-matrix environment must be substituted, see Eqs. (2.1.24) and (2.1.25).

7.3 Stability

As in the case of the single-stage amplifiers considered in Chapters 2 and 5, the n-stage amplifier is on the boundary of stability when the reduced determinant δ, as given by Eq. (7.2.4), becomes zero, and this depends on the magnitude of $T \exp(j\Theta)$. If all quantities $T \exp(j\Theta)$ are assumed to be identical, there exists a certain upper limit at which the amplifier is on the verge of self-oscillation. (As pointed out in the preceding chapters, Θ is determined exclusively by the transistor transfer properties, whereas T depends also on the tuned circuit dampings. Hence, when the type of transistor to be used and its working point have been chosen and the operating frequency is known, then Θ is fixed, but T is still variable.)

Denoting the boundary of stability of the n-stage amplifier by ${}_nT_g$ and assuming moreover that all double-tuned bandpass filters of the amplifier are identical, ${}_nT_g$ can be calculated from Eq. (7.3.1). In this expression the quantities x_p and x_s represent the normalized frequencies of the primary and secondary of the bandpass filters respectively:

$$\delta = \begin{vmatrix} 1+jx_p & -q^2 & 0 & -- & 0 & 0 & 0 \\ 1 & 1+jx_s & {}_nT_g\cdot\exp(j\Theta) & -- & 0 & 0 & 0 \\ 0 & 1 & 1+jx_p & -- & 0 & 0 & 0 \\ - & - & - & -- & - & - & - \\ - & - & - & -- & - & - & - \\ 0 & 0 & 0 & -- & 1+jx_s & {}_nT_g\cdot\exp(j\Theta) & 0 \\ 0 & 0 & 0 & -- & 1 & 1+jx_p & -q^2 \\ 0 & 0 & 0 & -- & 0 & 1 & 1+jx_s \end{vmatrix} = 0 \quad (7.3.1)$$

By writing out δ as given by Eq. (7.3.1), an n^{th} order polynomial with complex coefficients is obtained. This means that, for an n-stage amplifier, there are n values of ${}_nT_g$ for a given value of Θ.

In the case of a two-stage amplifier Eq. (7.3.1) leads to a quadratic in ${}_2T_g$ with complex coefficients, which can be solved analytically. For $x_p = x_s = x$ ($r = 1$) the result is:

$${}_2T_g \exp(j\Theta) = \left\{1 + \frac{q^2}{1+x^2} + jx\left(1 - \frac{q^2}{1+x^2}\right)\right\} (1 + jx \pm jq). \quad (7.3.2)$$

In Fig. 7.4 this expression has been plotted in the complex plane for $q = 1$. Values of x are indicated on the curves. Apparently, there are two values of ${}_2T_g$ for every value of Θ that gives rise to instability phenomena at different values of x. Because x is variable over a wide range of values the smallest value of ${}_2T_g$ must be considered as the boundary of stability. This boundary, which is indicated by shading in Fig. 7.4, consists of parts of both mathe-

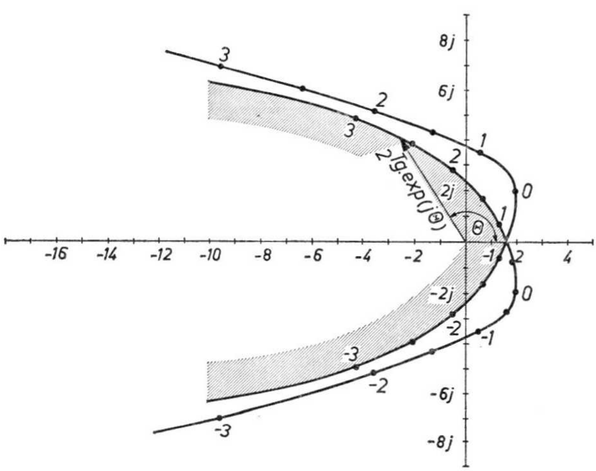


Fig. 7.4. The mathematical expression of the boundary of stability in a two-stage amplifier leads to two curves as drawn in this figure for $q = 1$. The boundary of instability which is important in practical amplifier design is the curve indicated by shading.

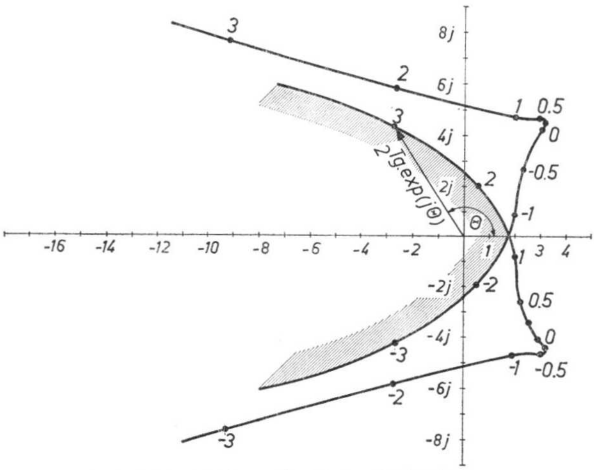


Fig. 7.5. Mathematical boundaries in a two-stage amplifier with $q^2 = 2$.

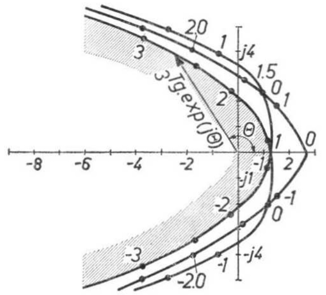


Fig. 7.6. Boundary of stability in a three-stage amplifier with $q^2 = 1$.

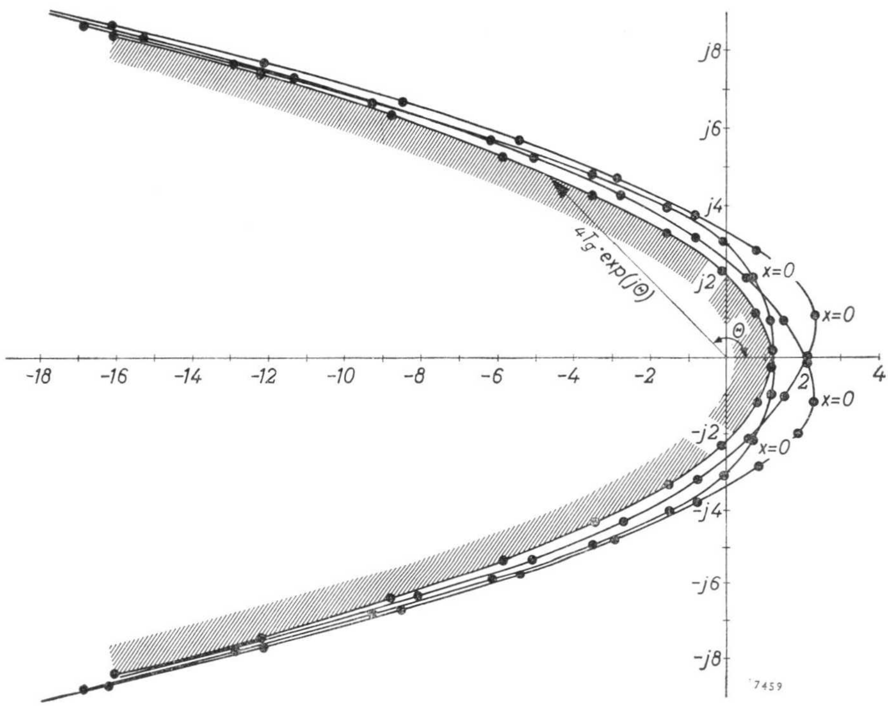


Fig. 7.7. Boundaries of stability for a four-stage amplifier with identical T's and Θ's and q's; $r = 1$ and $q^2 = 1$. The points for $x = 0$ of the various curves are indicated. Other values of x are marked by means of dots placed at intervals of 0.5.

matical boundaries. Fig. 7.5 gives the mathematical boundaries of stability for the same amplifier but now with the coefficient of coupling of the double-tuned bandpass filters equal to $q^2 = 2$.

In Figs. 7.6 and 7.7 the mathematical boundaries of stability for a three and four-stage amplifier with $q^2 = 1$ and $r = 1$ are plotted. The practical boundary of stability is again indicated by shading. For amplifiers with double-tuned bandpass filters consisting of three or more stages an analytical calculation of the boundaries of stability is no longer possible. These boundaries have been calculated by means of an electronic computer using an iterative method.

In Fig. 7.8 the practical boundary of stability of the four-stage amplifier is plotted for $q^2 = 2$, together with that of a single-stage amplifier with two single-tuned bandpass filters ($q^2 = 0$) for which:

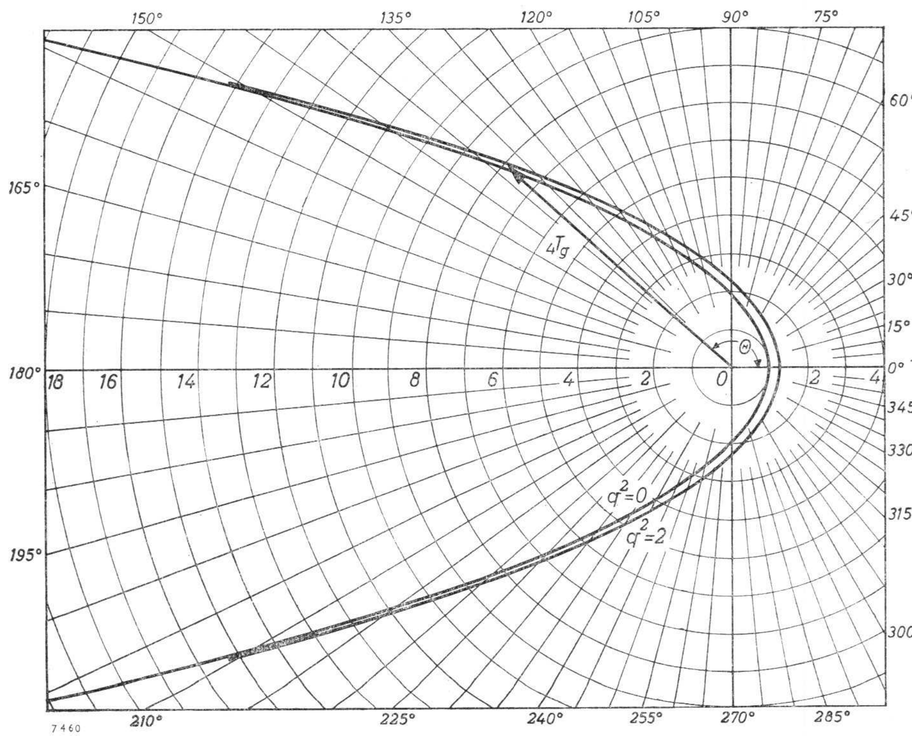


Fig. 7.8. Boundaries of stability of a four-stage amplifier for $q^2 = 2.0$ and of a single-stage amplifier with two single-tuned circuits ($q^2 = 0$).

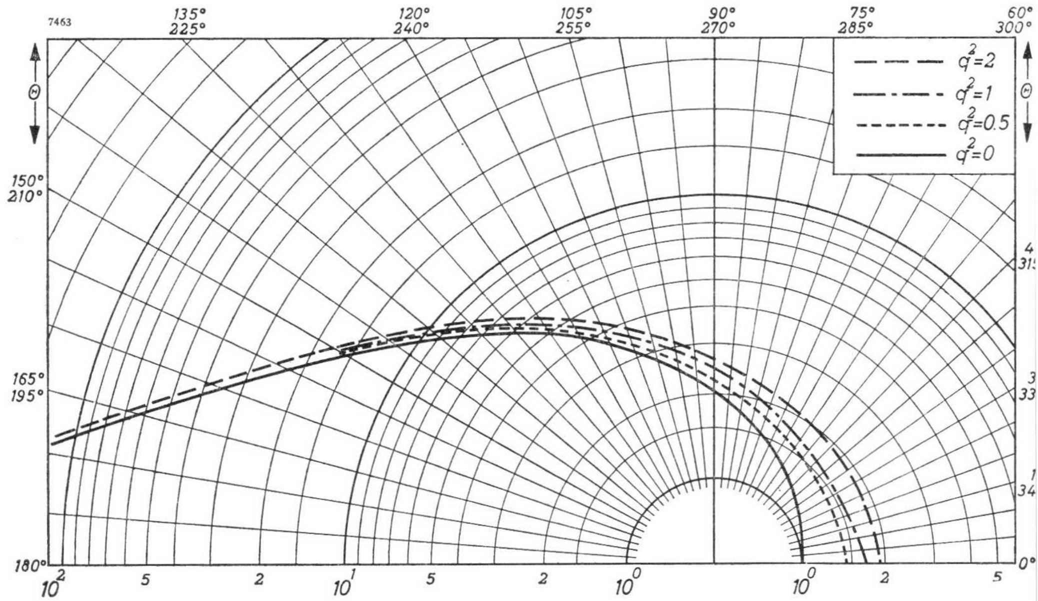


Fig. 7.9. Boundaries of stability of a two-stage amplifier for $r = 1$ and several values of q^2. The fully drawn curve is applicable to a single-stage amplifier with two single-tuned circuits. This figure clearly shows that this curve closely approaches the exact curves and may therefore be considered as an approximate boundary of stability that is sufficiently accurate for most practical cases.

$$T_g = \frac{2}{1 + \cos \Theta} \tag{7.3.2}$$

(see Section 2.2). Practical boundaries of stability for values of q^2 smaller than 2 lie even closer to that applicable to the single-stage amplifier. The curve for T_g given by the simple expression (7.3.2) thus very nearly coincides with the practical boundaries of stability for this four-stage amplifier, irrespective of the value of q^2. The same considerations hold for the three-stage amplifier and, to a lesser extent, for the two-stage amplifier. This also follows from Figs. 7.9, 7.10 and 7.11 in which the boundary of stability according to Eq. (7.3.2) is plotted together with the exact boundaries.

As already mentioned in Chapters 2 and 5, the parabola representing expression (7.3.2) may therefore be considered as the basic boundary of stability for almost every bandpass amplifier (with double-tuned bandpass filters).

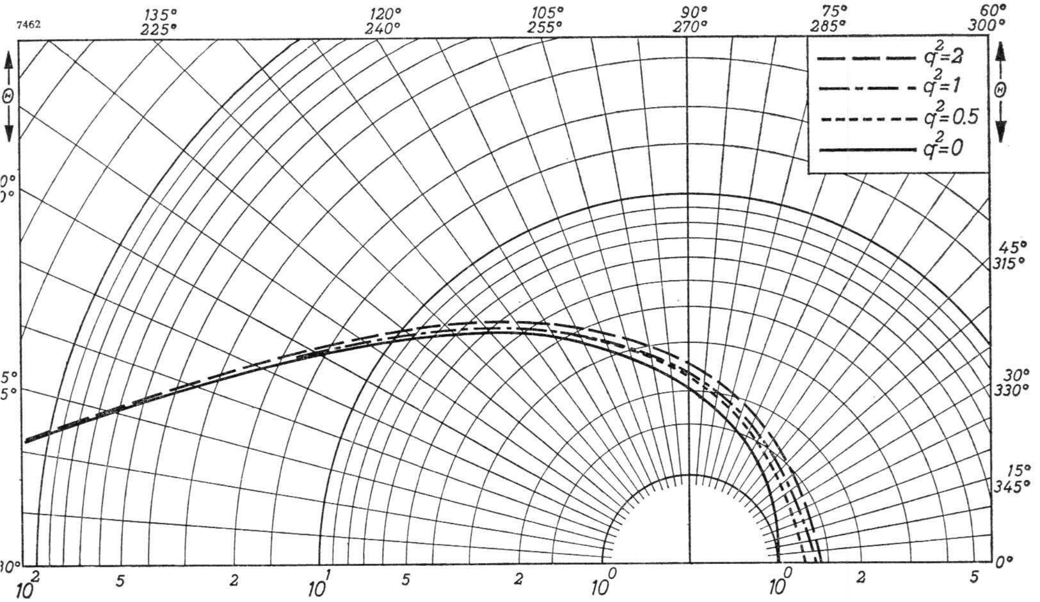


Fig. 7.10. As Fig. 7.9, but for a three-stage amplifier.

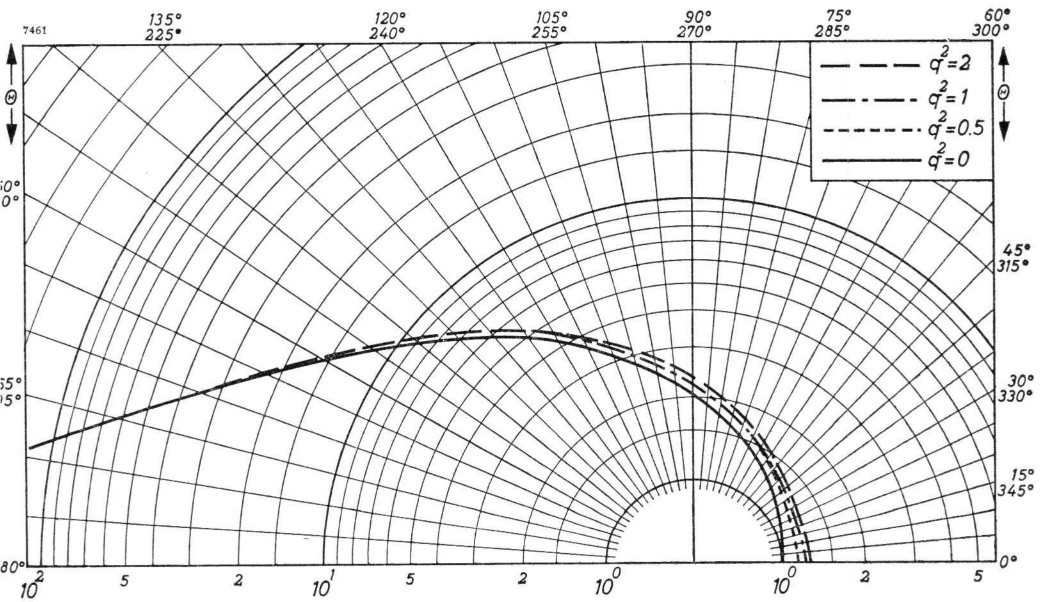


Fig. 7.11. As Fig. 7.9, but for a four-stage amplifier.

It approximates T_g with sufficient accuracy for most practical I.F. amplifier designs.

In those cases which require a more accurate value of T_g use can be made of the graphs plotted in Figs. 7.9, 7.10 and 7.11 which give ${}_nT_g$ for two-, three- and four-stage amplifiers respectively.

7.4 Tuning Procedure

The n-stage amplifier can be aligned either according to tuning method A, B or C as explained in detail in Section 5.7.

Analogous to the method outlined in sub-sections 2.3.7 and 5.7.7, tuning correction terms $p_1x' + p_2x''$ are introduced for the n-stage amplifier. In so doing the reduced determinant for the n-stage amplifier becomes:

$$
{}_n\delta = \begin{vmatrix}
y_{2n+2} & -q^2{}_{n+1} & 0 & -- & 0 & 0 & 0 \\
1 & y_{2n+1} & T_n\cdot\exp(j\Theta) & -- & 0 & 0 & 0 \\
0 & 1 & y_{2n} & -- & 0 & 0 & 0 \\
- & - & - & -- & - & - & - \\
- & - & - & -- & - & - & - \\
0 & 0 & 0 & -- & y_3 & T_1\cdot\exp(j\Theta_1) & 0 \\
0 & 0 & 0 & -- & 1 & y_2 & -q_1{}^2 \\
0 & 0 & 0 & -- & 0 & 1 & y_1
\end{vmatrix}, \qquad (7.4.1)
$$

in which y stands for $1 + j\,(x + p_1x' + p_2x'')$.

It can be derived from this expression that the tuning correction terms and the minor determinants for tuning methods B and C become as given in Table 7.1. Using ${}_n\delta$ as given by Eq. (7.4.1) and the tuning correction terms from this table, together with the values of p_1 and p_2 as indicated in the table in section 5.7.7 (page 145), the performance of the n-stage amplifier can be calculated for each of the three methods of tuning.

7.5 Gain

7.5.1 TRANSDUCER GAIN

The transducer gain of n-stage amplifiers with double-tuned bandpass filters, defined at the tuning frequency ($x = 0$) is again given by:

$$ {}_n\Theta_t = 4\, G_S G_L \cdot |{}_nZ_{t0}|^2, \qquad (7.5.1) $$

in which:

TABLE 7.1. GENERAL FORMULAE FOR n-STAGE AMPLIFIERS WITH $(n+1)$ DOUBLE-TUNED BANDPASS FILTERS

Tuning method B	Tuning method C
Major determinant: see Eq. (7.4.1), page 196.	
Minor determinants at the tuning frequency; $r = 1, 2, 3, \ldots n$:	
$P_{1M} = 1$	$Q_{(2n+2)M} = 1$
$P_{2rM} = P_{(2r-1)M} + q_r^2 P_{(2r-2)M}$	$Q_{(2r+1)M} = Q_{(2r+2)M} + q^2{}_{(r+1)} Q_{(2r+3)M}$
$P_{(2r+1)M} = P_{2rM} - T_r \cos \Theta_r \cdot P_{(2r-1)M}$	$Q_{2rM} = Q_{(2r+1)M} - T_r \cos \Theta_r \cdot Q_{(2r+2)M}$
$P_{(2n+2)M} = P_{(2n+1)M} + q^2{}_{n+1} P_{(2n-2)M}$	$Q_{1M} = Q_{2M} + q_1^2 Q_{3M}$
Tuning correction terms; $r = 1, 2, 3, \ldots n$:	
$x_1' = 0$	$x''_{2n+2} = 0$
$x_{2r}' = 0$	$x''_{2r+1} = 0$
$x'_{2r+1} = T_r \sin \Theta_r \cdot \dfrac{P_{(2r-1)M}}{P_{2rM}}$	$x_{2r}'' = T_r \sin \Theta_r \dfrac{Q_{(2r+2)M}}{Q_{(2r+1)M}}$
$x'_{2n+2} = 0$	$x_1'' = 0$

Transducer gain:

$$_n\Phi_t = 4 \frac{G_S}{G_{2n+2}} \cdot \frac{G_L}{G_1} \cdot \prod_{m=1}^{m=n} {}_m(T \cdot N) \cdot \prod_{m=1}^{m=n+1} q_m{}^2 \cdot \frac{1}{|_n\delta_0|^2},$$

or: $$_n\Phi_t = \prod_{m=1}^{m=n} {}_m\Phi_{uM} \cdot \prod_{m=1}^{m=n+1} {}_m\Phi_{tb} \cdot {}_n\Phi_f ,$$

where: $$_n\Phi_f = \frac{1}{|_n\delta_0|^2} \cdot \prod_{m=1}^{m=n+1} (1 + q_m{}^2)^2 .$$

$$_nZ_{t0} = \frac{\prod_{m=1}^{m=n} {}_m y_{21} \cdot \prod_{m=1}^{m=n+1} jq_m}{\left(\prod_{m=1}^{m=2n+2} G_m \right)^{1/2} \cdot {}_n\delta_0}, \tag{7.5.2}$$

where $_n\delta_0$ follows from Eq. (7.4.1) by putting $x = 0$. The quantities p_1 and p_2 occurring in the expression for $_n\delta_0$ depend on the tuning method and follow from the table on page 145.

In an analogous way to that employed in Section 5.8, the transducer gain $_n\Phi_t$ can be split up into the maximum unilateralized power gains Φ_{uM} of the transistors, the transducer losses Φ_{tb} of the double-tuned bandpass filters and a factor $_n\Phi_f$ which accounts for the losses caused by the real part of the feedback of the transistors in the amplifier. Hence:

$$ {}_n\Phi_t = \prod_{m=1}^{m=n} {}_m\Phi_{uM} \cdot \prod_{m=1}^{m=n+1} {}_m\Phi_{tb} \cdot {}_n\Phi_f, \tag{7.5.3} $$

in which:

$$ {}_n\Phi_f = \frac{1}{|{}_n\delta_0|^2} \cdot \prod_{m=1}^{m=n+1} (1 + q_m{}^2)^2 . \tag{7.5.4} $$

With Eq. (5.8.18) the transducer gain of the n-stage amplifier can alternatively be expressed by:

$$ {}_n\Phi_t = \frac{4\,G_S}{G_{2n+2}} \cdot \frac{G_L}{G_1} \cdot \prod_{m=1}^{m=n} {}_m(T.N) \cdot \prod_{m=1}^{m=n+1} q_m{}^2 \cdot \frac{1}{|{}_n\delta_0|^2} . \tag{7.5.5} $$

7.5.2 GAIN PER STAGE

7.5.2.1 Voltage Gain

For amplifiers of which the constants are expressed in the admittance matrix environment the voltage gain per stage can readily be calculated. The voltage at the input terminals of the r^{th} transistor of the amplifier is denoted by v_{2r+1} (see Fig. 7.2). From Eqs. (7.2.1), (7.2.2) and (7.2.4) it follows, using Cramer's Rule:

$$ v_{2r+1} = i_S \cdot \frac{\prod_{m=1}^{m=2r} G_m}{\prod_{m=1}^{m=2n+2} G_m} \cdot \frac{P_{2r}}{P_{2n+2}} \cdot \prod_{m=r+1}^{m=n+1} {}_mY_{21} \cdot \prod_{m=r+1}^{m=n} {}_my_{21} . \tag{7.5.6} $$

A corresponding relation can be derived for the input voltage of stage $(r - 1)$ (denoted by v_{2r-1}). The voltage gain $(V.G.)_r$ between the input terminals of the r^{th} and the $(r - 1)^{\text{th}}$ transistor then becomes:

$$ (V.G.)_r = \left|\frac{v_{2r+1}}{v_{2r-1}}\right| = \frac{1}{G_{2r} \cdot G_{2r-1}} \cdot \left|\frac{P_{2r-2}}{P_{2r}}\right| \cdot |{}_rY_{21}| \cdot |{}_ry_{21}| . \tag{7.5.7} $$

With ${}_rY_{21} = q_r\sqrt{G_{2r} \cdot G_{2r-1}},$ (7.5.8)

Eq. (7.5.7) becomes:

$$ (V.G.)_r = \frac{1}{\sqrt{G_{2r} \cdot G_{2r-1}}} \cdot \left|\frac{P_{2r-2}}{P_{2r}}\right| \cdot q_r \cdot |{}_ry_{21}| . \tag{7.5.9} $$

Now we may write:

$$ \begin{aligned} P_{2r} &= P_{2r-1} + q_r{}^2 P_{2r-2}, \\ &= P_{2r-2} - T_{r-1} \exp(j\Theta_{r-1}) \cdot P_{2r-3} + q_r{}^2 P_{2r-2}, \\ &= (1 + q_r{}^2) P_{2r-2} - T_{r-1} \exp(j\Theta_{r-1}) P_{2r-3}. \end{aligned} $$

Hence:

$$\frac{P_{2r-2}}{P_{2r}} = \frac{1}{1 + q_r^2 - T_{r-1} \exp(\mathrm{j}\Theta_{r-1}) \cdot \dfrac{P_{2r-3}}{P_{2r-2}}}. \quad (7.5.10)$$

Substituting Eq. (7.5.10) into Eq. (7.5.9) gives:

$$(V.G.)_r = \frac{1}{\sqrt{G_{2r} \cdot G_{2r-1}}} \cdot \frac{q_r}{1 + q_r^2} \cdot |_r y_{21}| \cdot \left| \frac{1 + q_r^2}{1 + q_r^2 - T_{r-1} \exp(\mathrm{j}\Theta_{r-1}) \cdot \dfrac{P_{2r-3}}{P_{2r-2}}} \right|. \quad (7.5.11)$$

For $T_{r-1} = 0$ (no feedback in transistor $(r - 1)$) this becomes:

$$(V.G.)_r = (Z_t)_r \cdot |_r y_{21}|. \quad (7.5.12)$$

Here $(Z_t)_r$ denotes the transimpedance of the r^{th} double-tuned bandpass filter at the tuning frequency (see Appendix III). The last factor in Eq. (7.5.11) thus accounts for the extra admittance due to the feedback of the transistor loading the r^{th} double-tuned bandpass filter.

7.5.2.2 *Power Gain*

The power gain Φ_r of the r^{th} stage of the amplifier is given by:

$$\Phi_r = {}_r\Phi_{u\mathrm{M}} \cdot {}_r\Phi_{tb} \cdot \Phi_{fr}. \quad (7.5.13)$$

In this expression ${}_r\Phi_{u\mathrm{M}}$ denotes the maximum unilateralized power gain of the r^{th} transistor, ${}_r\Phi_t$ denotes the transducer losses of the double-tuned bandpass filter following the r^{th} transistor and Φ_{fr} denotes the losses attributed to the feedback of the r^{th} transistor.

The feedback losses Φ_{fr} are caused by the extra input admittance of the transistor due to its feedback. In analogy with Section 6.6.2.2 the extra input admittance can be calculated as:

$$-G_{2r+1} \cdot T_r \exp(\mathrm{j}\Theta_r) \cdot \frac{P_{2r-1}}{P_{2r}}.$$

The total admittance at the input terminals of the rth transistor therefore becomes:

$$G_{2r+1} \left\{ 1 + q_{r+1}^2 - T_r \exp(\mathrm{j}\Theta_r) \cdot \frac{P_{2r-1}}{P_{2r}} \right\},$$

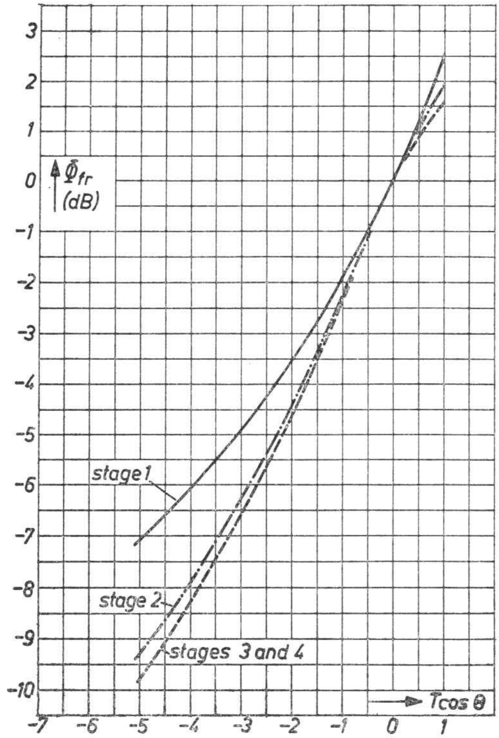


Fig. 7.12. Feedback losses (or gain) of the four stages of a four-stage amplifier as a function of $T \cos \Theta$. All stages have identical elements and the coupling coefficient of the double-tuned bandpass filter equals $q^2 = 1$. Furthermore, it is assumed that tuning method B is applied.

and the losses due to the feedback become:

$$\Phi_{fr} = \frac{(1 + q^2{}_{r+1})^2}{\left| 1 + q^2{}_{r+1} - T_r \exp(\mathrm{j}\Theta_r) \dfrac{P_{2r-1}}{P_{2r}} \right|^2}. \tag{7.5.14}$$

For tuning method B, this expression reduces to:

$$\Phi_{fr} = \frac{(1 + q^2{}_{r+1})^2}{\left(1 + q^2{}_{r+1} - T_r \cos \Theta_r \cdot \dfrac{P_{(2r-1)\mathrm{M}}}{P_{2r\mathrm{M}}} \right)^2}. \tag{7.5.15}$$

In Fig. 7.12 the feedback losses of each stage of a four-stage amplifier with identical stages and $q^2 = 1$ are plotted as a function of $T \cos \Theta$. It is assumed that the amplifier is tuned according to method B so that Eq. (7.5.15) is applicable. It follows from these curves that the feedback losses of stages 2, 3 and 4 are almost equal and larger than those of stage 1. The small differences in feedback losses between stages 2, 3 and 4 can be explained by considering that in a chain of amplifying stages the influence of one stage on the foregoing stage becomes identical for all stages if the chain is infinitely long. The difference in losses between stage 1 and stage 2 is therefore larger than that between stage 2 and stage 3, which is in turn larger than that between stage 3 and stage 4.

7.6 Response Curve

7.6.1 THE COMPLEX RESPONSE CURVE

The reduced determinant ${}_n\delta$ derived for an n-stage amplifier is a complex function of the normalized frequency x with T and Θ as parameters. In order to ascertain the complex response curve (cf. sub-section 2.5.1) it is necessary to relate the values of x of the various tuned circuits to a normalized value for the complete amplifier.

Since this book is confined to synchronously tuned amplifiers [1]) the β-values of the various resonant circuits are all identical. The x-values can then simply be related by incorporating the Q-values of all resonant circuits into a normalized Q-value for the complete amplifier. Denoting this normalized frequency for the complete amplifier by x gives:

$$ {}_n\delta(x) = R_e\,\{{}_n\delta(x)\} + \mathrm{j}I_m\,\{{}_n\delta(x)\}. \tag{7.6.1}$$

By plotting ${}_n\delta$ in the complex plane, the complex response curve of the n-stage amplifier is obtained.

7.6.2 THE AMPLITUDE RESPONSE CURVE

In accordance with sub-section 2.5.2, the normalized amplitude response curve of the n-stage amplifier is given by:

$$ \left|\frac{{}_n\delta_0}{{}_n\delta}\right|, \tag{7.6.2}$$

in which ${}_n\delta$ is given by Eq. (7.4.1) and ${}_n\delta_0$ is equal to ${}_n\delta$ at $x = 0$.

---

[1]) Although the amplifier is synchronously tuned, it is inherent to tuning methods B and C that the circuits resonate at different frequencies.

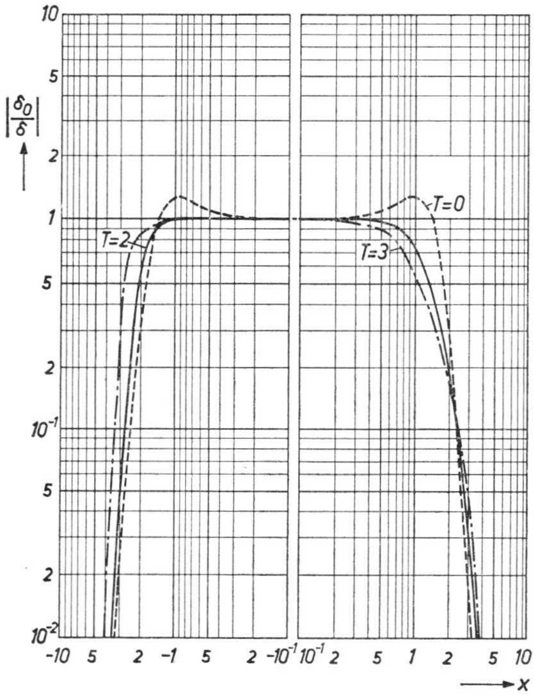


Fig. 7.13. Amplitude response curves of a three-stage amplifier with four identical double-tuned bandpass filters, applicable to tuning methods B and C. $\Theta = 225°$, $r = 1$ and $q^2 = 2$.

Fig. 7.13 shows the amplitude response curve of a three-stage amplifier of which $q^2 = 2$ and $r = 1$. The regeneration phase angle is taken to be 225°. These curves have been calculated for $T = 0$, $T = 2$ and $T = 3$. They are applicable to both tuning methods B and C.

Comparison of the curves for $T = 0$ and $T = 2$ clearly shows that the humps of the curve for $T = 0$, which are to be attributed to the overtransitional coupling of the double-tuned bandpass filters, have disappeared at $T = 2$. This is due to $T \cos \Theta$ assuming a negative value (cf. Section 5.9).

7.6.3 THE AMPLITUDE RESPONSE CURVE FOR A LARGE VALUE OF THE REGENERATION COEFFICIENT

Fig. 7.14 represents the amplitude response curve for the same amplifier as in sub-section 7.6.2 but now for $T = 8$ and for tuning method A. According to Fig. 7.10 the value of T_g for this case amounts to $T_g = 8$. This implies that for $T = 8$ the amplitude response must become infinite which, also appears from Fig. 7.14 for $x = -2.9$. According to sub-section 7.3 there are three mathematical boundaries of stability for this three-stage amplifier. This also follows from Fig. 7.14 which indicates instability phenomena at two other frequencies.

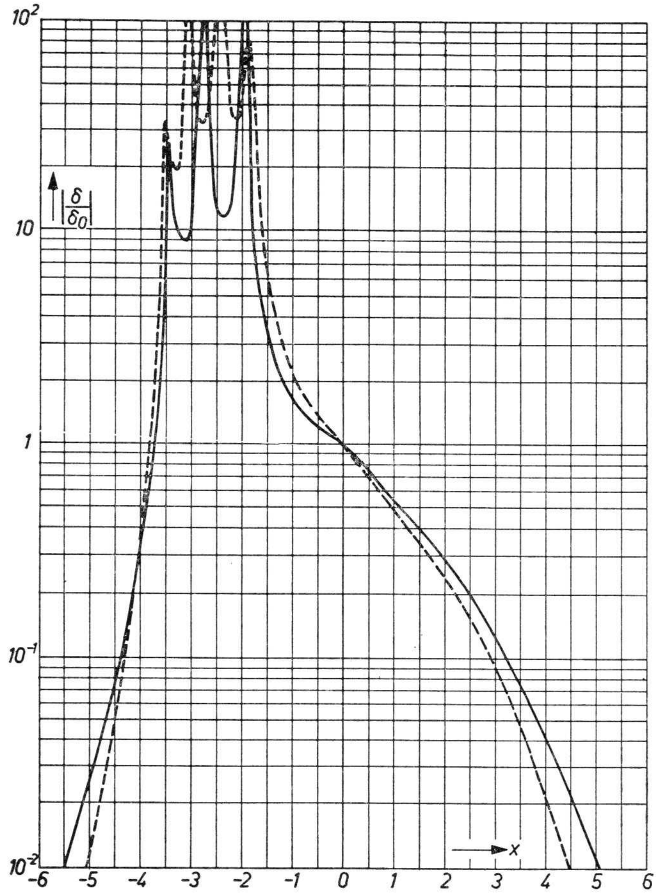


Fig. 7.14. Amplitude response curve for the same amplifier as in Fig. 7.13 but now for a value of regeneration coefficient almost at the boundary of stability ($T = 8$) of the amplifier. According to the theory presented in sub-section 7.3 instability phenomena should occur at three different frequencies, which is clearly illustrated in this figure. The curve has been calculated for tuning method A for which also Fig. 7.10 is valid. The dashed curve is valid for a four-stage amplifier with the same combination of parameters. Now, instability phenomena are present at four different frequencies.

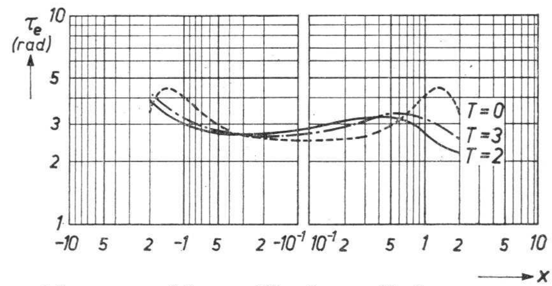


Fig. 7.15. Envelope delay curves of the amplifier the amplitude response curves of which are given in Fig. 7.13.

7.6.4 THE ENVELOPE DELAY CURVE

As shown in section 2.5, the envelope delay of an amplifier is given by the expression:

$$t_e = \tau_e \frac{2Q}{\omega_0}, \tag{7.6.3}$$

in which τ_e represents the reduced envelope delay:

$$\tau_e = \frac{\Delta\varphi}{\Delta x}. \tag{7.6.4}$$

Now $\Delta\varphi$ is defined as the difference between the φ-values calculated from:

$$\varphi = \tan^{-1} \frac{I_m \{{}_n\delta(x)\}}{R_e \{{}_n\delta(x)\}}, \tag{7.6.5}$$

provided the values of x are not too far apart. The difference between the x-values obviously corresponds to Δx.

Fig. 7.15 shows the reduced envelope delay curve for the same amplifier as the one the response curves of which were given in Fig. 7.13. The graph reveals that the envelope delay curves are slightly flattened due to the presence of feedback.

7.7 Table of Formulae

In Table 7.1 given on page 197, general formulae are set out for the n-stage amplifier with $(n + 1)$ double tuned bandpass filters. The determinant for ${}_n\delta$ given by Eq. (7.4.1) on page 196 is applicable to these general formulae.

In this table the index r denotes either the r^{th} transistor or the r^{th} double-tuned bandpass filter of the amplifier, starting to count in both cases at the output side of the amplifier. In Fig. 7.16 the quantities P_M (valid for tuning method B) have been plotted for an amplifier consisting of identical stages with $q^2 = 1$.

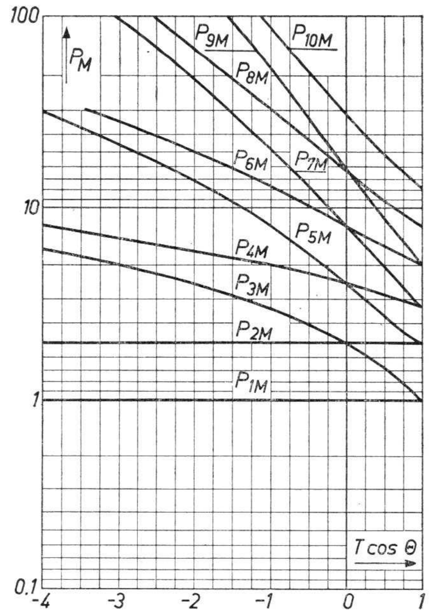


Fig. 7.16. Plot of P_M (minor determants in case of tuning method B) has been plotted as a function of T cos Θ for an amplifier consisting of identical stages and $q^2 = 1$,

The equations given in the tables for the minor determinants (required for calculating the tuning correction factors) are recurrent relations. This implies that for multi-stage amplifiers the final results become extremely complicated unless these amplifiers consist of identical stages, as is often the case.

BIBLIOGRAPHY

[7.1] P. BURA, *Regeneration Effects in Double-Tuned Bandpass Amplifiers*, 1957, I.R.E. Nat. Conv. Record, Pt. 2, p. 107-119.

[7.2] G. E. VALLEY and H. WALLMAN, *Vacuum Tube Amplifiers*, Chapter VI, McGraw Hill, New York 1958.

[7.3] Z. JENOLEK and R. S. SIDOROWICZ, *Bandpass Amplifiers*, Wireless Eng., Vol. **31**; p. 84-99, 1954.

[7.4] C. J. McCLUSKEX, *Bandpass Transistor Amplifiers*, Electronic Tech., Vol. **38**, p. 183-186, 1961.

CHAPTER 8

MULTI-STAGE AMPLIFIERS WITH ARBITRARY TYPES OF INTERSTAGE COUPLING NETWORKS

8.1 General

In Chapters 6 and 7 multi-stage amplifier are considered employing only single-tuned or only double-tuned bandpass filters. There is however no limitation to the method of analyzing the amplifier on account of the types of coupling networks between the various stages. The interstage networks may either be single-tuned bandpass filters (as employed exclusively in Chapter 6), double-tuned bandpass filters (as employed exclusively in Chapter 7) or multiple-tuned bandpass filters. Also complicated networks may be used as interstage coupling devices.

The method of analysis in all cases amounts to determining the definite admittance matrix (assuming that all active and passive networks contained in the amplifier are expressed in the admittance matrix environment) of the amplifier according to the method considered in Appendix I using the normalized detuning concept of specifying the admittance of the tuned circuits. Then this definite admittance matrix may be simplified by introducing regeneration coefficients for the transistors and coupling factors for the double-tuned or multiple-tuned bandpass filters.

When all bandpass filters of the amplifier consist of one or more single-tuned circuits which are parallel-tuned, all tuned circuit admittances will appear in the main diagonal of the definite admittance matrix, whereas all forward and reverse transfer admittances of the transistors and bandpass filters will appear in the diagonals adjacent to the main diagonal. If, furthermore, no couplings are present in the amplifier between the various stages except those via the interstage coupling networks, the definite admittance matrix will contain zero entries at all places except at the three diagonals mentioned.

The order of the admittance matrix will be equal to the number of nodal points in the amplifier. In the cases considered above, the order of the matrix thus equals the number of tuned circuits contained in the amplifier.

When one or more complicated interstage coupling networks are used in the amplifier it is often convenient to determine the definite admittance matrices of these networks separately before considering the whole amplifier. Sometimes it will be advisable to reduce the order of these separate matrices to 2×2 before incorporating them in the definite admittance matrix of the complete amplifier. The matrix reduction method is considered in Appendix I.

Up to now all amplifier analyses are confined to synchronous tuning of the various tuned circuits using either of the methods A, B or C. In the case of non-synchronous tuning of the various circuits "frequency shift terms", v, must be introduced in the tuned circuits admittances. These frequency shift terms relate the tuning frequencies of the various circuits to that of a reference circuit.

For tuning methods B and C, which are applicable only in the case of synchronous tuning, tuning correction terms, x', must be added to the tuned circuit admittances. These tuning correction terms can be calculated according to the methods outlined in the preceding chapters.

When the (loaded) quality factors of the tuned circuits of the amplifier are not identical, these quality factors must be related to the quality factor of a reference circuit using certain proportionality factors.

The various steps in arriving at the representation of the admittances o the tuned circuits in the general amplifier determinant may be summarized as follows:

Consider the r^{th} tuned circuit of the amplifier and assume that the amplifier is tuned according to method A. Then its relative admittance may be written as:

$$1 + jx_r. \tag{8.1.1}$$

If the amplifier is tuned either according to methods B or C, a tuning correction term x', the value of which is generally different for the two tuning methods, appears in the expression for the relative admittance of the tuned circuit, namely:

$$1 + j(x_r + x_r'). \tag{8.1.2}$$

Now the relative admittance of this tuned circuit must be related to that of the reference circuit. Let the quality factor of this reference circuit be denoted Q_{ref} and its relative detuning by:

$$\beta_{ref} = \frac{f}{f_{0,ref}} - \frac{f_{0,ref}}{f}. \tag{8.1.3}$$

Then the normalized detuning of this circuit becomes:

$$x_{ref} = Q_{ref} \cdot \beta_{ref}. \tag{8.1.4}$$

When the rth tuned circuit resonates at a frequency f_1 we may put:

$$\beta_1 = \frac{f_1}{f_{0,\mathrm{ref}}} - \frac{f_{0,\mathrm{ref}}}{f_1}. \tag{8.1.5}$$

Together with the quality factor Q_r of this circuit, we can define a frequency shift term:

$$v_r = Q_r \cdot \beta_1, \tag{8.1.6}$$

which adds to the relative admittance of the circuit as:

$$1 + \mathrm{j}(x_r + v_r + x_r'). \tag{8.1.7}$$

Next the quality factor Q_r of the rth tuned circuit is related to that of the reference circuit by:

$$a_r = \frac{Q_r}{Q_{\mathrm{ref}}}. \tag{8.1.8}$$

Then the relative admittance of the rth tuned circuit becomes with Eq. (8.1.7):

$$y_r = 1 + \mathrm{j}\{a_r(x_{\mathrm{ref}} + v_r) + x_r'\}. \tag{8.1.9}$$

If the general amplifier determinant has been written down and the various correction terms are included in the representation of the relative admittance of the tuned circuits as shown in Eq. (8.1.9), the determinant can be evaluated as a function of the normalized detuning of the reference circuit. This evaluation then yields information regarding the stability, the gain and the amplitude response as well as the envelope delay curves of the amplifier in the same manner as considered in the preceding chapters.

8.2 n-Stage Amplifier with n Double-Tuned Bandpass Filters and One Single-Tuned Bandpass Filter

A very important class of multi-stage amplifiers is that in which the output bandpass filter is a single-tuned circuit and the coupling networks between the various stages are double-tuned bandpass filters. This type of amplifier is mostly used in those cases in which the amplifier drives a detector circuit and an optimum match between amplifier and detector circuit is of prime importance.

According to the method outlined in the preceding section the general determinant for this type of amplifier can be derived. The reduced form of this determinant is given by Eq. (8.2.1), in which it is assumed that all tuned circuits are tuned synchronously either according to methods A, B or C:

$$
{}_n\delta = \begin{vmatrix}
y_{2n+1} & -q_n^2 & 0 & - & - & 0 & 0 & 0 \\
1 & y_{2n} & T_n\exp(\mathrm{j}\Theta_n) & - & - & 0 & 0 & 0 \\
0 & 1 & y_{2n-1} & - & - & 0 & 0 & 0 \\
- & - & - & - & - & - & - & - \\
- & - & - & - & - & - & - & - \\
0 & 0 & 0 & - & - & y_3 & -q_1^2 & 0 \\
0 & 0 & 0 & - & - & 1 & y_2 & T_1\exp(\mathrm{j}\Theta_1) \\
0 & 0 & 0 & - & - & 0 & 1 & y_1
\end{vmatrix} \tag{8.2.1}
$$

TABLE 8.1 GENERAL FORMULAE FOR n-STAGE AMPLIFIERS WITH n DOUBLE-TUNED AND ONE SINGLE-TUNED BANDPASS FILTERS.

<table>
<tr><th>Tuning method B</th><th>Tuning method C</th></tr>
<tr><td colspan="2">Major determinant: see Eq. (8.2.1).
Minor determinants at the tuning frequency; $r = 1, 2, 3, \ldots n$:</td></tr>
<tr><td>$P_{1\mathrm{M}} = 1$
$P_{2r\mathrm{M}} = P_{(2r-1)\mathrm{M}} - T_r\cos\Theta_r \cdot P_{(2r-2)\mathrm{M}}$
$P_{(2r+1)\mathrm{M}} = P_{2r\mathrm{M}} + q_r^2 P_{(2r-1)\mathrm{M}}$</td><td>$Q_{1\mathrm{M}} = Q_{2\mathrm{M}} - T_1\cos\Theta_1 \cdot Q_{3\mathrm{M}}$
$Q_{2r\mathrm{M}} = Q_{(2r+1)\mathrm{M}} + q_{(r+1)}^2 Q_{(2r+2)\mathrm{M}}$
$Q_{(2r+1)\mathrm{M}} = Q_{(2r+2)\mathrm{M}} - T_r\cos\Theta_r \cdot Q_{(2r+3)\mathrm{M}}$
$Q_{(2n+1)\mathrm{M}} = 1$</td></tr>
<tr><td colspan="2">Tuning correction factors; $r = 1, 2, 3, \ldots n$:</td></tr>
<tr><td>$x_1' = 0$
$x_{2r}' = T_r\sin\Theta_r \cdot \dfrac{P_{(2r-2)\mathrm{M}}}{P_{(2r-1)\mathrm{M}}}$
$x_{2r+1}' = 0$</td><td>$x_1'' = T_1\sin\Theta_1 \cdot \dfrac{Q_{3\mathrm{M}}}{Q_{2\mathrm{M}}}$
$x_{2r}'' = 0$
$x_{(2r+1)}'' = T_r\sin\Theta_r \cdot \dfrac{Q_{(2r+3)\mathrm{M}}}{Q_{(2r+2)\mathrm{M}}}$
$x_{(2n+1)}'' = 0$</td></tr>
<tr><td colspan="2">Transducer gain:
$${}_n\Phi_t = \frac{4G_S}{G_{2n+1}} \cdot \frac{G_L}{G_1} \cdot \prod_{m=1}^{m=n} {}_m(T \cdot N) \cdot \prod_{m=1}^{m=n} \cdot q_m^2 \cdot \frac{1}{|{}_n\delta_0|^2},$$
or:
$${}_n\Phi_t = \prod_{m=1}^{m=n} {}_m\Phi_{u\mathrm{M}} \cdot \prod_{m=1}^{m=n} {}_m\Phi_{tb} \cdot \Phi_i' \cdot \Phi_{mm} \cdot {}_n\Phi_f,$$
where: $\Phi_{tb} = (1 - w_p)(1 - w_s) \cdot \left(\dfrac{2q}{1+q^2}\right)^2$, (transducer losses double-tuned bandpass filter)
$\Phi_i' = (1 - w_1)^2$, (insertion losses single-tuned bandpass filter)
$\Phi_{mm1} = \dfrac{(g_{22} + G_L)^2}{4g_{22}G_L}$, mismatch losses across the single-tuned bandpass filter.
and $${}_n\Phi_f = \frac{1}{|{}_n\delta_0|^2} \cdot \prod_{m=1}^{m=n} (1 + q_m^2)^2$$</td></tr>
</table>

The transimpedance of this type of amplifier can be derived as:

$$Z_{t,n} = \frac{\prod_{m=1}^{m=n} {}_m y_{21} \cdot \prod_{m=1}^{m=n} jq_m}{\left[\prod_{m=1}^{m=2n\times 1} G_m\right]^{\frac{1}{2}} \cdot G_1 \cdot {}_n\delta}, \tag{8.2.2}$$

from which the transducer gain can be calculated.

In Table 8.1 the most important results obtained when analyzing this type of amplifier are compiled, using the same scheme as for the n-stage amplifiers with $n + 1$ double-tuned bandpass filters as analyzed in Chapter 7 (see Table 7.1).

Amplitude response and envelope delay curves of this type of amplifier may be obtained from the reduced determinant of Eq. (8.2.1) in the usual manner.

BIBLIOGRAPHY

[8.1] P. BURA, *Regenreation Effects in Double-tunned Bandpass, Amplifiers*, 1957, I. R. E. Nat. Conv. Record, Pt. 2, p. 107-119.

[8.2] G. E. VALLEY and H. WALLMAN, *Vacuum Tube Amplifiers*, Chapter VI, McGraw Hill, New York 1958.

[8.3] Z. JENOLEK, and R. S. SIDOROWICZ, *Bandpass Amplifiers*, Wireless Eng., Vol. **31**, p. 84-99, 1954.

[8.4] C. J. MCCLUSKEY, *Bandpass Transistor Amplifiers*, Electronic Techn. Vol. **38**, p. 183-186, 1961,

CHAPTER 9

AMPLIFIERS WITH DOUBLE-TUNED BANDPASS FILTERS WITH COMPLEX COUPLING COEFFICIENTS

In transistor bandpass amplifiers, unless special precautions are taken, some asymmetry in the amplitude response curve always occurs due to the internal feedback of the transistors employed. This asymmetry may be compensated by a special method of adjusting the various resonant circuits of the amplifier. This, however, requires complicated tuning procedures and, moreover, reduces the obtainable gain.

Another method of compensating this asymmetry, which does not have the drawbacks of the special tuning methods is the use of double-tuned bandpass filters with complex coupling. These are bandpass filters in which the coupling system contains resistive as well as reactive elements.

To analyze this method of achieving symmetrical amplitude response curves a single stage amplifier will first be considered. Then the analysis will be extended to multi-stage amplifiers.

9.1 Conditions for Symmetry of Response Curve in a Single-Stage Amplifier

As already referred to, the asymmetry of the response curve of an amplifier in which amplifying elements with internal feedback are employed can always be compensated by means of a special method of adjusting the various resonant circuits. In general, however, a trial and error method will be required which does not lend itself to a mathematical analysis. This implies that it is not possible to predict, on a theoretical basis, the performance of such an amplifier. These systems will therefore not be considered further in this book.

Another method of achieving symmetry is the use of synchronously tuned double-tuned bandpass filters with complex coupling. This method, which lends itself well to a mathematical analysis, will be developed in the following sub-sections.

9.1.1 THE SINGLE-STAGE AMPLIFIER WITH TWO SINGLE-TUNED BANDPASS FILTERS

In Fig. 9.1 a simplified diagram of a one-stage amplifier with two single-tuned circuits has been given. This type of amplifier has been analyzed in

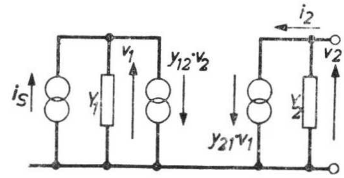


Fig. 9.1. Equivalent circuit diagram of a single-stage amplifier with two single-tuned bandpass filters.

detail in Chapter 2, in which it was concluded that its transimpedance function could be written as:

$$Z_t = -\frac{y_{21}}{G_1 G_2 \cdot \delta}, \tag{9.1.1}$$

and

$$\delta = (1 + jx_1)(1 + jx_2) - T\exp(j\Theta), \tag{9.1.2}$$

assuming that tuning method A has been applied.

If y_{21} is assumed to be constant over the passband considered, the amplitude response curve will be symmetrical with respect to the centre of the passband if $|\delta|$ is symmetrical with respect to $x_1 = x_2 = 0$.

The first part of the expression for δ, see Eq. (9.1.2), represents a parabola in the complex plane which is symmetrical with respect to the real axis. In Fig. 2, this parabola has been plotted for $x_1 = x_2 = x$. Also the vector

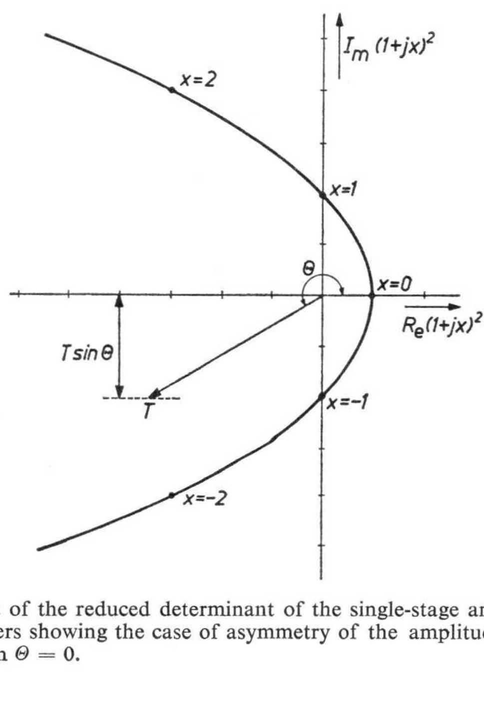


Fig. 9.2. Polar plot of the reduced determinant of the single-stage amplifier with single tuned bandpass filters showing the case of asymmetry of the amplitude response curve. For symmetry, $T\sin\Theta = 0$.

$T \exp(j\Theta)$ has been shown. Since δ equals the distance between the parabola and the extremity of T (see Section 2.5) only a symmetrical response curve will be obtained for $T \sin \Theta = 0$.

Furthermore, since the parabola is fitted with a frequency scale with $x = 0$ at the real axis, it appears that maximum amplitude response (minimum value of $|\delta|$) occurs at $x < 0$ if $T \sin \Theta < 0$ and at $x > 0$ if $T \sin \Theta > 0$. This suggests that it would be possible to achieve a symmetrical response curve if a passive four-terminal network is incorporated in the amplifier with a regeneration coefficient which is the complex conjugate of $T \exp \Theta$. Then a certain amount of "left asymmetry" would be compensated by the same amount of "right asymmetry".

9.1.2 THE SYMMETRICAL AMPLIFIER STAGE

In Fig. 9.3 an equivalent circuit diagram of an amplifier stage which would have the supposed symmetry is given. It consists of two four-terminal networks coupled together.

To distinguish between the two four-terminal networks, the current sources of the first network are denoted by capital Y's whereas those of the second network are denoted by lower case y's.

For the circuit of Fig. 9.3 we may write:

$$\left\| \begin{matrix} i_S \\ 0 \\ 0 \end{matrix} \right\| = \left\| \begin{matrix} Y_3 & Y_{12} & 0 \\ Y_{21} & Y_2 & y_{12} \\ 0 & y_{21} & Y_1 \end{matrix} \right\| \cdot \left\| \begin{matrix} v_3 \\ v_2 \\ v_1 \end{matrix} \right\| . \tag{9.1.3}$$

By putting

$$Y = G(1 + jx), \tag{9.1.4}$$

and

$$x_1 = x_2 = x_3 = x, \tag{9.1.5}$$

the determinant of Eq. (9.1.3) becomes:

$$\Delta = G_1 G_2 G_3 \cdot \delta, \tag{9.1.6}$$

and

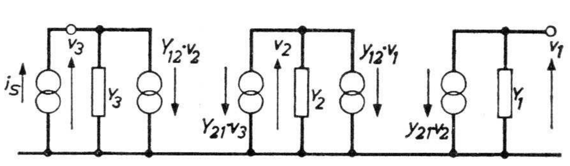


Fig. 9.3. Equivalent circuit diagram of an asymmetry-compensated single-stage amplifier.

$$\delta = \begin{vmatrix} 1 + jx_3 & T_2\exp(j\Theta_2) & 0 \\ 1 & 1 + jx_2 & T_1\exp(j\Theta_1) \\ 0 & 1 & 1 + jx_1 \end{vmatrix} . \tag{9.1.7}$$

In this equation T_2, Θ_2 and T_1, Θ_1 are the regeneration coefficients and phase angles of the first and the second four-terminal networks.

By writing out the reduced determinant we obtain with Eq. (9.1.5):

$$\delta = (1 + jx)\{(1 + jx)^2 - T_1\exp(j\Theta_1) - T_2\exp(j\Theta_2)\}. \tag{9.1.8}$$

Since $|\delta|$ represents the amplitude response curve of the amplifier, $|\delta|$ must be symmetrical with respect to $x = 0$ for a symmetrical response curve. The first factor of $|\delta|$ according to Eq. (9.1.8), $|1 + jx|$, is symmetrical and hence it is required that the second factor $|(1 + jx)^2 - T_1\exp(j\Theta_1) - T_2\exp(j\Theta_2)|$ also has this symmetry. This second factor is represented in Fig. 9.4. The term $(1 + jx)^2$ is again the parabola, and it follows that for symmetry it is required that:

$$T_1 \sin \Theta_1 + T_2 \sin \Theta_2 = 0 . \tag{9.1.9}$$

This is, indeed, in accordance with the assumption made in the preceding subsection.

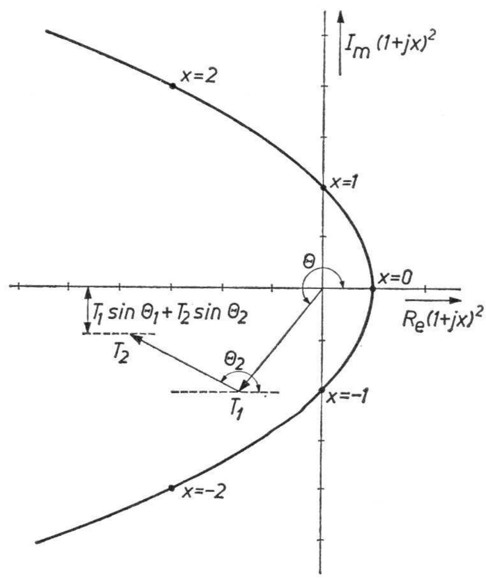


Fig. 9.4. Polar plot of the asymmetrical factor in the reduced determinant of the amplifier arrangement of Fig. 9.3. For symmetry, $T_1 \sin \Theta_1 + T_2 \sin \Theta_2 = 0$.

9.1.3 GENERAL CONDITIONS FOR SYMMETRY

In the preceding sub-section the conditions for symmetry are derived for a single-stage amplifier with $x_1 = x_2 = x_3$. In general, however, x_1, x_2 and x_3 will be different due to different quality factors of these tuned circuits. We therefore put:

$$\left.\begin{aligned} x_1 &= cx, \\ x_2 &= bx, \\ \text{and} \quad x_3 &= ax. \end{aligned}\right\} \tag{9.1.10}$$

By writing out the reduced determinant of Eq. (9.1.7) we then obtain:

$$\begin{aligned} \delta = &- x^2(ab + ac + bc) + x(aT_1 \sin \Theta_1 + cT_2 \sin \Theta_2) \\ &+ 1 - T_1 \cos \Theta_1 - T_2 \cos \Theta_2 \\ &+ \mathrm{j}[- abcx^3 + x(a + b + c - aT_1 \cos \Theta_1 - cT_2 \cos \Theta_2) \\ &- T_1 \sin \Theta_1 - T_2 \sin \Theta_2]. \end{aligned} \tag{9.1.11}$$

We now put:

$$\left.\begin{aligned} ab + ac + bc &= - A, \\ aT_1 \sin \Theta_1 + cT_2 \sin \Theta_2 &= B, \\ 1 - T_1 \cos \Theta_1 - T_2 \cos \Theta_2 &= C, \\ abc &= - D, \\ a + b + c - aT_1 \cos \Theta_1 - cT_2 \cos \Theta_2 &= E, \\ T_1 \cos \Theta_1 + T_2 \sin \Theta_2 &= - F. \end{aligned}\right\} \tag{9.1.12}$$

Then δ can be written as:

$$\delta = Ax^2 + Bx + C + \mathrm{j}(Dx^3 + Ex + F), \tag{9.1.13}$$

and $|\delta|^2$ as:

$$\begin{aligned} |\delta|^2 = \; & D^2x^6 + (A^2 + 2DE)x^4 + (2AB + 2DF)x^3 + (B^2 + 2AC + E^2)x^2 \\ & (2BC + 2EF)x + C^2 + F^2. \end{aligned} \tag{9.1.14}$$

For symmetry the terms with odd powers of x must vanish. Hence:

$$\left.\begin{aligned} BC + EF &= 0, \\ \text{and} \quad AB + DF &= 0. \end{aligned}\right\} \tag{9.1.15}$$

Condition (9.1.15) is fulfilled for:

$$B = F = 0, \tag{9.1.16}$$

or for:

$$A = C = D = E = 0. \tag{9.1.17}$$

The second condition, Eq. (9.1.17), cannot be satisfied because A and D cannot become zero, see Eq. (9.1.12). The first condition, Eq. (9.1.16), leads with Eq. (9.1.12) to:

$$\left.\begin{aligned} &aT_1 \sin \Theta_1 + cT_2 \sin \Theta_2 = 0, \\ \text{and} \qquad & \\ &T_1 \sin \Theta_1 + T_2 \sin \Theta_2 = 0. \end{aligned}\right\} \tag{9.1.18}$$

It then follows that for symmetry:

$$\left.\begin{aligned} &a = c, \\ \text{and} \qquad & \\ &T_1 \sin \Theta_1 + T_2 \sin \Theta_2 = 0. \end{aligned}\right\} \tag{9.1.19}$$

Expression (9.1.19) thus represents the general conditions for symmetry of the response curve of a single-stage amplifier. It is required that the regeneration coefficients of the two four-terminal networks are their complex conjugates and that the quality factors of the input and output-tuned circuits are equal.

An alternative method of deriving the conditions for symmetry is the following: Again assume that $x_1 = x_2 = x_3 = x$ and write Eq. (9.1.8) as:

$$\delta = (1 + jx)^3 + (1 + jx)(m + jn). \tag{9.1.20}$$

Here $m + jn$ stand for $-T_1 \exp(j\Theta_1) - T_2 \exp(j\Theta_2)$. To determine the quantity δ, the two terms of Eq. (9.1.20) must be added vectorially. The first term is always symmetrical around $x = 0$. The vectorial sum of the first and the second factor can then only be symmetrical if the real axis of the polar diagram of the second term coincides with that of the first factor. This is the case for $n = 0$, or, generally, if the co-factors of the $(1 + jx)$ terms are real.

9.1.4 PRACTICAL REALIZATION OF THE SYMMETRICAL AMPLIFIER STAGE

For a symmetrical response curve of the single-stage amplifier under consideration, condition (9.1.19) must be satisfied, i.e.

$$T_1 \sin \Theta_1 + T_2 \sin \Theta_2 = 0.$$

If we assume $T_1 \cos \Theta_1 = T_2 \cos \Theta_2$ (as is required for symmetry of a multi-stage amplifier, see following section) it follows that:

$$\Theta_2 = 2\pi - \Theta_1 + 2k\pi. \tag{9.1.21}$$

The angle Θ_1 follows from the admittance parameters of the transistor at the chosen d.c. operating point and, therefore, has a fixed value. This means that the passive four-terminal network of Fig. 9.3 must be so arranged that equality (9.1.21) is satisfied. Hence:

$$\arg Y_{12} + \arg Y_{21} = 2 \arg Y_{21} = -\Theta + (k+1)2\pi,$$

or:

$$\arg Y_{21} = -\frac{\Theta_1}{2} + (k+1)\pi, \qquad (9.1.22)$$
$$k = 0, 1, 2, 3, \ldots .$$

For all values of Θ_1 of the transistor, arg Y_{21} (of the passive four-terminal network) may be situated in the 2nd or 3rd quadrant which implies that the required symmetry can be achieved by taking for the first four-terminal network a double-tuned bandpass filter in which a resistance is added to the coupling elements. Examples of such bandpass filters are given Figs. 9.5a and 9.5b (for these bandpass filters $k = 0$ in Eq. (9.1.22)).

9.1.5 AMPLITUDE RESPONSE CURVE

The amplitude response curve of the symmetrical amplifier stage becomes with Eq. (9.1.14), taking into account Eqs. (9.1.16) and (9.1.19):

$$|\delta| = \{D^2x^6 + (A^2 + 2DE)x^4 + (2AC + E^2)x^2 + C^2\}^{\frac{1}{2}} . \quad (9.1.23)$$

For identical tuned circuits ($a = b = c = 1$) and $T_2 = T_1$, this becomes with Eq. (9.1.12):

$$|\delta| = \{x^6 + (3 + 4T\cos\Theta)x^4 + (3 + 4T^2\cos^2\Theta)x^2 + (1 - 2T\cos\Theta)^2\}^{\frac{1}{2}} \quad (9.1.24)$$

Further consideration of Eq. (9.1.8) in combination with Eq. (9.1.9) or the expressions given by Eqs. (9.1.23) and (9.1.24) reveals that the amplitude response curve of the symmetrical amplifier stage is identical to that of a triple-tuned bandpass filter, for which:

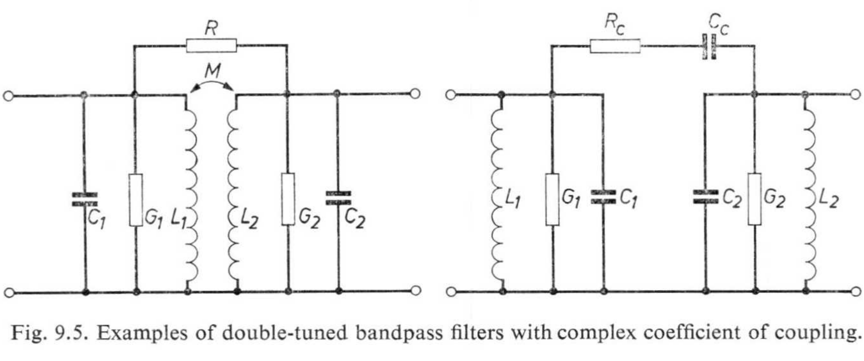


Fig. 9.5. Examples of double-tuned bandpass filters with complex coefficient of coupling.

$$|\delta| = \{x^6 + (3 - 4q^2)x^4 + (3 + 4q^4)x^2 + (1 + 2q^2)^2\}^{\frac{1}{2}}. \qquad (9.1.25)$$

Then:

$$q_1{}^2 = q_2{}^2 = -T\cos\Theta. \qquad (9.1.26)$$

For the amplitude response curve of this amplifier stage the same remarks can therefore be made as are applicable to the triple-tuned bandpass filter, see Bibliography [9-1] and [9-2]. It can hence be concluded that the amplitude response curve of amplifier arrangement under consideration has the following properties:

a) symmetry, provided the quality factors of input and output tuned circuits are equal ($a = c$);
b) single-humped top for equal quality factors of the three tuned circuits ($a = b = c$);
c) triple-humped top when the quality factor of the second tuned circuit is large compared with those of the first and the third tuned circuit ($b > a = c$). The three humps become equal for a particular value of $T_1 \cos \Theta_1 = T_2 \cos \Theta_2$;
d) no flat topped response curve can be obtained assuming that tuning method A is applied.

In Fig. 9.6 the amplitude response curves of the single amplifier stage are represented assuming $a = b = c = 1$ and $T_1 = T_2 = T$ for $T \cos\Theta = 0, 1,$

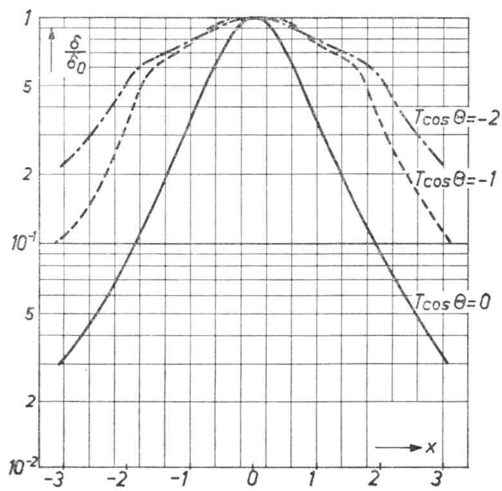


Fig. 9.6. Amplitude response curves for a single-stage-asymmetry-compensated amplifier. The curve for $T \cos \Theta = 0$ is valid for the case of a cascade of three single-tuned bandpass filters coupled by means of unilateral devices.

and 2. The well known properties of the triple-tuned bandpass filter referred to above are evidently present in this set of curves.

Condition c for three equal humps can generally not be met in transistor amplifiers because, in view of stability or otherwise, $b \approx a$.

The round-off character of the top of the amplitude response curve limits the application of this amplifier arrangement to cases in which a flat-topped response curve is not essential. Furthermore, this type of symmetry-compensated amplifier may advantageously be used in cases where a very flat envelope delay curve is required as will become apparent form the curves given for the three and four-stage amplifiers considered in the following section.

9.2 Multi-Stage Amplifier with Double-Tuned Bandpass Filters with Complex Coupling Coefficients

9.2.1 CONDITIONS FOR SYMMETRY OF RESPONSE CURVE

In Fig. 9.7 the equivalent circuit diagram of a two stage amplifier with two double-tuned bandpass filters and one single-tuned bandpass filter is given. Before deriving the conditions for symmetry of amplitude response curve we will make the following assumptions:

a) the two transistors are identical and their regeneration coefficients are denoted by $T_1 \exp(\mathrm{j}\Theta_1)$.
b) the two double tuned bandpass filters are identical and their complex coefficients of coupling are denoted by $T_2 \exp(\mathrm{j}\Theta_2)$.
c) all tuned circuits of which the bandpass filters are composed are identical and their admittances are denoted by $Y = G(1 + \mathrm{j}x)$.

The reduced determinant for this amplifier can then be written as:

$$\delta = \begin{vmatrix} y & b & 0 & 0 & 0 \\ 1 & y & a & 0 & 0 \\ 0 & 1 & y & b & 0 \\ 0 & 0 & 1 & y & a \\ 0 & 0 & 0 & 1 & y \end{vmatrix}, \tag{9.2.1}$$

in which

$$y = 1 + \mathrm{j}x, \tag{9.2.2}$$

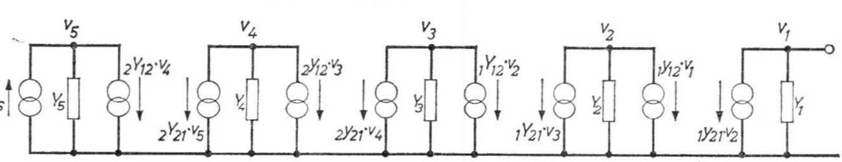


Fig. 9.7. Equivalent circuit diagram of a two-stage asymmetry-compensated amplifier.

$$a = T_1 \exp(j\Theta_1), \tag{9.2.3}$$

and

$$b = T_2 \exp(j\Theta_2). \tag{9.2.4}$$

By writing out δ, we obtain:

$$\delta = y^5 - y^3(a + b) + y(a^2 + ab + b^2). \tag{9.2.5}$$

Since y is a symmetrical function of x, the condition for symmetry of δ around $x = 0$ is that the coefficients of the terms y and y^3 are real, see also sub-section 9.1.3.

For symmetry:

$$I_m(a + b) = 0, \tag{9.2.6}$$

$$I_m(a^2 + ab + b^2) = 0. \tag{9.2.7}$$

After substitution of Eqs. (9.2.3) and (9.2.4) it follows from Eqs. (9.2.6) and (9.2.7):

$$\left.\begin{aligned} &\Theta_2 = -\Theta_1, \\ \text{and} \quad & \\ &T_1 = T_2. \end{aligned}\right\} \tag{9.2.8}$$

Extending the above analysis, it may be concluded that a multistage amplifier has a symmetrical amplitude response curve if the asymmetry of the amplitude response curve of the amplifier due to the feedback of each transistor is compensated by an asymmetry of opposite direction of a double-tuned bandpass filter with complex coupling coefficient. For an amplifier consisting of identical "stages", the regeneration coefficients and regeneration phase angles of transistors and double-tuned bandpass filters must meet with the conditions expressed by Eq. (9.2.8).

9.2.2 RESPONSE CURVE

In Fig. 9.8 the amplitude response curves of a three and a four-stage amplifier with complex coupling coefficients in the double-tuned bandpass filters are plotted. The curves are valid for $\Theta_1 = 210°$ and $\Theta_2 = 150°$ and $T_1 = T_2 = T = 3.5$ and $T = 7.0$ respectively. Fig. 9.9 gives the corresponding envelope delay curves.

It appears that the envelope delay curves are substantially flat over the range of normalized detunings considered.

9.3 Stability

Instability in an amplifier occurs if its reduced determinant becomes zero. In the assymetry-compensated single-stage amplifier this is the case for (see

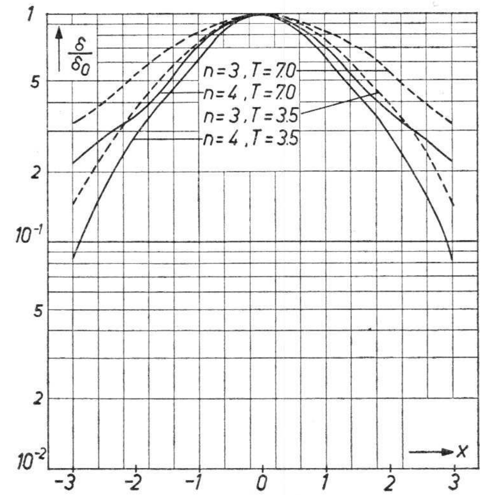


Fig. 9.8. Amplitude response curve for three and four-stage asymmetry compensated amplifiers with transistor regeneration phase angles $\Theta=210°$ and $T=3.5$, respectively $T=7.0$.

sub-section 9.1.2):

$$\delta = (1 + jx)\{(1 + jx)^2 - 2T\cos\Theta\} = 0, \tag{9.3.1}$$

assuming equal quality factors for the tuned circuits and equal values for the regeneration coefficients of the active and passive four-terminal networks.

It follows from Eq. (9.3.1) that instability occurs at

$$T\cos\Theta = \frac{1}{2}. \tag{9.3.2}$$

This means that instability in this amplifier is only possible if Θ is situated in the first or the fourth quadrant.

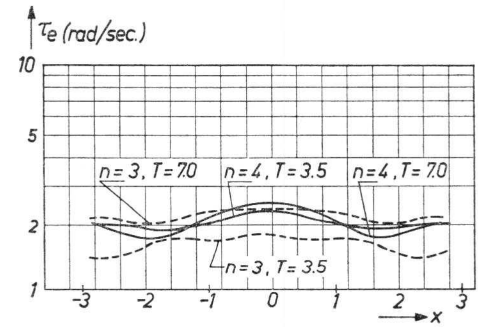


Fig. 9.9. Envelope delay curves for the amplifier arrangements of Fig. 9.8.

If this single-stage amplifier is to be protected against instability by a stability factor s, we obtain the condition:

$$T\cos\Theta \leqslant \frac{1}{2s}. \tag{9.3.3}$$

A similar condition can be derived for an asymmetry-compensated multistage amplifier.

The stability condition given by Eq. (9.3.3) is, however, only valid in an amplifier in which the asymmetry due to the feedback of the transistors is exactly compensated. In practical amplifiers the complex regeneration coefficients of the transistors and the double-tuned bandpass filters will spread around a certain average value. Due to these spreads a considerable decrease of the value of s according to Eq. (9.3.3) is possible. To accomodate with these spreads it is generally advisable to choose T smaller than or equal to the value $T_g = 2/(1 + \cos\Theta)$, the value of the regeneration coefficient at the "basic" boundary of stability as considered in the preceding chapters. If $T = T_g/s'$, in which s' expressses the amount of protection against instability due to spreads, we obtain as a second requirement:

$$T \leqslant \frac{2}{s'(1+\cos\Theta)}. \tag{9.3.4}$$

For a safe design of an asymmetry-compensated amplifier, both conditions (9.3.3) and (9.3.4) should thus be satisfied.

9.4 Transducer Gain

The transducer gain of the amplifier is again given by

$$\Phi_{t,n} = 4G_S G_L \cdot |Z_{t,n}|^2, \tag{9.4.1}$$

in which for an n-stage amplifiers with n transistors and n double tuned bandpass filters and one single-tuned bandpass filter, see Chapter 8:

$$Z_{t,n} = \frac{\prod\limits_{m=1}^{m=n} {}_m y_{21} \cdot \prod\limits_{m=1}^{m=n} {}_m Y_{21}}{\prod\limits_{m=1}^{m=2n+1} G_m \cdot \delta_n}. \tag{9.4.2}$$

If the regeneration coefficients of all transistors are denoted by T_{tr} and those of the double-tuned bandpass filters by T_{bf}, we obtain for the transducer gain of the n-stage amplifier:

$$\Phi_{t,n} = 4\,\frac{G_S}{G_{2n+1}} \cdot \frac{G_L}{G_1} \cdot N^n \cdot T_{tr}{}^n \cdot T_{bf}{}^n \cdot \frac{1}{|\delta_n|^2}\,. \tag{9.4.3}$$

For $T_{tr} = T_{bf}$ it can be calculated that at the centre frequency of the asymmetry-compensated amplifier:

for $n = 1$:

$$\delta_{1,0} = 1 - 2T\cos\Theta, \tag{9.4.4}$$

for $n = 2$:

$$\delta_{2,0} = 1 - 4T\cos\Theta + 3T^2\cos^2\Theta, \tag{9.4.5}$$

for $n = 3$:

$$\delta_{3,0} = 1 - 6T\cos\Theta + 10T^2\cos^2\Theta - 4T^3\cos^3\Theta, \tag{9.4.6}$$

and for n = 4:

$$\delta_{4,0} = 1 - 8T\cos\Theta + 21T^2\cos^2\Theta - 20T^3\cos^3\Theta + 5T^4\cos^4\Theta. \tag{9.4.7}$$

With these expressions $\Phi_{t,n}$ can be calculated. Obviously, the transducer gain at the tuning frequency of this asymmetry-compensated amplifier is equal to that of the amplifier with a single tuned bandpass filter at the output considered in Section 8.2, when tuned according to method B and $q^2 = -T\cos\Theta$.

BIBLIOGRAPHY

[9.1] B. G. DAMMERS, J. HAANTJES, J. OTTE and H. VAN SUCHTELEN, *Application of the Electron Tube in Radio Receivers and Amplifiers, I, Electronic Valves IV*, Philips Technical Library, Centrex Publishing Company, Eindhoven, 1948.

[9.2] R. FELDTKELLER, *Hochfrequenz Bandfilter*, Hirzel, Stuttgart, 1953.

[9.3] B. D. H. TELLEGEN, *Coupled Circuits*, Philips Research Rep. **2**, p. 1-19, 1947.

CHAPTER 10

STAGGERED TUNING IN TRANSISTOR BANDPASS AMPLIFIERS

A well known technique in selective amplifiers equipped with valves is the application of the staggered tuning principle. This technique which utilizes simple single-tuned interstage networks, enables a desired bandpass characteristic to be obtained with gain levels comparable with those achievable with more complicated interstage networks. The success of the staggered tuning technique in valve amplifiers rests on the fact that here the amplification is limited by the maximally possible tuned circuit impedance. This maximum tuned circuit impedance depends on the required bandwidth of the circuit and the minimum value of tuning capacitance of which the lower limit is set by the parasitic capacitances present across the circuit. In a stagger-tuned amplifier the quality factors of the individual tuned circuits can be made larger than in a synchronously tuned amplifier with the same overall bandwidth to such an extent that the overall amplification of the stagger-tuned amplifier is larger (see Bibliography [10.2]).

In selective amplifiers with transistors the impedance levels of the tuned circuits are generally much lower than in the valve case. The lower impedances are required for reasons of stability of each stage of the amplifier and/or to make all transistors of a given type interchangeable in the amplifier, taking into account their spreads in parameters (see Chapter 11). No increase in (power) gain can hence be obtained by narrowing the bandwidth of the various tuned circuits and, in fact, a reduction in gain occurs when staggering the tuning of the interstage networks. This implies that, generally, staggered tuning offers no advantages in transistorized amplifiers. Therefore synchronous tuning of the various tuned circuits is used almost exclusively.

Despite the lower gain the staggered tuning principle is sometimes used in transistorized amplifiers to achieve a better response characteristic compared with the synchronously tuned case. Unless all stages of the amplifier are perfectly neutralized the design calculations of, say, flat-staggered doubles or triples become extremely complex because of the feedback inherent to the transistors. These calculations are therefore considered to be beyond the scope of this book.

The determinant method on which the analysis of the synchronously tuned amplifiers considered is based is however also suitable for non-synchronously tuned amplifiers, as already referred to in Chapter 8. This method may therefore also be applied in the analysis of the performance of staggered amplifiers as soon as the staggering scheme, the tuning frequencies of the individual circuits and their quality factors are known.

For the case of $\Theta = 270^\circ$ (valve case) Jenolek and Sidorowicz (see Bibliography [10.1]) have investigated the influence of the feedback on the amplitude response curve for staggered pairs and staggered triples also using a determinant method. In this analysis various staggering sequences are considered. The quality factors and tuning frequencies of the individual tuned circuits were determined using the normal stagger diagram for unilateral amplifiers.

From this analysis it follows that even at small values of the regeneration coefficient T a severe distortion of the amplitude response curve occurs which is mainly to be attributed to the deterioration of stability of each stage due to the cascade of stages (see Chapter 6). This deterioration depends, obviously, on the sequence of the resonant frequencies of the individual tuned circuits in the amplifier. It might be concluded from the analysis mentioned that the staggered tuning technique can only successfully be employed in a transistor amplifier if the stability factor of each stage is made very large ($s > 20$). This may be achieved either by neutralization or by sufficiently damping the transistors at their input and output terminals. In the latter case, as already referred to, the power gain will generally be considerably less than in the case of synchronous tuning.

CHAPTER 11

SPREADS IN TRANSISTOR AMPLIFIERS

In designing practical bandpass amplifiers the spreads and tolerances of the properties of the active as well as the passive devices to be used must be taken into account. The design must be such that the performance of the amplifier remains within allowable limits over the range of possible spreads of transistor parameters as well as component tolerances.

We will restrict ourselves to investigating in some detail the consequences of the deviation of the admittance parameters of the transistors to be used in the amplifier from the nominal values. Tolerances in the circuitry external to the transistors will only be considered in as far as they influence the spreads in the transistor admittance parameters.

Parameter spreads of the transistors affect the stability of the amplifier as well as the gain and the response curve. Because an amplifier is useless in practice unless it is adequately stable, the design based on stability taking into account spreads will be considered in detail. The consequences of transistor parameter spreads on gain and response curve will not be considered in detail because for this investigation actual transistor parameters should be taken into account. This thus leads to different parameters for each transistor of the amplifier. When the amplifier determinant is written down taking into account these parameters, questions regarding gain and response curve can be answered after evaluation of the determinant. Such procedures are, however, outside the scope of this book because they only yield results for specific cases which cannot be used in general. In Book II, an example of such an investigation is given for a three-stage vision I.F. amplifier of a television receiver.

As already referred to, only transistor admittance parameters will be considered in this chapter. Investigations for other parameter systems may be carried out by using similar methods.

11.1 Stability

As already pointed out, in amplifier designs for a certain type of transistor care must be taken that all transistors of that type are interchangeable without impairing the performance of the amplifier too much due to spreads in transistor parameters. Usually it will be accepted that when other transistors of the same type are inserted in the amplifier, the amplifier must be realigned. It is, however, not acceptable that the amplifier can become unstable with transistors with parameters satisfying the published data inserted in it. Therefore the stability of the amplifier has to be considered for transistors with a combination of parameters which set the severest stability requirements to the amplifier.

As will be obvious from the preceding chapters such a transistor has minimum values of g_{11} and g_{22}, maximum values of $|y_{12}|$ and $|y_{21}|$ and an angle $\Theta = \varphi_{12} + \varphi_{21}$ such that cos Θ has a maximum value.

We will denote a minimum value of a parameter by adding a suffix m and a maximum value by adding a suffix M. No extra suffix denotes a nominal or typical parameter value.

In the amplifier design on stability for a nominal transistor a certain stability factor $s > 1$ was taken into account, see sub-section 2.2.4.

For the design on stability of an amplifier in which transistors with a combination of extreme parameters as mentioned above are assumed to be inserted we will allow that s reduces to $s = 1$. Then the amplifier is on the boundary of stability. This is allowable for the following reasons:

> A single transistor with a combination of extreme parameters as mentioned, or as it further will be referred to, an "extreme transistor", never occurs in practice. Therefore the stability factor of an amplifier equipped with practical transistors will always be larger than unity when it is allowed that s reduces to $s = 1$ for the "extreme transistor".

Based on these principles the influences of spreads of the transistor parameters will now be investigated by considering a single-stage amplifier with two single-tuned bandpass filters. This simple type of amplifier has been chosen because of the straightforwardness of the analysis and, moreover, because its boundary of stability may be considered as a general boundary of stability for all selective amplifier configurations, see Chapters 2, 5, 6 and 7.

Because the influences of the parameter spreads on the stability present themselves somewhat differently in neutralized and non-neutralized amplifiers, the two cases will be dealt with separately.

11.2 Non-Neutralized Amplifiers

To investigate the influences of the transistor parameter spreads in a non-neutralized amplifier we will consider the spreads of the various parameters separately. In this way a clear picture of the influence of each parameter will be obtained.

In investigating the various spreads in the non-neutralized amplifier use will be made of the T-plane representation of the amplifier stability.

11.2.1 SPREADS IN Y_{12}

According to Chapter 2, we may write for a single-stage amplifier with a stability factor s:

$$(1 + jx)^2 - sT\exp(j\Theta) = 0. \tag{11.2.1}$$

At the boundary of stability $s = 1$ and $T = T_g$.
Since

$$T = \frac{|y_{12}| \cdot |y_{21}|}{G_1 G_2}, \tag{11.2.2}$$

and

$$\Theta = \varphi_{12} + \varphi_{21}, \tag{11.2.3}$$

Eq. (11.2.1) can also be written as:

$$sT\exp(j\varphi_{12}) = (1 + jx)^2 \cdot \exp(-j\varphi_{21}), \tag{11.2.4}$$

or

$$|y_{12}| \exp(j\varphi_{12}) \cdot s \cdot \frac{|y_{21}|}{G_1 G_2} = (1 + jx)^2 \cdot \exp(-j\varphi_{21}). \tag{11.2.5}$$

In investigating the influences of the spreads in y_{12} we will assume in this sub-section that the parameters y_{11}, y_{22} and y_{21} are constant.

The influence of the spreads in y_{12} can clearly be seen from Fig. 11.1 which is a plot of the right-hand side of Eq. (11.2.4) in the complex $T\exp(j\varphi_{12})$ plane. The vector $T\exp.(j\varphi_{12})$ represents the nominal case. The modulus of y_{12} is assumed to spread between

$$|y_{12}| + \Delta|y_{12}| = |y_{12M}|,$$

and (11.2.6)

$$|y_{12}| - \Delta|y_{12}| = |y_{12m}|,$$

wheras the phase angle φ_{12} spreads between

$$\varphi_{12} + \Delta\varphi_{12} = \varphi_{12M},$$

and (11.2.7)

$$\varphi_{12} - \Delta\varphi_{12} = \varphi_{12m}.$$

The spreads in $|y_{12}|$ lead to extreme values of the regeneration coefficient T:

$$T_M = |y_{12M}| \cdot \frac{|y_{21}|}{G_1 G_2},$$

and (11.2.8)

$$T_m = |y_{12m}| \cdot \frac{|y_{21}|}{G_1 G_2}.$$

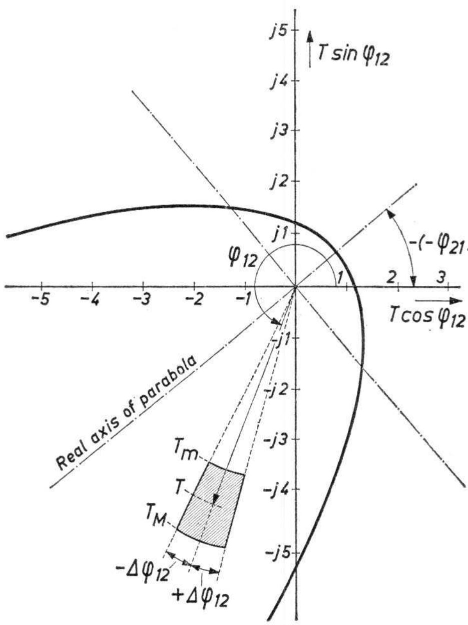


Fig. 11.1. Location of spread area of the transistor feedback admittance y_{12} in the complex T plane. It is assumed that y_{21}, g_{11} and g_{22} are not subjected to spreads so that spreads in T are only due to spreads in y_{12}. The real axis of the parabola denoting the boundary of stability is shifted over an angle $-\varphi_{21}$ with respect to the real axis of the T plane so that spreads in φ_{12} can easily be incorporated.

The shaded area in Fig. 11.1, bounded by T_m, T_M, $\varphi_{12\mathrm{m}}$ and $\varphi_{12\mathrm{M}}$, is thus the area in which the regeneration coefficient T will be situated taking into account spreads of y_{12}. Obviously the stability factor of the amplifier is smallest for the combination T_M and $\varphi_{12\mathrm{M}}$.

11.2.2 SPREADS IN Y_{21}

If also the parameter y_{21} is subjected to spreads, the situation becomes slightly more complex. If the angle φ_{21} spreads between

$$\varphi_{21} + \Delta\varphi_{21} = \varphi_{21\mathrm{M}},$$

and (11.2.9)

$$\varphi_{21} - \Delta\varphi_{21} = \varphi_{21\mathrm{m}},$$

the angle through which the real axis of the parabola must be rotated with respect to the axis $T \cos \varphi_{12}$ varies between these two values as shown in Fig. 11.2.

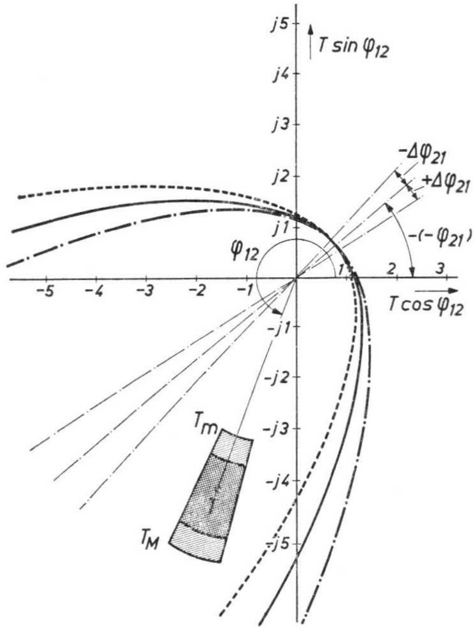


Fig. 11.2. Spreads in $|y_{21}|$ enlarge the spread area of T. The maximum value of T is obtained for a combination of maximum values of $|y_{12}|$ and $|y_{21}|$ whereas the minimum value holds for a combination of minimum values of $|y_{12}|$ and $|y_{21}|$. The double hatched spread area is considered to be due to spreads in y_{12} only. Spreads in φ_{21} present themselves as spreads in the location of the symmetry axis of the parabola.

Assuming $|y_{21}|$ to spread between

$$|y_{21}| + \Delta|y_{21}| = |y_{21M}|,$$

and (11.2.10)

$$|y_{21}| - \Delta|y_{21}| = |y_{21m}|,$$

the regeneration coefficient has extreme values of:

$$\left.\begin{aligned} T_M &= \frac{|y_{12M}| \cdot |y_{21M}|}{G_1 G_2}, \\ T_m &= \frac{|y_{12m}| \cdot |y_{21m}|}{G_1 G_2}. \end{aligned}\right\} \tag{11.2.11}$$

and

These extreme values are indicated in Fig. 11.2. The double hatched area in this figure refers to the spread area of y_{12} whereas the single hatched area represents the additional spread of $|y_{21}|$. The spreads in φ_{21} become apparent from the rotation of the parabola in the T exp $(j\varphi_{12})$ plane.

11.2.3 COMBINATION OF SPREADS IN y_{12} AND y_{21}

The influences of spreads of the y_{12} and y_{21} parameters of the transistors on the stability of the amplifier as illustrated separately in Fig. 11.1 and Fig. 11.2 can also be expressed in a combined form in the $T \exp j\Theta$ plane as illustrated in Fig. 11.3. The extreme values of the angle Θ then become:

$$\Theta + \Delta\varphi_{12} + \Delta\varphi_{21} = \Theta_M,$$

and (11.2.12)

$$\Theta - \Delta\varphi_{12} - \Delta\varphi_{21} = \Theta_m.$$

The extreme values T_M and T_m of the regeneration coefficient are given by Eq. (11.2.11).

The double hatched area in Fig. 11.3 again refers to the spread area of y_{12} whereas the single hatched area indicates the additional spreads due to y_{21}.

The severest case with respect to stability in Fig. 11.3. is the combination of T_M and Θ_M.

11.2.4 SPREADS IN g_{11} AND g_{22}

Until now it has been assumed that the conductances g_{11} and g_{22} are not subjected to spreads. In practical transistors these parameters also spread

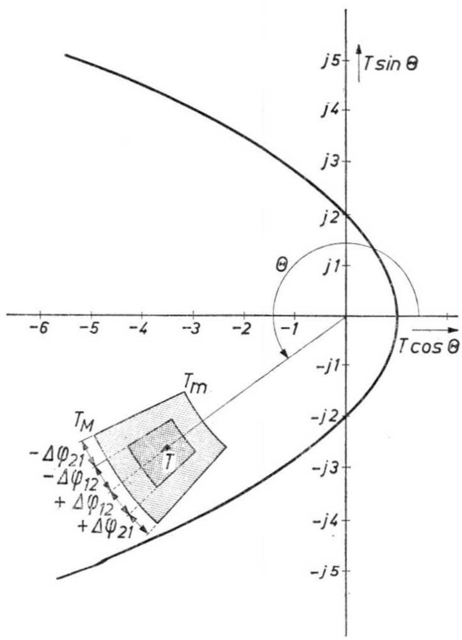


Fig. 11.3. Combined influence of the spreads in y_{12} and y_{21} in the $T \exp (j\Theta)$ plane.

around their nominal values and hence influence the value of the regeneration coefficient T.

In the single-stage amplifier under consideration we have

$$\left.\begin{aligned} G_1 &= g_{11} + G_1^* + G_S, \\ \text{and} \qquad G_2 &= g_{22} + G_2^* + G_L. \end{aligned}\right\} \tag{11.2.13}$$

For minimum values $g_{11\mathrm{m}}$ and $g_{22\mathrm{m}}$, G_1 and G_2 become:

$$\begin{aligned} G_{1\mathrm{m}} &= g_{11\mathrm{m}} + G_1^* + G_S, \\ G_{2\mathrm{m}} &= g_{22\mathrm{m}} + G_2^* + G_L \end{aligned} \tag{11.2.14}$$

After the design on stability for the nominal transistor has been carried out the quantities $(G_1^* + G_s)$ and $G_2^* + G_2)$ are known and $G_{1\mathrm{m}}$ and $G_{2\mathrm{m}}$ can be calculated. The extreme values for T then follow from:

$$\left.\begin{aligned} T_\mathrm{M} &= \frac{|y_{12\mathrm{M}}| \cdot |y_{21\mathrm{M}}|}{G_{1\mathrm{m}} \cdot G_{2\mathrm{m}}}, \\ \text{and} \qquad T_\mathrm{m} &= \frac{|y_{12\mathrm{m}}| \cdot |y_{21\mathrm{m}}|}{G_{1\mathrm{M}} \cdot G_{2\mathrm{M}}}. \end{aligned}\right\} \tag{11.2.15}$$

In Fig. 11.4 the spread area of T taking into account the extreme values of all admittance parameters is shown. The shaded regions indicate the extra increase of the spread area due to the spreads in g_{11} and g_{22}.

The spreads of the susceptive parts of y_{11} and y_{22} need not to be taken into account because they are incorporated in the tuned circuit susceptances when tuning the amplifier.

11.2.5 PRACTICAL DESIGN PROCEDURE WITH INTERCHANGEABILITY CHECK

In designing practical amplifiers the design is first carried through for transistors with nominal values of parameters. According to the introductory section of this chapter it must then be ascertained whether for the so-called "extreme transistor" the stability factor s_m is larger than unity or not.

If $s_m > 1$ all transistors of the type under consideration may be inserted in the amplifier without any risk of instability. If $s_\mathrm{m} < 1$ some of the transistors of the given type may give rise to instability when inserted in the amplifier. To remedy this, s_m must be increased to unity by increasing the tuned circuit dampings or the source and load dampings.

This then means that the stability factor for the nominal transistor becomes larger than was initially provided for.

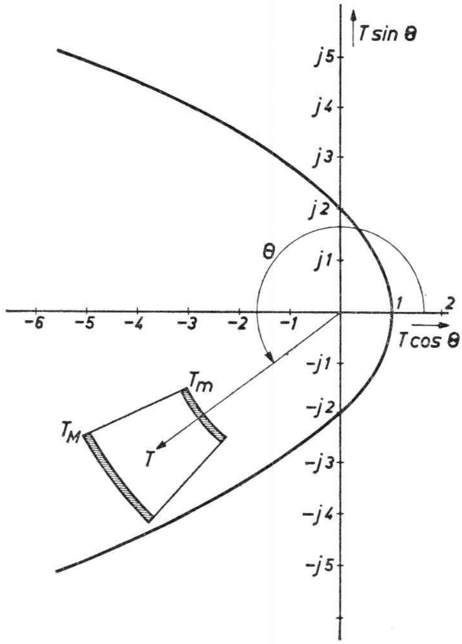


Fig. 11.4. Further enlargement of the spread area of T due to spreads in the dampings g_{11} and g_{22}. The shaded area indicates the influences of these parameters.

In order to check the interchangeability conditions the smallest value of T_g that may occur in the amplifier must be determined. Therefore that extreme value of Θ (Θ_{M} in common emitter connection and Θ_m in common base connection) must be taken that gives the minimum value for:

$$T_{g\mathrm{m}} = \frac{2}{1 + \cos \Theta}\,. \tag{11.2.16}$$

After T_M has been determined from Eq. (11.2.15) the minimum value of the stability factor s_{m} follows from:

$$s_{\mathrm{m}} = \frac{T_{g\mathrm{m}}}{T_{\mathrm{M}}}\,.$$

11.2.6 EXAMPLE

To illustrate the theory presented in the preceding subsections a single-stage amplifier will be designed with respect to stability. The transistor to be used in the amplifier is assumed to be of a type of which the admittance parameters are as given in Table 11.1.

TABLE 11.1 ADMITTANCE PARAMETERS

	minimum value	nominal value	maximum value	unit
g_{11}	2.5	5	10	m℧
$\|y_{12}\|$	50	100	200	μ℧
φ_{12}	260	265	270	°
$\|y_{21}\|$	75	100	130	m℧
φ_{21}	300	315	330	°
g_{22}	50	100	200	μ℧

For the nominal case:

$$\Theta = \varphi_{12} + \varphi_{21} = 220^\circ$$

$$T_g = \frac{2}{1 + \cos \Theta} = 8.6.$$

For $s = 4$, $T = 2.15$

Then $$G_1 G_2 = \frac{|y_{12} y_{21}|}{T} = 4.65 \cdot 10^{-6}\,℧^2.$$

Assuming $\dfrac{G_1}{G_2} = \dfrac{g_{11}}{g_{22}}$, it follows that

$$G_1 = \sqrt{\frac{g_{11}}{g_{22}} G_1 G_2} = 15.3 \text{ m℧}$$

and $G_2 = 300\ \mu℧$.

As $G_1 = g_{11} + G_1^* + G_S, \quad G_1^* + G_S = 10.3$ m℧;

also $G_2 = g_{22} + G_2^* + G_L, \quad G_2^* + G_L = 200\ \mu℧$.

For reasons of interchangeability we must take into account

$$g_{11\text{m}}, \quad |y_{12\text{M}}|, \quad \varphi_{12\text{M}}, \quad |y_{21\text{M}}|, \quad \varphi_{21\text{M}} \text{ and } g_{22\text{m}},$$

This leads to:

$$\Theta_\text{M} = \varphi_{12\text{M}} + \varphi_{21\text{M}} = 240^\circ,$$

$$T_{g\text{m}} = \frac{2}{1 + \cos \Theta_\text{M}} = 4,$$

$$G_{1\text{m}} = g_{11\text{m}} + G_1^* + G_S = 12.8 \text{ m℧},$$

$$G_{2\text{m}} = g_{22\text{m}} + G_2^* + G_L = 250\ \mu℧,$$

Furthermore, $$T_\text{M} = \frac{|y_{12\text{M}}||y_{21\text{M}}|}{G_{1\text{m}} \cdot G_{2\text{m}}} = 8.1.$$

This gives for the minimum stability factor

$$s_{\mathrm{m}} = \frac{T_{g\mathrm{m}}}{T_{\mathrm{M}}} = 0.5.$$

A stability factor $s_{\mathrm{m}} = 0.5$ indicates that the amplifier may become unstable for a transistor of the type under consideration with an unfavourable combination of parameters. To meet with the interchangeability criterion s_m must be increased to $s_{\mathrm{m}} = 1$ by increasing $(G_1^* + G_S)$ and $(G_2^* + G_L)$.

For $s_{\mathrm{m}} = 1$, $T_{\mathrm{M}} = T_{g\mathrm{m}} = 4$ and $G_{1\mathrm{m}} \cdot G_{2\mathrm{m}} = 6.5 \cdot 10^{-6}\ \mho^2$.

Assuming $\dfrac{g_{11\mathrm{m}}}{g_{22\mathrm{m}}} = \dfrac{G_{1\mathrm{m}}}{G_{2\mathrm{m}}}$ it follows that:

$$G_{1\mathrm{m}} = 18\ \mathrm{m}\mho,$$

and

$$G_{2\mathrm{m}} = 360\ \mu\mho.$$

This yields $\quad G_1^* + G_S = 15.5\ \mathrm{m}\mho,$

and $\quad G_2^* + G_L = 310\ \mu\mho.$

For the nominal case we then find:

$$G_1 = 20.5\ \mathrm{m}\mho \text{ and } G_2 = 410\ \mu\mho.$$

The regeneration coefficient then becomes:

$$T = \frac{100 \cdot 10^{-3} \cdot 100 \cdot 10^{-6}}{20.5 \cdot 10^{-3} \cdot 410 \cdot 10^{-6}} = 1{\cdot}2,$$

and the stability factor:

$$s = \frac{8.6}{1.2} = 7.2.$$

11.3 Neutralized Amplifiers

As regards spreads in transistor parameters two methods of neutralization must be considered which were already referred to in Chapter 3 as "perfect neutralization" and "fixed-component neutralization".

In the case of perfect neutralization the y_{12} parameter of every transistor of the given type which is inserted in the amplifier will be exactly neutralized by adjusting the components of the neutralizing network. This implies that we need *not* to consider the stability of this type of amplifier in view of the transistor parameter spreads.

In practical amplifier constructions, however, the y_{12} parameters of the transistors are subjected to variations during life or due to environmental conditions. These variations must be catered for by sufficiently large values of the dampings at the transistor input- and output terminals. To determine the required value of the product of these dampings the same method may be followed as for the amplifier with fixed neutralizing components considered in the next section.

11.4 Amplifiers with Fixed-Component Neutralization

In Chapter 3 various methods for neutralizing the reverse transmission of signals through a transistor are considered. For perfect neutralization the neutralizing components must have values which are different for each transistor because of the spreads in the reverse transmission properties. As already referred to in Section 3.6, in practical amplifier constructions fixed-component neutralization is employed. Then perfect neutralization is achieved for a transistor which has a particular value of y_{12}. Transistors having different values of y_{12} are either over-neutralized or under-neutralized.

The aim of the following sub-section is to investigate which value of y_{12} can best be perfectly neutralized by the fixed-component network taking into account the spreads in the four transistor admittance parameters as well as the spreads in the components of the neutralizing network. After having found the best values for the neutralizing components it must be assured that all transistors of the type considered are interchangeable in the amplifier with this neutralizing network without giving rise to instability phenomena.

11.4.1 COMPONENT VALUES OF THE NEUTRALIZING NETWORK

11.4.1.1 No Spreads in the Neutralizing Components

To determine the values of the fixed components of the neutralizing network we will, in the first instance, disregard the spreads in the neutralizing components themselves and only take into account the transistor parameter spreads.

As y_{12} is the parameter to be neutralized the effect of a neutralizing network can best be illustrated by expressing the stability conditions of the amplifier in the $|y_{12}| \exp(j\varphi_{12})$ plane. This can be done by rewriting Eq. (11.2.5) as:

$$|y_{12}| \exp(j\varphi_{12}) = \frac{1}{s} \frac{G_1 G_2}{|y_{21}|} (1 + jx)^2 \exp(-j\varphi_{21}). \quad (11.4.1)$$

The right-hand side of the expression can be represented by a parabola in the y_{12} plane. Its focus is located at the origin of the y_{12} plane and its axis of symmetry is shifted over an angle $(-\varphi_{21})$ with respect to the real axis. The

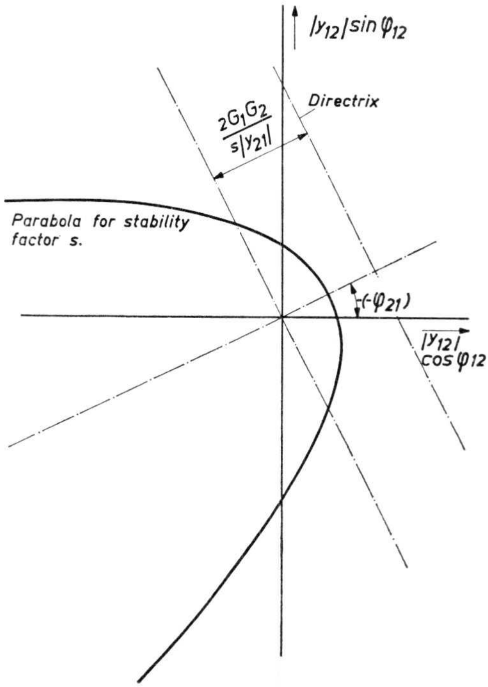


Fig. 11.5. Location of the stability parabolas in the complex y_{12} plane.

directrix of the parabola is located at a distance $2\,G_1G_2/s|y_{21}|$ from the origin. In Fig. 11.5 such a parabola has been constructed. The location of the parabola in the y_{12} plane is thus dependent on the values of y_{21}, G_1, G_2 and s and for each set of values of these parameters a new parabola must be constructed. For values of y_{12} on the parabola for $s = 1$ the amplifier is on the verge of oscillation for the particular values of y_{21}, G_1 and G_2 assumed. For values of y_{12} located outside this parabola the amplifier is unstable.

In Fig. 11.6 the spread area of y_{12} of a particular transistor has been shown. Assuming certain values of y_{21}, G_1 and G_2 and taking $s = 1$ the parabola thus represents the boundary of stability. Also a parabola for $s = 2$ has been shown. It follows that for a large number of transistors of this type the amplifier is unstable if no further measures are taken.

As already referred to, one of the measures that can be taken is the application of a neutralizing network. Assuming that this network has a transfer admittance $|Y_{12N}|\exp(\mathrm{j}\varphi_{12N})$ and that its influence on the y_{21}, y_{11} and y_{22} parameters of the neutralized transistor four-terminal network is negligible, the effect of the neutralizing network may be represented as shown in Fig.

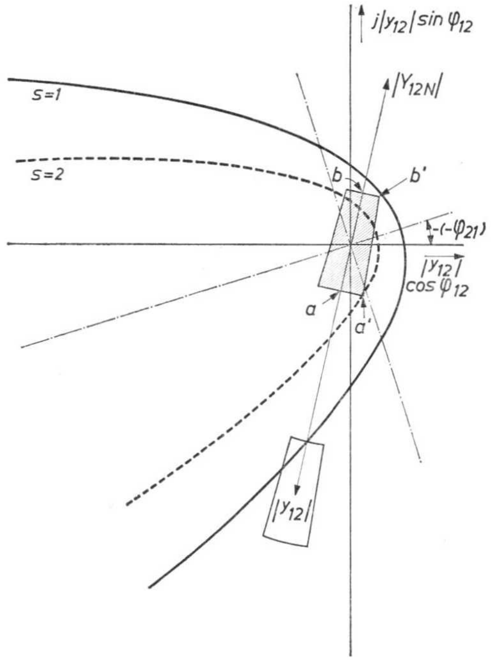


Fig. 11.6. Location of spread area of y_{12} of the transistor and the feedback admittance $|Y_{12N}|$ of the neutralizing network in the y_{12} plane. The hatched area represents the remaining spread area of y_{12} after neutralization with Y_{12N}.

11.6. Here, the vector Y_{12N} represents the transfer admittance of the neutralizing network. The hatched area represents the remainder of the feedback of the transistor after neutralization of an amount Y_{12N} of it.

It follows that, due to this particular choice of the value of Y_{12N}, for over-neutralized transistors the stability factor of the amplifier is much smaller than for under-neutralized transistors. The value of Y_{12N} should preferably be chosen such that the stability factor in the over-neutralized case is equal to that in the under-neutralized case. In sub-section 9.6.2 this value has been calculated, assuming zero spreads in the phase angles φ_{12} and φ_{21}. After neutralization with this value the points a and b appear on the same parabola for a certain value of s, see Fig. 11.7. Strictly speaking, spreads in φ_{12} should also be taken into account. This requires that the points a' and b' are both situated on a parabola for a certain value of s (different from that for the points a and b). Moreover, spreads in y_{21} should also be taken into account. Consideration of the spreads in φ_{12} and φ_{21}, however, only leads to second order variations of the (nominal) value of Y_{12N}. The assumption of nominal values for φ_{12} and φ_{21} for determining the nominal value of Y_{12N} is therefore justified.

As follows from Fig. 11.7 the value of the transistor feedback admittance which is perfectly neutralized lies between the minimum value and the average value of y_{12}. This means that most transistors of a given type are under-neutralized.

11.4.1.2 Spreads in the Neutralizing Components

In practice normal capacitors and resistors are used for the components of the neutralizing networks. This implies that we have to take into account the tolerances of these components which are equal to 10%, say. Furthermore, spreads may occur due to spreads in the transformer ratio of the phase inverting transformer. The component tolerances influence both magnitude and phase of the neutralizing admittance whereas the spreads in transformer ratio only affect the magnitude.

As it is probable that the spreads in the magnitude are larger and, hence, more important than the spreads in phase, the magnitude spreads will be considered first.

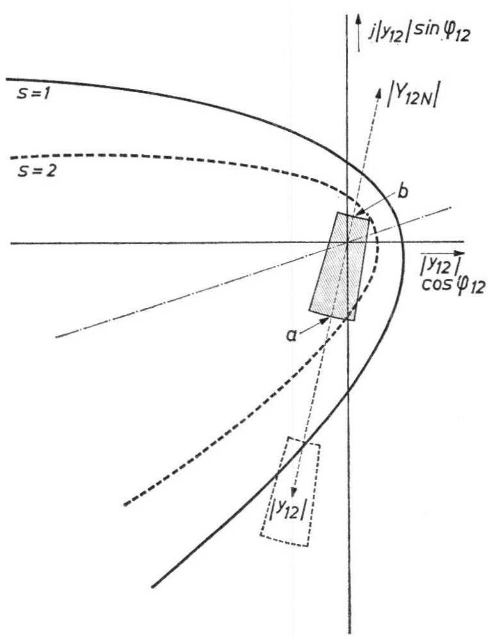


Fig. 11.7. Location of the remaining spread area of y_{12} for equal stability in the extreme over- and under-neutralized cases.

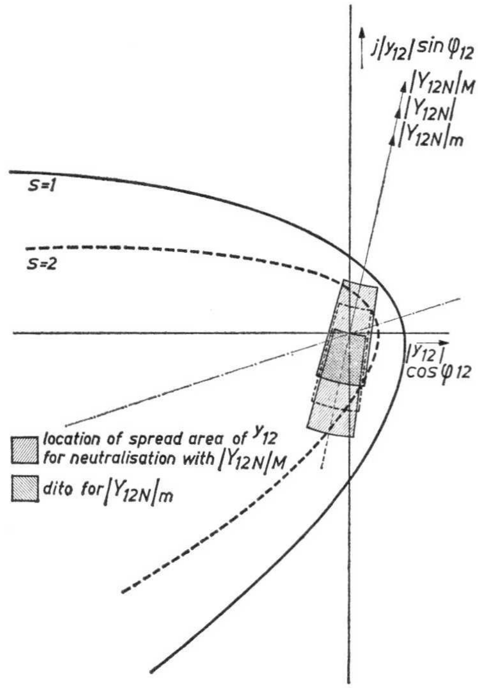


Fig. 11.8. Effects of spreads in $|Y_{12N}|$.

In Fig. 11.8 the effects of spreads in magnitude of Y_{12N} are shown. The spread area of y_{12} indicated by the dashed lines is valid for the correct nominal value of Y_{12N} as found in Fig. 11.7. The hatched spread areas apply to the cases with a positive spread, $|Y_{12N\mathrm{M}}|$, and with a negative spread, $|Y_{12N\,\mathrm{m}}|$. It follows from the figure that for the over-neutralized and under-neutralized cases the stability factor is again different.This means that with a fixed-component neutralizing network the nominal value of Y_{12N} must be determined by taking into account the spreads in y_{12} of the transistor as well as the spread in Y_{12N} of the neutralizing network.

As the spreads in $|Y_{12N}|$ are usually specified as a certain percentage of deviation from the nominal value, difficulties occur when it is attempted to take these spreads into account. This is due to the fact that, to determine the nominal value of Y_{12N}, the absolute magnitudes of the spreads are required whereas for the spreads in Y_{12N} only the relative values are know. (The absolute values depend on the nominal value to be found.) For practical amplifiers the correct nominal value of Y_{12N} can be determined with sufficient accuracy by assuming that the absolute spreads in Y_{12N} are equal to the percentage spread multiplied by the minimum value of the transistor feedback admittance y_{12}. This might be seen as follows: In the preceding sub-section we have found that due to spreads in the transistor feedback admittance only the correct value of $|Y_{12N}|$ lies between $|y_{12}|$ and $|y_{12m}|$, assuming y_{21} to be situated in the fourth quadrant. As, due to the neutralizing network, the total spread in both directions increases, the correct, nominal, value of $|Y_{12N}|$ will approach more closely the value $|y_{12\mathrm{m}}|$. For y_{21} situated in the second quadrant, $|Y_{12N}|$ more closely approaches $|y_{12\mathrm{M}}|$. To determine the nominal value of $|Y_{12N}|$ the absolute spead of $|Y_{12N}|$ obtained in this way should be added to the absolute spreads in $|y_{12}|$ of the transistor.

These points will be elucidated by means of an example: let a transistor have an average feedback admittance $|y_{12}|$ of 100 $\mu℧$ which spreads between 50 $\mu℧$ and 150 $\mu℧$. Then $|\Delta y_{12}| = 150 - 50 = 100\ \mu℧$. Let furthermore the neutralizing network have a spread of 20%. We assume that $|\Delta Y_{12N}|$ is equal to 20% of 50 $\mu℧$ or $\Delta Y_{12N} = 20\ \mu℧$. The total spread to be taken into account then becomes $100 + 20 = 120\ \mu℧$. If we assume that the transistor has a forward transfer admittance of 100 m℧ at the average with spreads from 80 m℧ to 120 m℧ and that the average value of Θ equals $\Theta = 225°$, we obtain with Eq. (3.6.8) for the nominal value of $|Y_{12N}|$:

$$|Y_{12N}| = 48\ \mu℧.$$

We have now taken into account spreads in $|y_{12}|$, $|y_{21}|$ and $|Y_{12N}|$ for finding the optimum nominal value for the magnitude of the feedback admittance of the neutralizing network. This value yields the same stability factor in the extreme over-neutralized and under-neutralized cases and we are therefore able to design the amplifier with a certain stability factor s for these extreme cases. The parameters mentioned are, however, not the only spread parameters which contribute to the stability of the amplifier. Spreads in φ_{12}, φ_{21}, g_{11}, g_{22} and φ_{12N} should also be considered. To investigate whether the influences of these spreads are tolerable the interchangeability criterion referred to in the introductory section of this chapter will be applied.

11.4.2 SPREADS IN φ_{12N}

In the preceding sub-section we have investigated how the spread area of the transistor feedback admittance y_{12} is influenced by the spreads in $|Y_{12N}|$. In this sub-section we will take into account spreads in the phase angle φ_{12N} of the feedback admittance of the neutralizing network.

In Fig. 11.9 the y_{12} spread area obtained in Fig. 11.8 drawn for a properly chosen value of Y_{12N} (dashed lines) is shown. Also the spread area of Y_{12N}

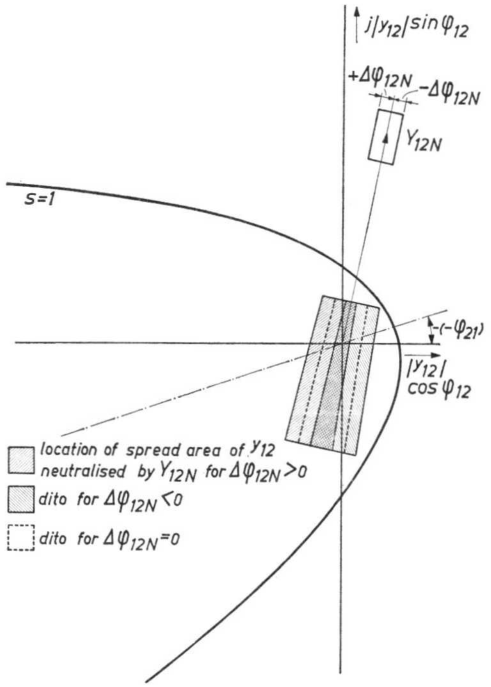


Fig. 11.9. Combined effect of spreads in modulus and argument of Y_{12N}.

which is assumed to be rectangular, has been indicated. The spread in φ_{12N} enlarges the remaining spread area of y_{12} as shown in the figure. It follows that when y_{21} is situated in the fourth quadrant (common emitter connection) negative spreads of φ_{12N} seriously decrease the stability of the amplifier whereas positive spreads of φ_{12N} increases the stability. For y_{21} situated in the second quadrant (common base connection) the reverse is true.

11.4.3 SPREADS IN $|y_{21}|$, g_{11} AND g_{22}

According to sub-section 11.4.1.1 the distance between the directrix of the parabola representing the boundary of stability in the y_{12} plane and the origin equals $\dfrac{2G_1G_2}{|y_{21}|}$. If G_1 and G_2 are calculated for minimum values of g_{11} and g_{22} (G_{1m} and G_{2m}) and the maximum value is taken for $|y_{21}|$, a parabola is obtained which represents the stability boundary in this extreme case. To achieve interchangeability the total spread area of y_{12} must be located inside this parabola. Fig. 11.10 represents a case in which this condition is met for the nominal value of φ_{21} (drawn curve).

11.4.4 SPREADS IN φ_{21}

According to sub-section 11.2.2 spreads in φ_{21} can be taken into account by shifting the axis of symmetry of the parabola as shown in Fig. 11.10. It appears that for this particular choice of parameters a part of the spread area of y_{12} is located outside the parabola valid for the positive spread of φ_{21} (dashed curve). The amplifier oscillates with transistors having y_{12} located outside the parabola. To remedy this the value of the damping product G_{1m} G_{2m} must be increased by increasing the tuned circuit dampings. Then the parabola becomes wider and encloses a larger part of the y_{12} plane.

11.4.5 INTERCHANGEABILITY CHECK AND SUMMARY

It will be apparent from the preceding sub-sections that the check on interchangeability of transistors in a neutralized amplifier can best be carried out graphically. After the optimum value of the feedback admittance of the

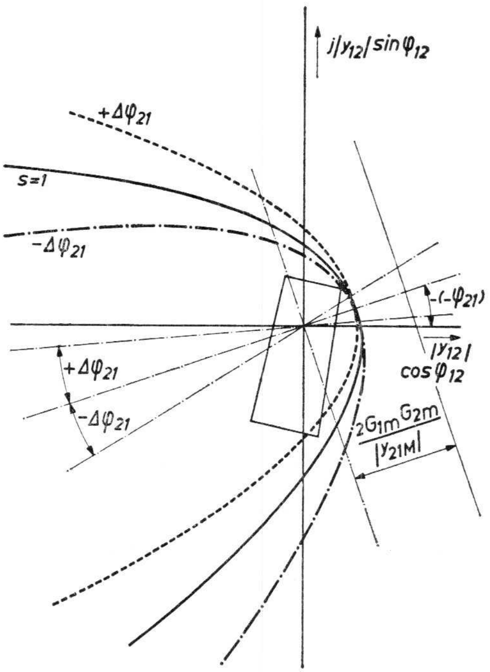


Fig. 11.10. Interchangeability check. The spread area of y_{12} including the spreads of Y_{12N} should in any case be situated inside the parabola for $s = 1$ in the y_{12} plane taking into account the most unfavourable combination of parameters. It follows that in this particular case the amplifier may become unstable for a number of transistors of the given type because part of the spread area of y_{12} is situated outside the parabola for $s = 1$ and a positive spread of φ_{21}.

neutralizing network has been calculated according to the method outlined in sub-sections 11.4.1.1 and 11.4.1.2, the remaining spread area of y_{12} can be drawn on a piece of properly scaled graph paper. Then two parabolas are constructed having the symmetry axes shifted with respect to the real axis of the y_{12} plane over angles equal to $-(-\varphi_{21m})$ and $-(-\varphi_{21M})$. The directrices of the two parabolas are at a distance $\frac{2G_{1m}G_{2m}}{|y_{21}|_M}$ from the origin of the y_{12} plane. If the spread area of y_{12} is located inside the two parabolas all transistors of the given type are interchangeable in the amplifier without any risk of oscillations. If not, the parabolas must be enlarged and this can be achieved by an increase of the dampings G_{1m} and/or G_{2m}.

In carrying out the interchangeability check, possible variations of the transistor feedback admittance which may occur during life and those which are due to environmental conditions should also be taken into account.

11.5 Stability of Multi-Stage Amplifiers

11.5.1 AMPLIFIERS WITH SINGLE-TUNED BANDPASS FILTERS AS INTER-STAGE COUPLING DEVICES

In multi-stage amplifiers with single-tuned bandpass filters the relative influences of the transistor parameter spreads on the stability are the same as those considered in the preceding sub-sections except for the fact that the reduction of the stability factors due to the cascade of stages should be taken into account. This has been dealt with in Chapter 6.

In carrying out the interchangeability checks a factor u_n should be taken into account in constructing the parabola which represents the boundary of stability.

11.5.2 AMPLIFIERS WITH DOUBLE-TUNED BANDPASS FILTERS AS INTER-STAGE COUPLING DEVICES

In multi-stage amplifiers with double-tuned bandpass filters as considered in Chapter 6 the boundary of stability can, with sufficient accuracy, be approximated by the parabola considered in the preceding sub-sections. This implies that for these amplifiers the results of the interchangeability analysis are immediately applicable.

For the single stage amplifier with double-tuned bandpass filters, considered in Chapter 5, the boundary of stability differs appreciably from the parabola. For this type of amplifier the parabola should be replaced by the proper polar diagram representing the boundary of stability, especially if large accuracies are required.

BIBLIOGRAPHY

[11.1] Z. JENOLEK and R. S. SIDOROWICZ, *Bandpass Amplifiers*, Wireless Eng. Vol. **31**, p. 84-99, 1954.

[11.2] G. E. VALLEY and H. WALLMAN, *Vacuum Tube Amplifiers*, Chapter VI, McGraw Hill, New York, 1958.

[11.3] A. G. W. UITJENS, *Television Receiver Design, Monograph I, I.F. Stages, Electronic Valves, Vol. VIII-A*, Philips Technical Library, Centrex Publishing Company, Eindhoven, 1953.

CHAPTER 12

EFFECTS OF NON-IDEAL TRANSFORMERS FOR COUPLING TRANSISTORS AND BANDPASS FILTERS

In bandpass amplifiers the transistors are nearly always employed in the common emitter or common base configurations. This implies that their input admittance is usually large compared with that of practically realizable bandpass filters, whereas the output admittance of the transistors is in the same order of magnitude as that of the bandpass filters.

For stability, variations in response curve due to spreads of transistor parameters or for other reasons the admittance presented by the bandpass filters at the transistor terminals must be equal to or larger than the driving point admittances y_{11} and y_{22} of the transistors. This requires the use of impedance transforming devices at the input terminals of the transistors. These transformations may be achieved by either two winding transformers, tapping of the tuning inductance of the bandpass filters (auto-transformers) or tapping of the tuning capacitance of the bandpass filters. The last two methods are referred to as inductive tapping and capacitive tapping respectively.

In this chapter we will consider the influences of these "transformers" on the performance of the amplifier as far as stability is concerned. Special attention will be paid to deviations from the case of the ideal transformer (i.e. a device providing impedance transformation with a real transformer ratio and nothing else).

12.1 Stability of an Amplifier Stage with Practical Impedance Transforming Networks

In an amplifier in which potentially unstable active elements are employed, stability is achieved either by suitably dimensioning the immittances presented to the terminals of each active element or by means of unilateralization. We will restrict ourselves to the case of transistors in the admittance matrix environment in which stability is ensured by means of sufficient damping at the transistor terminals. These dampings are provided by bandpass filters connected to the transistor either directly (at the output side) or by means of impedance transforming networks (at the input side) as already referred to.

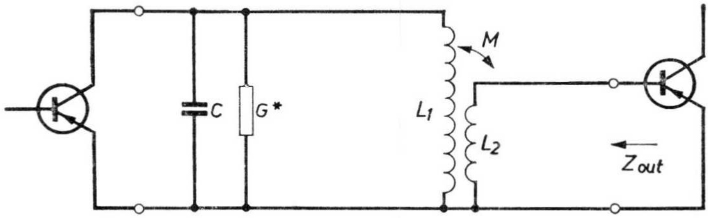


Fig. 12.1. Single-tuned interstage coupling network providing impedance transformation between the top of the tuned circuit and the input terminals of the transistor by means of a double-winding transformer.

To design these bandpass filters with respect to stability of the amplifier, the effects of the non-ideal conditions of the impedance transforming networks on the dampings presented to the transistor must be taken into account.

12.1.1 OUTPUT IMPEDANCE OF PRACTICAL IMPEDANCE TRANSFORMING NETWORKS

To enable the effects of the non-ideal conditions of the impedance transforming networks on the amplifier stability to be investigated equivalent circuit diagrams will be derived. These diagrams will enable us to obtain a qualitative insight into these effects and, by substituting circuit values, to decide whether or not they must be taken into account in the design of the amplifier. The various impedance transforming networks will first be considered in connection with single-tuned bandpass filters after which the analysis will be extended to double-tuned bandpass filters.

12.1.2 THE DOUBLE-WINDING TRANSFORMER

In Fig. 12.1 a single-tuned bandpass filter is used as the coupling networks between two transistors of an amplifier. The tuning inductance has been provided with an extra winding to achieve the required impedance transformation between the bandpass filter and the transistor and vice-versa.

When we put:

$$n^2 = \frac{L_2}{L_1}, \tag{12.1.1}$$

and

$$k = \frac{M}{\sqrt{L_1 L_2}}, \tag{12.1.2}$$

an equivalent circuit diagram for this bandpass filter as shown in Fig. 12.2 is obtained, see sub-section 3.3.3. It includes an ideal transformer of ratio $1 : nk$.

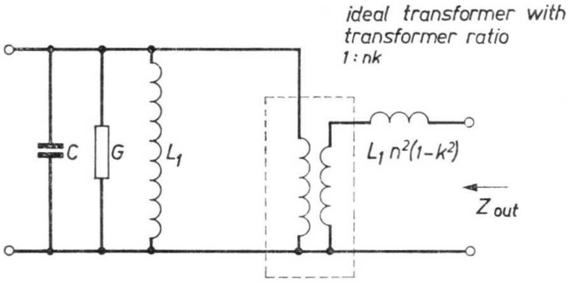


Fig. 12.2. Equivalent circuit diagram of the interstage network of Fig. 12.1. The non-ideal transformer has been replaced by an ideal transformer and a series inductance.

For further analysis of this equivalent circuit it is assumed that all capacitances are contained in C and that all dampings (of the bandpass filter, not including those of the transistor connected to L_2 in Fig. 12.1) are contained in G_i. Inspection of Fig. 12.2 then shows that the impedance of the bandpass filter seen at the output terminals equals:

$$Z_{out} = n^2k^2 \frac{1}{G(1 + \mathrm{j}x)} + \mathrm{j}\omega L_1 n^2(1 - k^2), \tag{12.1.3}$$

or:

$$Z_{out} = n^2k^2 \frac{1}{G(1 + \mathrm{j}x)} + j\frac{\omega}{\omega_0} \cdot \omega_0 L_1 n^2(1 - k^2). \tag{12.1.4}$$

Expression (12.1.4) reveals that the output impedance of the circuit of Fig. 12.2 compared with the case of an ideal transformer is increased by an amount

$$\mathrm{j}\omega_0 L_1 n^2(1 - k^2) = \mathrm{j}\omega_0 L_2(1 - k^2), \tag{12.1.5}$$

assuming

$$\frac{\omega}{\omega_0} \approx 1.$$

When plotted in the complex plane the impedance Z_{out} expressed by Eq. (12.1.4) consists of a circle of diameter n^2k^2/G the origin of which is situated at the top of a vector $\mathrm{j}\omega_0 L_1 n^2(1 - k^2)$ on the imaginary axis, see Fig. 12.3.

12.1.3 THE AUTO-TRANSFORMER

Fig. 12.4 shows a single-tuned bandpass filter with a tap on the tuning inductance which provides an impedance transformation between the top of the bandpass filter and the terminal (tap) to which the transistor has to be connected. The tapped tuning inductance may be replaced by an arrangement of inductances as shown in Fig. 12.5.

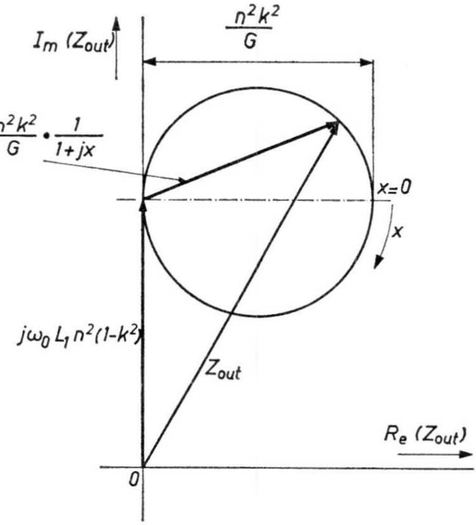


Fig. 12.3. Polar diagram showing the variation of the output impedance of a practical double-winding transformer as a function of the normalized frequency x. The effect of the series inductance is represented by the vector $j\omega_0 L_1 n^2(1 - k^2)$.

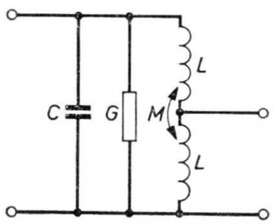


Fig. 12.4. Single-tuned interstage network in which impedance transformation is obtained by tapping the tuning inductance.

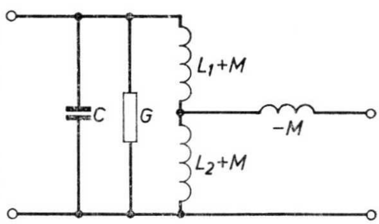


Fig. 12.5. Equivalent circuit diagram for the interstage network of Fig. 12.4.

In order to calculate the impedance seen when looking into the output terminals of Fig. 12.5, we consider the four-terminal network containing only the elements C, G, $L_1 + M$ and $L_2 + M$, (see Fig. 12.6) for which the admittance parameters are equal to:

$$\left.\begin{aligned} Y_{11} &= G + j\omega C + \frac{1}{j\omega(L_1 + M)}, \\ Y_{12} &= Y_{21} = -\frac{1}{j\omega(L_1 + M)}, \\ Y_{22} &= \frac{1}{j\omega L_p}, \end{aligned}\right\} \qquad (12.1.6)$$

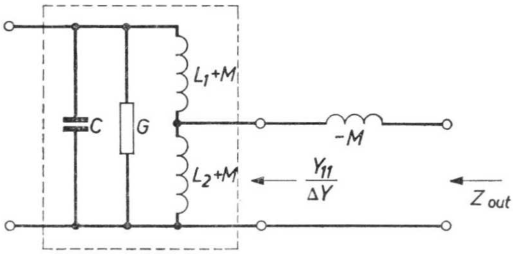


Fig. 12.6. As Fig. 12.5. The effects of the part of the diagram indicated by the dashed lines on the output impedance are calculated separately after which the inductance $-M$ is added to obtain the total output impedance.

in which

$$L_p = \frac{(L_1 + M)(L_2 + M)}{L_c}, \tag{12.1.7}$$

and

$$L_c = L_1 + L_2 + 2M. \tag{12.1.8}$$

Furthermore we put:

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21},$$

for which it follows from Eq. (12.1.6):

$$\Delta Y = \frac{1}{j\omega L_p}\left(G + j\omega C + \frac{1}{j\omega L_c}\right),$$

or:

$$\Delta Y = \frac{1}{j\omega L_p} \cdot G(1 + jx). \tag{12.1.9}$$

The output impedance of the complete network of Fig. 12.6 then becomes:

$$Z_{out} = -j\omega M + \frac{Y_{11}}{\Delta Y},$$

$$= -j\omega M + \frac{1}{1 + jx}\left\{j\omega L_p + \frac{1}{G}\left(\frac{L_p}{L_1 + M} - \omega^2 L_p C\right)\right\}. \tag{12.1.10}$$

Now $\frac{L_p}{L_1 + M} - \omega^2 L_p C$ equals

$$\frac{L_2 + M}{L_c}\left\{1 - \left(\frac{\omega}{\omega_0}\right)^2 \cdot \frac{L_1 + M}{L_c}\right\},$$

which for $\frac{\omega}{\omega_0} \approx 1$ reduces to:

$$\left(\frac{L_2 + M}{L_c}\right)^2 = n^2, \text{ say.} \tag{12.1.11}$$

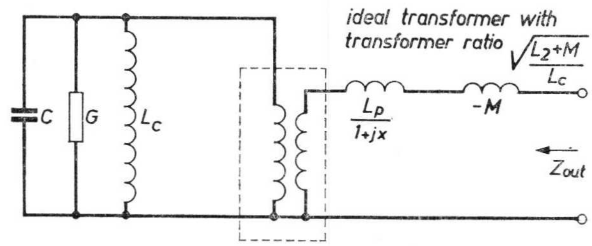


Fig. 12.7. Complete equivalent circuit diagram of a single-tuned interstage network with tapped tuning inductance.

The quantity n as defined here denotes the voltage transformation ratio of the unloaded transformer.

If $p^2 = \frac{L_2}{L_1}$, n^2 becomes:

$$n^2 = \left(\frac{p^2 + kp}{1 + p^2 + 2kp}\right)^2. \tag{12.1.12}$$

Combining Eqs. (12.1.10) and (12.1.11) we obtain:

$$Z_{out} = \frac{1}{1+\mathrm{j}x}\left(\frac{n^2}{G} + \mathrm{j}\omega_0 L_p\right) - \mathrm{j}\omega_0 M. \tag{12.1.13}$$

For normalizing Z_{out} we put:

$$\omega_0 L_p G_n \frac{1}{n^2} = Q_{spr}. \tag{12.1.14}$$

Then Eq. (12.1.13) becomes:

$$Z_{out} = \frac{n^2}{G(1+\mathrm{j}x)}\left(1 + \mathrm{j}Q_{spr}\right) - \mathrm{j}\omega_0 M. \tag{12.1.15}$$

Here, Q_{spr} denotes the quality factor of the spread inductande L_p.

When Z_{out}, according to Eq. (12.1.13), is plotted in the complex plane, a polar diagram as shown in Fig. 12.8 is obtained. It consists of a vector $-\mathrm{j}\omega_0 M$ and a circle representing $\frac{1}{1+\mathrm{j}x}\left(\frac{n^2}{G} + \mathrm{j}\omega_0 L_p\right)$ with diameter $\left\{\left(\frac{n^2}{G}\right)^2 + \omega_0{}^2 L_p\right\}^{\frac{1}{2}}$. To construct this diagram it has arbitrarily been assumed that $\frac{\omega_0 L_p G}{n^2} = 0.4$ and $\frac{\omega_0 M G}{n^2} = 0.05$. The polar diagram for Z_{out} being based on Eq. (12.1.13), is only correct as long as the approximation

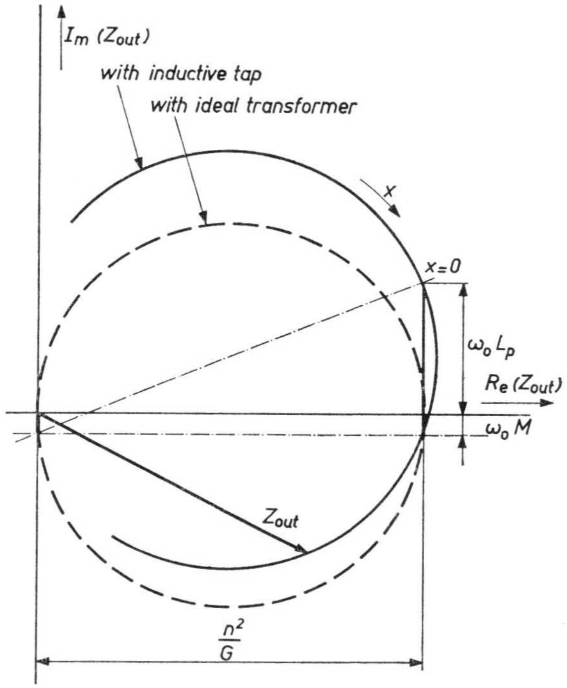


Fig. 12.8. Polar diagram of the output impedance of the circuit of Fig. 12.7. The construction of this diagram is based on Eq. (12.1.13).

$\frac{\omega}{\omega_0} \approx 1$ is justified. This is the case for not too large values of the normalized detuning x.

It clearly follows from the polar plot that when $\omega_0 M$ and $\omega_0 L_p$ are not small compared with n^2/G a considerable discrepancy in magnitude and frequency dependency between the practical transformer and an ideal one occurs. For an ideal transformer the polar diagram of Z_{out} is, obviously, a circle with its origin at 0 and of diameter n^2/G as shown by the dashed curve in the diagram.

12.1.4 THE CAPACITIVE TAP

Fig. 12.9 represents a capacitively tapped single-tuned bandpass filter. Analogous to the inductively tapped bandpass filter considered in the preceding sub-section, the capacitively tapped bandpass filter may also be considered as a four-terminal network. It can then easily be calculated that the output impedance follows from:

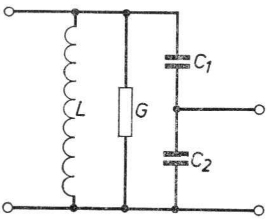


Fig. 12.9. Single-tuned interstage network with tapped tuning capacitance to provide impedance transformation.

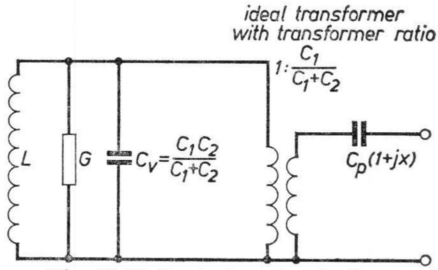


Fig. 12.10. Equivalent circuit diagram for the capacitively tapped interstage network of Fig. 12.9.

$$Z_{out} = \frac{n^2}{G} \cdot \frac{1}{1+\mathrm{j}x} + \frac{1}{\mathrm{j}\omega C_p} \cdot \frac{1}{1+\mathrm{j}x}, \tag{12.1.16}$$

or for $\frac{\omega}{\omega_0} \approx 1$ and after normalization:

$$Z_{out} = \frac{n^2}{G(1+\mathrm{j}x)} (1 - \mathrm{j}Q_{spr}). \tag{12.1.17}$$

In these expressions:

$$n^2 = \frac{C_1}{C_1 + C_2}, \tag{12.1.18}$$

and

$$C_p = C_1 + C_2,$$

$$Q_{spr} = \frac{G/n^2}{\omega_0 C_p}. \tag{12.1.20}$$

Fig. 12.10 shows an equivalent circuit of this bandpass filter based on Eq. (12.1.16) whereas Fig. 12.11 shows a polar diagram of the output impedance again taking an arbitrary value of $Q_{spr} = 0.4$. The construction of the polar plot of Z_{out} is based on Eq. (12.1.16).

12.1.5 SUMMARY ON PRACTICAL IMPEDANCE TRANSFORMING NETWORKS

According to the preceding sub-section a single-tuned bandpass filter with a practical impedance transforming network may, with regard to its output impedance, be represented as a single-tuned circuit with admittance $G(1+\mathrm{j}x)$, an ideal transformer with transformer ratio n and a series impedance Z. The ideal transformer ratio n together with the impedance Z form a new,

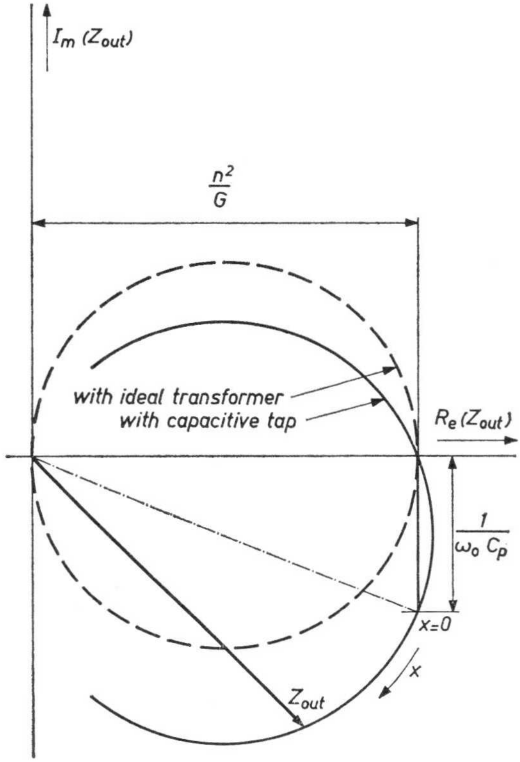


Fig. 12.11. Polar diagram of the output impedance of Fig. 12.10. The construction of this diagram is based on Eq. (12.1.16).

complex, transformer ratio transforming the impedance $\frac{1}{G(1 + jx)}$ of the parallel tuned circuit. For not too large values of Z (such that the approximation $\omega/\omega_0 \approx 1$ is justified) the new, transformed, polar diagram is again a circle with diameter $\frac{n^2}{G}\sqrt{1 + Q^2{}_{spr}}$ which is shifted with respect to the real axis of the complex plane over an angle φ; $\tan^{-1} \varphi = Q_{spr}$.
The quantities Q_{spr} for inductive and capacitive taps are given by Eqs. (12.1.14) and (12.1.20) respectively.

Furthermore it follows from the foregoing consideration that the effects of the non-idealness of the practical impedance transforming networks on the output impedance may be minimized by reducing the tuned circuit damping G and, in case of an inductive transformation, by making the coupling as tight as possible.

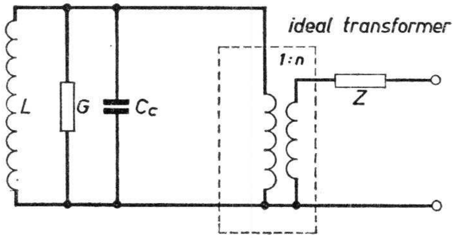


Fig. 12.12. General equivalent circuit diagram of a single-tuned interstage network taking into account practical methods for achieving impedance transformations. The impedance Z accounts for the spread capacitance or inductance of the practical transformer.

12.1.6 DOUBLE TUNED BANDPASS FILTERS WITH PRACTICAL IMPEDANCE TRANSFORMING NETWORKS

In an analogous way as for single-tuned bandpass filters it can be investigated how practical impedance transforming networks influence the output impedance. It then follows that the same complex transformer ratio as for single tuned bandpass filters acts upon the output impedance

$$\frac{1}{G_s\left(1 + \mathrm{j}x_s + \dfrac{q^2}{1 + \mathrm{j}x_p}\right)} \tag{12.1.21}$$

of the double-tuned bandpass filter.

In Fig. 12.13.a. a circuit diagram for a capacitively tapped double tuned bandpass filter is given. Fig. 12.13.b. represents the equivalent circuit whereas

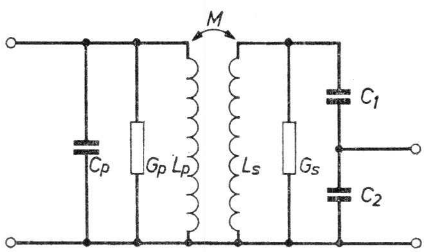


Fig. 12.13.a. Double-tuned bandpass filter with capacitively tapped secondary.

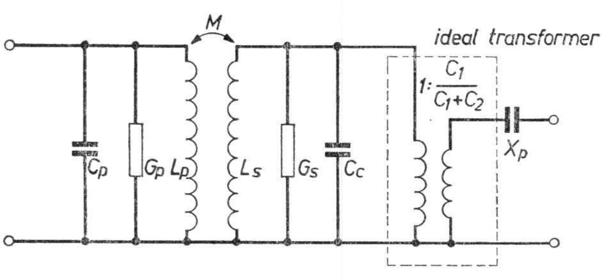


Fig. 12.13.b. Equivalent circuit diagram of the interstage network of Fig. 12.13.a.

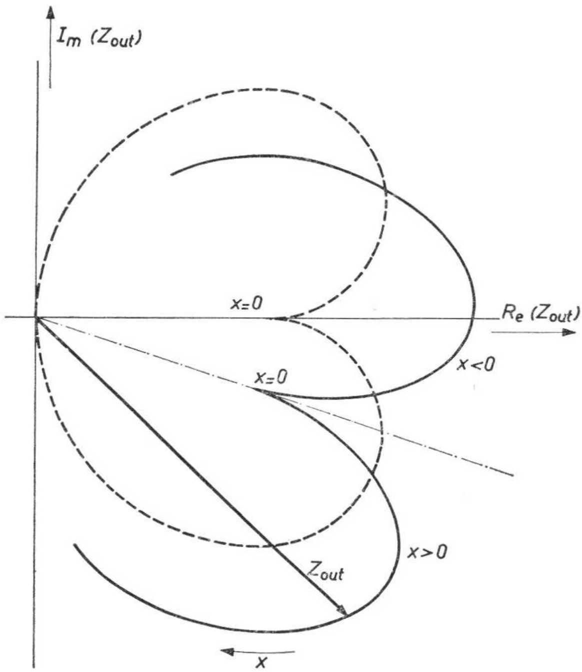


Fig. 12.14. Polar diagram of the output impedance of the double-tuned bandpass filter represented in Fig. 12.13 b.

Fig. 12.14 gives a polar diagram for Z_{out} assuming $q^2 = 1$ and $Q_{spr} = 0.4$. The dashed curve is valid in the case of an ideal transformer (with $Q_{spr} = 0$).

12.1.7 BOUNDARIES OF STABILITY IN A SINGLE-STAGE AMPLIFIER WITH PRACTICAL TRANSFORMERS

Using polar impedance diagrams the boundary of stability of an amplifier stage can be ascertained in the same way as shown in subsections 5.6.2 and 5.7.8 for polar admittance diagrams. With impedance diagrams $1/T_g$ is found instead of T_g which was obtained from constructions using polar admittance diagrams.

In Fig. 12.15 the construction is presented for determining the boundary of stability of a single stage amplifier with two single tuned bandpass filters and $\Theta = 270°$. The input bandpass filter is assumed to be connected to the transistor input by means a transformer with either an inductive — or an capacitive spread reactance. Again the arbitrary value of $Q_{spr} = 0.4$ has been taken.

The case of an ideal transformer is also shown. The values obtained for T_g in these three cases are tabulated below.

TABLE 12.1 BOUNDARY OF STABILITY OF AN AMPLIFIER STAGE

Case	Line of intersection	T_g
inductive tap	OA	1.25
ideal transformer	OB	2.00
capacitive tap	OC	2.50

It follows from the table that an amplifier stage designed with a certain value of T, i.e. with a certain value of the stability factor s, assuming ideal

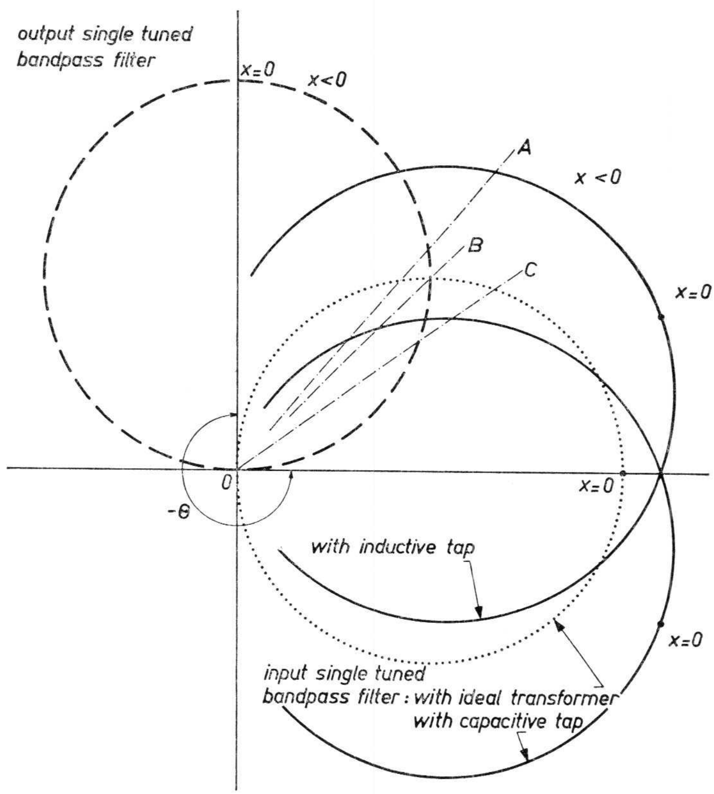


Fig. 12.15. Polar diagram for determining the boundary of stability (T_g) in a single-stage amplifier with two single-tuned bandpass filters and $\Theta = 270°$. The line OA intersects the polar impedance diagrams of the output bandpass filter and that of the input bandpass filter with an inductive tap. The products of the line lengths OA' and OA'' equals the reciprocal of T_g. The lines OB and OC are valid for input bandpass filters with an ideal transformer and with a capacitive tap respectively.

transformers to be employed becomes much less stable in the case of an inductive tap and much more stable in the case of a capacitive tap.

Considering the location of the polar plot in the complex plane of the output impedance of a practical transformer and the value of Θ, certain conclusions regarding the stability of the amplifier compared with the case of an ideal transformer can be drawn. These conclusions are summarized in Table 12.2:

TABLE 12.2 STABILITY OF AN AMPLIFIER STAGE

Case	$0 < \Theta < 180°$	$180° < \Theta < 360°$
inductive tap	increases	decreases
capacitive tap	decreases	increases

Depending on the location of Θ the stability of the amplifier increases by suitably choosing the kind of practical transformer. In practice, however, no advantage will generally be gained from this effect because for larger values of T a considerable deterioration of amplitude response curve of the amplifier occurs. In practical amplifier constructions it is therefore always attempted to make the transformers in such a way that their properties approach as nearly as possible those of an ideal transformer.

BIBLIOGRAPHY

[12.1] D. M. DUNCAN, *Mismatch Design of Transistor I.F. Amplifier*, Proc. I.R.E. Austr., p. 147-158, 1962.

[12.2] R. PAUL, *Zur Stabilität von Transistorbestückten Schmalbandverstärkerstufen*, Nachrichtent., **11**, p. 295-392, 1961.

APPENDIX I

APPLICATION OF MATRIX THEORY IN BANDPASS AMPLIFIER ANALYSIS

In this appendix a survey of the matrix theory of linear networks will be presented with special reference to the application in bandpass amplifier analyses. No attempt will be made to give an extensive treatment of the fundamental theory (see Bibliography I · 1, 3, 4, 6,13 and 14 to 17) but merely to state and to illustrate the basic rules governing the manipulation of matrices and to derive these matrices for various four terminal-networks thereof.

I.1 Matrix Algebra

A matrix equation may be considered as a symbolic method of writing a set of linear simultaneous equations. Consider for example the general simultaneous equations below:

$$\left.\begin{matrix} a_{11}\,x_1 + a_{12}\,x_2 + & \ldots\ldots\, a_{1n}\,x_n = y_1 \\ a_{21}\,x_1 + a_{22}\,x_2 + & \ldots\ldots\, a_{2n}\,x_n = y_2 \\ \ldots\ldots\ldots & \ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + & \ldots\ldots\, a_{mn}x_n = y_m \end{matrix}\right\} . \tag{I.1.1}$$

This set of equations may be expressed in symbolic form as:

$$\left\| \begin{matrix} a_{11} & a_{12} & — & — & — & a_{1n} \\ a_{21} & a_{22} & — & — & — & a_{2n} \\ — & — & — & — & — & — \\ — & — & — & — & — & — \\ a_{m1} & a_{m2} & — & — & — & a_{mn} \end{matrix} \right\| \cdot \left\| \begin{matrix} x_1 \\ x_2 \\ — \\ — \\ x_n \end{matrix} \right\| = \left\| \begin{matrix} y_1 \\ y_2 \\ — \\ — \\ y_m \end{matrix} \right\| , \tag{I.1.2}$$

or shorter as:

$$\|a\| \cdot \|x\| = \|y\| . \tag{I.1.3}$$

The quantities between the double bars are known as matrices which are, in fact, displays of information, as a comparison with Eq. (I.1.1) reveals.

Matrices may be manipulated as algebraic quantities taking into account some basic rules governing these manipulations. Sub-section I.1.2 presents these basic rules and in sub-section I.1.1 definitions are given of the various forms in which matrices may occur.

I.1.1 VARIOUS FORMS OF MATRICES

Matrices may occur in various special forms depending on the character of the information displayed. The forms which are of importance in our amplifier analyses and the terms used in connection with these matrix forms are defined below:

matrix : A rectangular array of $m.n.$ quantities arranged in m rows and n columns. A matrix cannot be evaluated.

determinant : A square array of n^2 quantities. A determinant can be evaluated by forming the sum of the products of the elements of any row or column and their respective co factors

$$\Delta = \sum_{j=1}^{j=n} a_{ij} \cdot A_{ij} . = \sum_{i=1}^{i=h} a_{ij} \cdot A_{ij} . \quad (I.1.4)$$

co factor : The co factor A_{ij} of the element a_{ij} of a determinant equals the product of the factor $(-1)^{i+j}$ and the (minor) determinant formed by deleting the row and column containing the element a_{ij} from the given determinant.

column matrix : A matrix consisting of one column of m elements.

row matrix : A matrix consisting of one row of n elements

square matrix : A matrix consisting of an equal number of columns and rows.

diagonal matrix : A matrix with all elements equal to zero except those in the principal diagonal.

unit matrix : A diagonal matrix with elements equal to unity in the principal diagonal.

null matrix or zero matrix : A matrix with all elements equal to zero.

determinant of a square matrix: A determinant whose array of elements is identical with the array of the matrix itself. Clearly only square matrices have determinants.

non-singular matrix : A matrix of which the determinant has a value different from zero.

singular matrix : A matrix of which the determinant vanishes after evaluation.

I.1.2.3 BASIC RULES OF MATRIX ALGEBRA

I.1.2.1 Equality

Two matrices are equal if their corresponding elements are equal;

$$||a|| = ||b||, \text{ if, and only if, } a_{ij} = b_{ij}. \tag{I.1.5}$$

I.1.2.2 Addition and Subtraction

The sum or difference of two matrices is another matrix of which the elements are equal to the sum or difference of the corresponding elements of the two matrices. This implies that the matrices to be added or subtracted must have the same number of columns and the same number of rows;

$$||a|| + ||b|| = ||c||, \text{ where } c_{ij} = a_{ij} + b_{ij}. \tag{I.1.6}$$

E.g.:

$$\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} + \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix} = \begin{Vmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{Vmatrix}.$$

I.1.2.3 Multiplication by a factor

If a matrix is multiplied by a factor, each element of the matrix is multiplied by that factor;

$$k \cdot ||a|| = ||b|| \text{ with } b_{ij} = k \cdot a_{ij}. \tag{I.1.7}$$

E.g.:

$$k \cdot \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} = \begin{Vmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{Vmatrix}.$$

I.1.2.4 Multiplication of Two Matrices

To obtain the element of the i^{th} row and the j^{th} column of a matrix $||c||$ which is the product of two matrices $||a||$ and $||b||$, the elements of the i^{th} row of $||a||$ are multiplied by the elements of the j^{th} column of $||b||$ and the results are summed;

$$||a|| \cdot ||b|| = ||c||, \text{ with } c_{ij} = \sum_{k=1}^{k=n} a_{ik} \cdot b_{kj}. \tag{I.1.8}$$

It is thus necessary that the second matrix has as many rows as the first has columns. If this is the case it is said that the matrices are *conformable in the order*. There is no limit to the number of rows of the first matrix or to the number of columns of the second matrix. E.g.:

$$\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix} = \begin{Vmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{Vmatrix}$$

$$\begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix} \cdot \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} = \begin{Vmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{Vmatrix}$$

I.1.2.5 Distributive Law

For matrices the distributive law is valid;

$$(||a|| + ||b||) \cdot ||c|| = ||a|| \cdot ||c|| + ||b|| \cdot ||c||. \tag{I.1.9}$$

I.1.2.6 Commutative Law for Multiplication

For matrices the commutative law for multiplication is generally *not* valid;

$$||a|| \cdot ||b|| \neq ||b|| \cdot ||a||.$$

This is also illustrated by the examples of point 4.

The non-validity of the commutative law implies that great care must be exercized in determining whether in a certain case pre-multiplication or post-multiplication is required.

I.1.2.7 Inversion

Inversion of a matrix $||a||$ leads to a matrix $||b||$ in which the element b_{ij} equals the quotient of the co factor A_{ji} of the element a_{ji} and the determinant of matrix $||a||$;

$$||a||^{-1} = ||b|| \text{ with } b_{ij} = \frac{A_{ji}}{\Delta_a}. \tag{I.1.10}$$

E.g.:

$$\|a\| = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} ; \|a\|^{-1} = \|b\| = \begin{Vmatrix} \frac{A_{11}}{\Delta_a} & \frac{A_{21}}{\Delta_a} \\ \frac{A_{12}}{\Delta_a} & \frac{A_{22}}{\Delta_a} \end{Vmatrix} = \frac{1}{(\Delta_a)^2} \begin{Vmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{Vmatrix},$$

in which Δ_a is the determinant of $\|a\|$. Since inversion of a matrix requires obtaining its determinant, inversion can exist only for square matrices.

I.1.2.8 Multiplication of a Matrix by its Inverse Matrix

Multiplying a matrix by its inverse matrix yields a unit matrix;

$$\|a\| \cdot \|a\|^{-1} = \|a^{-1}\| \cdot \|a\| = \|I\|. \tag{I.1.11}$$

E.g.:

$$\begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot \begin{Vmatrix} \frac{a_{22}}{\Delta_a} & -\frac{a_{12}}{\Delta_a} \\ -\frac{a_{21}}{\Delta_a} & \frac{a_{11}}{\Delta_a} \end{Vmatrix} = \begin{Vmatrix} \frac{a_{11}a_{22}}{\Delta_a} - \frac{a_{12}a_{21}}{\Delta_a} & \frac{-a_{11}a_{12}}{\Delta_a} + \frac{a_{11}a_{12}}{\Delta_a} \\ \frac{a_{21}a_{22}}{\Delta_a} - \frac{a_{22}a_{21}}{\Delta_a} & \frac{-a_{21}a_{12}}{\Delta_a} + \frac{a_{11}a_{22}}{\Delta_a} \end{Vmatrix}$$

$$= \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} = \|I\|.$$

I.1.2.9 Division by a Matrix

Division by a matrix must by carried out by multiplication by the inverse matrix. Consider, by way of example, the matrix equation:

$$\|i\| = \|y\| \cdot \|v\|.$$

Then:

$$\|y\|^{-1} \cdot \|i\| = \|y\|^{-1} \cdot \|y\| \cdot \|v\|,$$

$$= \|I\| \cdot \|v\|,$$

$$= \|v\|.$$

Hence:

$$\|v\| = \|y\|^{-1} \cdot \|i\|. \tag{I.1.12}$$

I.2 Matrix Equations of a General Four Terminal Network

For the general (linear) four-terminal network represented in Fig. I.1 six different pairs of simultaneous equations can be written down relating input quantities to output quantities and vice-versa. These equations written in matrix form are:

$$\begin{Vmatrix} v_1 \\ v_2 \end{Vmatrix} = \begin{Vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{Vmatrix} \cdot \begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix}; \qquad (I.2.1)$$

$$\begin{Vmatrix} i_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{Vmatrix} \cdot \begin{Vmatrix} v_1 \\ v_2 \end{Vmatrix}; \qquad (I.2.2)$$

$$\begin{Vmatrix} v_1 \\ i_2 \end{Vmatrix} = \begin{Vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{Vmatrix} \cdot \begin{Vmatrix} i_1 \\ v_2 \end{Vmatrix}; \qquad (I.2.3)$$

$$\begin{Vmatrix} i_1 \\ v_2 \end{Vmatrix} = \begin{Vmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{Vmatrix} \cdot \begin{Vmatrix} v_1 \\ i_2 \end{Vmatrix}; \qquad (I.2.4)$$

$$\begin{Vmatrix} v_1 \\ i_1 \end{Vmatrix} = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \cdot \begin{Vmatrix} v_2 \\ -i_2 \end{Vmatrix}; \qquad (I.2.5)$$

$$\begin{Vmatrix} v_2 \\ -i_2 \end{Vmatrix} = \begin{Vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{Vmatrix} \cdot \begin{Vmatrix} v_1 \\ i_1 \end{Vmatrix}. \qquad (I.2.6)$$

The matrices expressing the properties of the four-terminal network are termed respectively:

the *impedance matrix* in Eq. (I.2.1),
the *admittance matrix* in Eq. (I.2.2),
the *hybrid-h matrix* in Eq. (I.2.3),
the *hybrid-k matrix* in Eq. (I.2.4),
the *forward transfer matrix* in Eq. (I.2.5) and
the *reverse transfer matrix* in Eq. (I.2.6).

With the elements of any one matrix given, the elements of all other matrices may be calculated by algebraic methods. Since such calculations

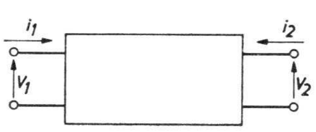


Fig. I.1. General four-terminal network.

are often required in matrix manipulations use can conveniently be made of tables of matrix and determinant interrelations as presented in Tables I.1 and I.2 (see Biblography [I . 12]).

I.3 Interconnection of Four-Terminal Networks

The various matrix equations of the preceding sub-section may advantageously be used to determine the resultant matrices when several four-terminal networks whose associated matrices are known are connected in various manners. In this sub-section we will consider two four-terminal networks connected either in:

a) series,
b) parallel,
c) series-parallel,
d) parallel-series or in
e) cascade.

Use will be made of that matrix which is fundamentally the most suitable for the kind of interconnection in question.

When connecting together four-terminal networks care must be taken that the networks are combined in such a way that the matrix equations of the individual networks remain valid after interconnection. This is the case if, and only if, the current entering one terminal of an input or output pair of an individual four-terminal network also emerges from the other terminal of the same pair after interconnection of the networks.

I.3.1 SERIES CONNECTION

In Fig. I.2 two four-terminal networks connected in series are represented. The networks I and II may be represented by the matrix equations:

$$||v'|| = ||z'|| \cdot ||i'||, \tag{I.3.1}$$

and

$$||v''|| = ||z''|| \cdot ||i''||. \tag{I.3.2}$$

For the combined four-terminal network, we have:

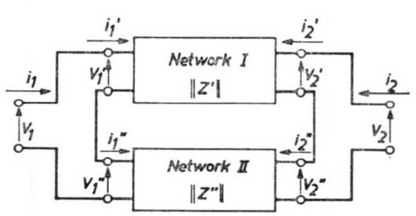


Fig. I.2. Series-connection of two four-terminal networks.

TABLE I.1. MATRIX INTERRELATIONS

to \ from	$\|\|z\|\|$	$\|\|y\|\|$	$\|\|h\|\|$	$\|\|k\|\|$	$\|\|a\|\|$	$\|\|b\|\|$
$\|\|z\|\|$	$\begin{matrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{matrix}$	$\begin{matrix} \frac{y_{22}}{\Delta y} & \frac{-y_{12}}{\Delta y} \\ \frac{-y_{21}}{\Delta y} & \frac{y_{11}}{\Delta y} \end{matrix}$	$\begin{matrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{k_{11}} & \frac{-k_{12}}{k_{11}} \\ \frac{k_{21}}{k_{11}} & \frac{\Delta k}{k_{11}} \end{matrix}$	$\begin{matrix} \frac{a_{11}}{a_{21}} & \frac{\Delta a}{a_{21}} \\ \frac{1}{a_{21}} & \frac{a_{22}}{a_{21}} \end{matrix}$	$\begin{matrix} \frac{b_{22}}{b_{21}} & \frac{1}{b_{21}} \\ \frac{\Delta b}{b_{21}} & \frac{b_{11}}{b_{21}} \end{matrix}$
$\|\|y\|\|$	$\begin{matrix} \frac{z_{22}}{\Delta z} & \frac{-z_{12}}{\Delta z} \\ \frac{-z_{21}}{\Delta z} & \frac{z_{22}}{\Delta z} \end{matrix}$	$\begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix}$	$\begin{matrix} \frac{1}{h_{11}} & \frac{-h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta k}{k_{22}} & \frac{k_{12}}{k_{22}} \\ \frac{-k_{21}}{k_{22}} & \frac{1}{k_{22}} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{a_{12}} & \frac{-\Delta a}{a_{12}} \\ \frac{-1}{a_{12}} & \frac{a_{11}}{a_{12}} \end{matrix}$	$\begin{matrix} \frac{b_{11}}{b_{12}} & \frac{-1}{b_{12}} \\ \frac{-\Delta b}{b_{12}} & \frac{b_{22}}{b_{12}} \end{matrix}$
$\|\|h\|\|$	$\begin{matrix} \frac{\Delta z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ \frac{-z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & \frac{-y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta y}{y_{11}} \end{matrix}$	$\begin{matrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{matrix}$	$\begin{matrix} \frac{k_{22}}{\Delta k} & \frac{-k_{12}}{\Delta k} \\ \frac{-k_{21}}{\Delta k} & \frac{k_{22}}{\Delta k} \end{matrix}$	$\begin{matrix} \frac{a_{12}}{a_{22}} & \frac{\Delta a}{a_{22}} \\ \frac{-1}{a_{22}} & \frac{a_{21}}{a_{22}} \end{matrix}$	$\begin{matrix} \frac{b_{12}}{b_{11}} & \frac{1}{b_{11}} \\ \frac{-\Delta b}{b_{11}} & \frac{b_{21}}{b_{11}} \end{matrix}$
$\|\|k\|\|$	$\begin{matrix} \frac{1}{z_{11}} & \frac{-z_{12}}{z_{11}} \\ \frac{z_{21}}{z_{11}} & \frac{\Delta z}{z_{11}} \end{matrix}$	$\begin{matrix} \frac{\Delta y}{y_{22}} & \frac{y_{12}}{y_{22}} \\ \frac{-y_{21}}{y_{22}} & \frac{1}{y_{22}} \end{matrix}$	$\begin{matrix} \frac{h_{22}}{\Delta h} & \frac{-h_{12}}{\Delta h} \\ \frac{-h_{21}}{\Delta h} & \frac{h_{11}}{\Delta h} \end{matrix}$	$\begin{matrix} k_{11} & k_{12} \\ k_{21} & k_{21} \end{matrix}$	$\begin{matrix} \frac{a_{21}}{a_{11}} & \frac{-\Delta a}{a_{11}} \\ \frac{1}{a_{11}} & \frac{a_{12}}{a_{11}} \end{matrix}$	$\begin{matrix} \frac{b_{21}}{b_{22}} & \frac{-1}{b_{22}} \\ \frac{\Delta b}{b_{22}} & \frac{b_{12}}{b_{22}} \end{matrix}$
$\|\|a\|\|$	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} \frac{-y_{22}}{y_{21}} & \frac{-1}{y_{21}} \\ \frac{-\Delta y}{y_{21}} & \frac{-y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} \frac{-\Delta h}{h_{21}} & \frac{-h_{11}}{h_{21}} \\ \frac{-h_{22}}{h_{21}} & \frac{-1}{h_{21}} \end{matrix}$	$\begin{matrix} \frac{1}{k_{21}} & \frac{k_{22}}{k_{21}} \\ \frac{k_{11}}{k_{21}} & \frac{\Delta k}{k_{21}} \end{matrix}$	$\begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{matrix}$	$\begin{matrix} \frac{b_{22}}{\Delta b} & \frac{b_{12}}{\Delta b} \\ \frac{b_{21}}{\Delta b} & \frac{b_{11}}{\Delta b} \end{matrix}$
$\|\|b\|\|$	$\begin{matrix} \frac{z_{22}}{z_{12}} & \frac{\Delta z}{z_{12}} \\ \frac{1}{z_{12}} & \frac{z_{11}}{z_{12}} \end{matrix}$	$\begin{matrix} \frac{-y_{11}}{y_{12}} & \frac{-1}{y_{12}} \\ \frac{-\Delta y}{y_{12}} & \frac{-y_{22}}{y_{12}} \end{matrix}$	$\begin{matrix} \frac{1}{h_{12}} & \frac{h_{11}}{h_{12}} \\ \frac{h_{22}}{h_{12}} & \frac{\Delta h}{h_{12}} \end{matrix}$	$\begin{matrix} \frac{-\Delta k}{k_{12}} & \frac{-k_{22}}{k_{12}} \\ \frac{-k_{11}}{k_{12}} & \frac{-1}{k_{12}} \end{matrix}$	$\begin{matrix} \frac{a_{22}}{\Delta a} & \frac{a_{12}}{\Delta a} \\ \frac{a_{21}}{\Delta a} & \frac{a_{11}}{\Delta a} \end{matrix}$	$\begin{matrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{matrix}$

TABLE I.2. DETERMINANT INTERRELATIONS

to \ from	Δz	Δy	Δh	Δk	Δa	Δb
Δz	Δz	$\dfrac{1}{\Delta y}$	$\dfrac{h_{11}}{h_{12}}$	$\dfrac{k_{22}}{k_{21}}$	$\dfrac{a_{12}}{a_{21}}$	$\dfrac{b_{12}}{b_{21}}$
Δy	$\dfrac{1}{\Delta z}$	Δy	$\dfrac{h_{22}}{h_{11}}$	$\dfrac{k_{11}}{k_{22}}$	$\dfrac{a_{21}}{a_{12}}$	$\dfrac{b_{21}}{b_{12}}$
Δh	$\dfrac{z_{11}}{z_{22}}$	$\dfrac{y_{22}}{y_{11}}$	Δh	$\dfrac{1}{\Delta k}$	$\dfrac{a_{11}}{a_{22}}$	$\dfrac{b_{22}}{b_{11}}$
Δk	$\dfrac{z_{22}}{z_{11}}$	$\dfrac{y_{11}}{y_{22}}$	$\dfrac{1}{\Delta h}$	Δk	$\dfrac{a_{22}}{a_{11}}$	$\dfrac{b_{11}}{b_{22}}$
Δa	$\dfrac{z_{12}}{z_{21}}$	$\dfrac{y_{12}}{y_{21}}$	$-\dfrac{h_{12}}{h_{21}}$	$-\dfrac{k_{12}}{k_{21}}$	Δa	$\dfrac{1}{\Delta b}$
Δb	$\dfrac{z_{21}}{z_{12}}$	$\dfrac{y_{21}}{y_{12}}$	$-\dfrac{h_{21}}{h_{12}}$	$-\dfrac{k_{21}}{k_{12}}$	$\dfrac{1}{\Delta a}$	Δb

$$\left.\begin{aligned} v_1 &= v_1' + v_1'', \\ v_2 &= v_2' + v_2'', \end{aligned}\right\} \tag{I.3.3}$$

and, provided there is no circulating current in the inner loop of the combination;

$$\left.\begin{aligned} i_1 &= i_1' = i_1'', \\ i_2 &= i_2' = i_2''. \end{aligned}\right\} \tag{I.3.4}$$

Hence:

$$||v|| = ||v'|| + ||v''|| = (||z'|| + ||z''||) \cdot ||i||. \tag{I.3.5}$$

By putting

$$||z'|| + ||z''|| = ||z||, \tag{I.3.6}$$

$||z||$ becomes, using the rule for the addition of matrices (see sub-section I.1.2.2):

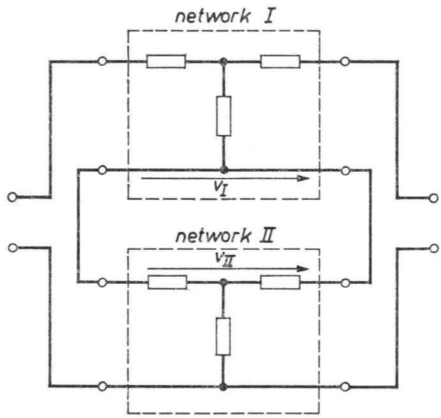


Fig. I.3. Non-permissible series connections of two four-terminal networks. The connection between the lower terminals of network I short-circuits the elements between the upper terminals of network II.

$$||z|| = \begin{Vmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{Vmatrix}. \tag{I.3.7}$$

The relations derived above are thus valid if the two networks are connected in a permissible manner. The difference between a permissible and a non-permissible interconnection is clearly illustrated by Figs. I.3 and I.4. In order that no circulating current will flow in the inner loop of the two series connected networks of Fig. I.2, the voltage between the lower (input and output) terminals of network I and that between the upper terminals of network II must be equal before interconnection. Clearly this condition is not fulfilled by the two networks of Fig. I.3 ($V_{\mathrm{I}} \neq V_{\mathrm{II}}$); the interconnection as shown is therefore not permissible. By rearranging the network II as shown in Fig. I.4 the interconnection becomes permissible (now $V_{\mathrm{I}} = V_{\mathrm{II}}$)

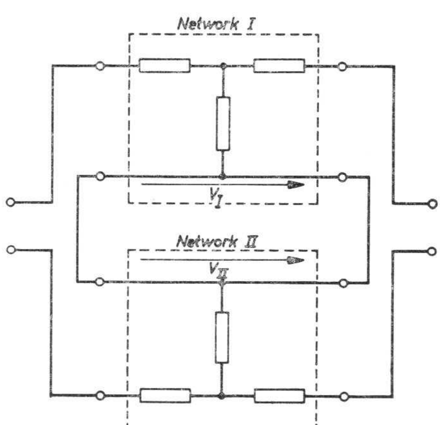


Fig. I.4. Series connection of the same networks as in Fig. I.3, but now in a permissible manner.

I.3.2 PARALLEL CONNECTION

Fig. I.5 shows two four-terminal networks connected in parallel. Let the networks I and II be characterized by:

$$\left.\begin{aligned} \|i_1'\| &= \|y'\| \cdot \|v'\|, \\ \text{and} \quad \|i_1''\| &= \|y''\| \cdot \|v''\|. \end{aligned}\right\} \tag{I.3.8}$$

Now:

$$\left.\begin{aligned} i_1 &= i_1' + i_1'', \\ i_2 &= i_2' + i_2'', \end{aligned}\right\} \tag{I.3.9}$$

provided there is no current unbalance in the combined network; i.e. there is no circulating current in the loop formed across the upper terminals of the pairs of terminals of both networks. Furthermore:

$$\left.\begin{aligned} v_1 &= v_1' = v_1'', \\ v_2 &= v_2' = v_2''. \end{aligned}\right\} \tag{I.3.10}$$

Hence, from Eqs. (I.3.8) to (I.3.10):

$$\begin{aligned} \|i\| &= \|i'\| + \|i''\| \\ &= (\|y'\| + \|y''\|) \cdot \|v\|. \end{aligned} \tag{I.3.11}$$

With

$$\|y\| = \|y'\| + \|y''\|, \tag{I.3.12}$$

$$\|y\| = \left\| \begin{matrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{matrix} \right\|. \tag{I.3.13}$$

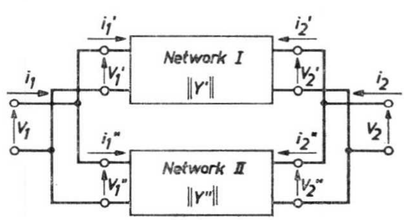


Fig. I.5. Parallel connection of two four-terminal networks.

An example of the technique of adding admittance matrices when paralleling four-terminal networks is found in the systematic formation of a double-tuned bandpass filter. The procedure is shown step by step in Fig. I.6.

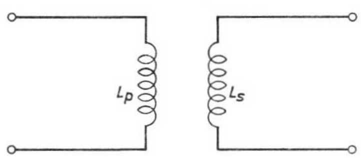
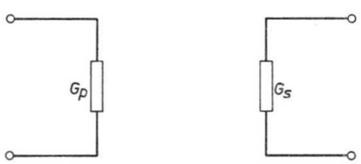
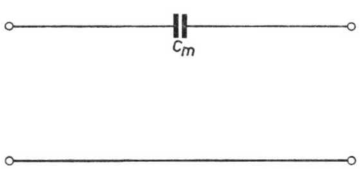
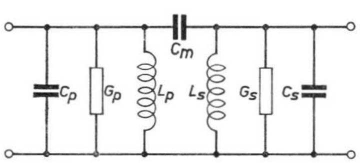
step no.	four-terminal network	admittance matrix $\|y\|$
1 prim. and sec. capacitances		$\begin{Vmatrix} j\omega C_p & 0 \\ 0 & j\omega C_s \end{Vmatrix}$
2 prim. and sec. inductances		$\begin{Vmatrix} \frac{1}{j\omega L_p} & 0 \\ 0 & \frac{1}{j\omega L_s} \end{Vmatrix}$
3 prim. and sec. dampings		$\begin{Vmatrix} G_p & 0 \\ 0 & G_s \end{Vmatrix}$
4 coupling capacitance		$\begin{Vmatrix} j\omega C_m & -j\omega C_m \\ -j\omega C_m & j\omega C_m \end{Vmatrix}$
5 complete double tuned bandpass filter		$\begin{Vmatrix} G_p+j\omega(C_p+C_m)+\frac{1}{j\omega L_p} & -j\omega C_m \\ -j\omega C_m & G_s+j\omega(C_s+C_m)+\frac{1}{j\omega L_s} \end{Vmatrix}$

Fig. I.6. Systematic formation of a double-tuned bandpass filter showing a step by step method of obtaining its admittance matrix.

I.3.3 SERIES-PARALLEL CONNECTION

Fig. I.7 presents a combination of two four-terminal networks connected in series at the input side and in parallel at the output side. Let the matrix equations of networks I and II be given by:

$$\left\| \begin{matrix} v_1' \\ i_2' \end{matrix} \right\| = \|h'\| \cdot \left\| \begin{matrix} i_1' \\ v_2'' \end{matrix} \right\|, \tag{I.3.14}$$

and

$$\left\| \begin{matrix} v_1'' \\ i_2'' \end{matrix} \right\| = \|h''\| \cdot \left\| \begin{matrix} i_1'' \\ v_2'' \end{matrix} \right\|. \tag{I.3.15}$$

In order that the interconnection of the networks as shown is permissible, no circulating current may flow in the loop across the terminals marked 1′, 2′, 2″ and 1″, see Fig. I.7. Then:

$$\left.\begin{aligned} v_1 &= v_1' + v_1'', \\ i_2 &= i_2' + i_2'', \\ \\ i_1 &= i_1' = i_1'', \\ v_2 &= v_2' = v_2'', \end{aligned}\right\} \tag{I.3.16}$$

$$\left\| \begin{matrix} v_1 \\ i_2 \end{matrix} \right\| = (\|h'\| + \|h''\|) \cdot \left\| \begin{matrix} i_1 \\ v_2 \end{matrix} \right\|, \tag{I.3.17}$$

and

$$\|h\| = \|h'\| + \|h''\| = \left\| \begin{matrix} h_{11}' + h_{11}'' & h_{12}' + h_{12}'' \\ \\ h_{21}' + h_{21}'' & h_{22}' + h_{22}'' \end{matrix} \right\|. \tag{I.3.18}$$

I.3.4 PARALLEL-SERIES CONNECTION

In Fig. I.8 a parallel-series connection of two four-terminal networks is shown. Let the matrix equations of the networks be given by:

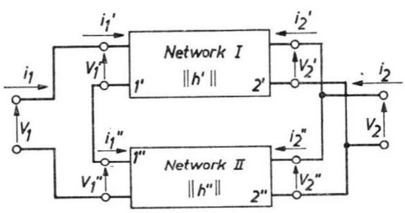


Fig. I.7. Series-parallel connection of two four-terminal networks.

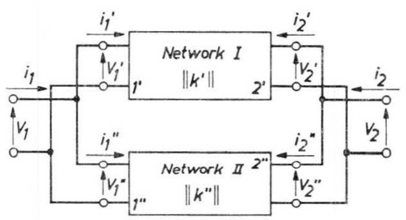


Fig. I.8. Parallel-series connection of two four-terminal networks.

$$\left\| \begin{matrix} i_1' \\ v_2' \end{matrix} \right\| = \|k'\| \cdot \left\| \begin{matrix} v_1' \\ i_2' \end{matrix} \right\|, \qquad \text{(I.3.19)}$$

and

$$\left\| \begin{matrix} i_1'' \\ v_2'' \end{matrix} \right\| = \|k''\| \cdot \left\| \begin{matrix} v_1'' \\ i_2'' \end{matrix} \right\|. \qquad \text{(I.3.20)}$$

If the interconnection of the networks is permissible (no circulating current flowing in the loop across the terminals marked 1′, 2′, 2″ and 1″ in Fig. I.8):

$$\left. \begin{aligned} i_1 &= i_1' + i_2'' , \\ v_2 &= v_2' + v_2'' , \\ v_1 &= v_1' = v_1'' , \\ i_2 &= i_2' = i_2'' , \end{aligned} \right\} \qquad \text{(I.3.21)}$$

$$\left\| \begin{matrix} i_1 \\ v_2 \end{matrix} \right\| = (\|k'\| + \|k''\|) \cdot \left\| \begin{matrix} v_1 \\ i_2 \end{matrix} \right\|, \qquad \text{(I.3.22)}$$

and $$\|k\| = \|k'\| + \|k''\| = \left\| \begin{matrix} k_{11}' + k_{11}'' & k_{12}' + k_{12}'' \\ k_{21}' + k_{21}'' & k_{22}' + k_{22}'' \end{matrix} \right\| . \qquad \text{(I.3.23)}$$

I.3.5 CASCADE CONNECTION

Fig. I.9 shows a cascade connection of two networks. Let the matrix equations of the networks be given by:

$$\left\| \begin{matrix} v_1' \\ i_1' \end{matrix} \right\| = \| a' \| \cdot \left\| \begin{matrix} v_2' \\ -i_2' \end{matrix} \right\|, \qquad \text{(I.3.24)}$$

and:

$$\left\| \begin{matrix} v_1'' \\ i_1'' \end{matrix} \right\| = \| a'' \| \cdot \left\| \begin{matrix} v_2'' \\ -i_2'' \end{matrix} \right\| . \qquad \text{(I.3.25)}$$

From inspection of Fig. I.9 it follows:

$$\left\| \begin{matrix} v_2' \\ -i_2' \end{matrix} \right\| = \left\| \begin{matrix} v_1'' \\ i_1'' \end{matrix} \right\| \qquad \text{(I.3.26)}$$

Hence, by substituting Eq. (I.3.25) into (I.3.24) taking into account Eq.(I.3.26)

$$\left\| \begin{matrix} v_1' \\ i_1' \end{matrix} \right\| = \| a' \| \cdot \| a'' \| \cdot \left\| \begin{matrix} v_2'' \\ -i_2'' \end{matrix} \right\| . \qquad \text{(I.3.27)}$$

Now $$\| a \| = \| a' \| \cdot \| a'' \| , \qquad \text{(I.3.28)}$$

which forms the new matrix of the two cascaded networks.

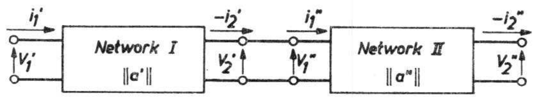


Fig. I.9. Cascade connection of two four-terminal networks.

Note that a cascade connection of networks is always permissible (a current entering a network via one terminal of a pair emerges from the other terminal of the pair also after cascading).

I.4 Admittance Matrix of a General n-Node Network

In the preceding sections various matrices for four-terminal networks are considered. In amplifier analyses, however, it is often convenient to consider the complete amplifier or parts of it as a network having n nodes of which at least three are accesible from outside the network. (The network is then of the three-terminal type of which the input- and output pairs have one terminal in common).

I.4.1 THE INDEFINITE ADMITTANCE MATRIX

In Fig. I.10 an n-node network is represented with voltages v_r applied to each of the terminals and currents i_r entering each of the terminals. The voltages v_r are all measured with respect to the same reference level.

If the n-node network only contains linear passive or active elements, it may be described by n independent simultaneous equations. These equations may be written down by considering that the current entering a certain node is a linear function of the voltages between this and all other nodes. For node r this equation reads:

$$\begin{aligned} i_r = y_{r1}(v_1 - v_r) + y_{r2}(v_2 - v_r) + \dots \\ + y_{r(r-1)}(v_{r-1} - v_r) + y_{r(r+1)}(v_{r+1} - v_r) + \dots \\ \dots + y_{rn}(v_n - v_r) \,. \qquad (I.4.1) \end{aligned}$$

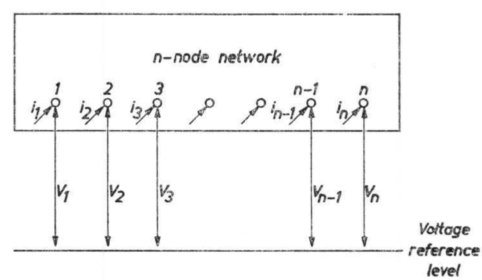


Fig. I.10. General n-node network.

This equation may also be written:

$$i_r = y_{r1} \cdot v_1 + y_{r2} \cdot v_2 + \ldots + y_{r(r-1)} \cdot v_{r-1} + y_{rr} \cdot v_r + \\ + y_{r(r+1)} \cdot v_{r+1} + \ldots + y_{rn} \cdot v_n \,. \qquad (I.4.2)$$

Apparently, in Eq. (I.4.2) a current $y_{rr}v_r$ has been intoduced which must be equal to the sum of the products of v_r and the corresponding admittances. Therefore:

$$y_{rr} = -\sum_{m=1}^{m=n} y_{rm} \,, \quad m \neq r \qquad (I.4.3)$$

which may also be written as:

$$\sum_{m=1}^{m=n} y_{rm} = 0 \,. \qquad (I.4.4)$$

Furthermore, according to Eq. (I.4.2):

$$i_r = \sum_{m=1}^{m=n} (y_{rm} \cdot v_m). \qquad (I.4.5)$$

The n equations with which the n-node network may be analyzed may be represented by the matrix:

$$||i|| = ||Y|| \cdot ||v|| \,. \qquad (I.4.6)$$

Here, $||Y||$ is a square matrix of order n which, according to Eq. (1.4.4), is singular. By way of example, the matrix $||Y||$ for a network having four nodes is given by:

$$||Y|| = \left\| \begin{matrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{matrix} \right\| \,. \qquad (I.4.7)$$

The admittance matrix $||Y||$ which after Shekel (Bibliography [I.12]) is defined as the *indefinite admittance matrix* [1]) has some special properties which will be discussed below.

Considering Eq. (I.4.4) it follows that each row of the matrix $||Y||$ must add to zero.

---

[1]) The matrix $||Y||$ is termed indefinite because its array of elements is defined relative to an arbitrary (indefinite) reference point.

Furthermore, by Kirchoffs first law the algebraic sum of all currents entering the nodes is zero. Thus:

$$\sum_{r=1}^{r=n} i_r = 0\,. \tag{I.4.8}$$

With Eq. (I.4.5) we then obtain:

$$\sum_{r=1}^{r=n} \left\{ \sum_{m=1}^{m=n} (y_{rm} \cdot v_m) \right\} = 0\,;$$

or:

$$\sum_{m=1}^{m=n} \left\{ \sum_{r=1}^{r=n} (y_{rm} \cdot v_m) \right\} = 0\,. \tag{I.4.9}$$

Since v_m is completely independent of r, Eq. (I.4.9) can only be satisfied if:

$$\sum_{r=1}^{r=n} y_{rm} = 0\,. \tag{I.4.10}$$

This implies that each column of the matrix $||Y||$ must also add to zero.

If two nodes of the network have the same voltage applied, it is permissible to interconnect these nodes. Then the current entering the combined node is equal to the sum of the currents entering the separate nodes. The indefinite admittance matrix of the new network may therefore be obtained by adding the corresponding elements of the columns and rows of the nodes which are combined. If, for example, the nodes 3 and 4 of the four-node network described by Eq. (I.4.7) are combined, the indefinite admittance matrix of the network becomes:

$$\begin{Vmatrix} y_{11} & y_{12} & y_{13} + y_{14} \\ y_{21} & y_{22} & y_{23} + y_{24} \\ y_{31} + y_{41} & y_{32} + y_{42} & y_{33} + y_{34} + y_{43} + y_{44} \end{Vmatrix}\,. \tag{I.4.11}$$

If the n-node network contains isolated nodes, that are nodes which have no electrical connection with other nodes, the indefinite admittance matrix contains columns and rows with zero elements corresponding to these nodes. This may become apparent from the following reasoning: An isolated node draws no external current. This necessitates a row of zero entries to ensure that the currents constituting the node current are zero irrespective of the voltages of other nodes. Furthermore, the voltage of an isolated node can have no influence upon the currents of other nodes, and so the column corresponding to such a node must have all zero entries.

I.4.2 THE DEFINITE ADMITTANCE MATRIX

The indefinite admittance matrix as considered in the preceding sub-section may be regarded as an array of elements displaying the properties of an n-node network with respect to some arbitrary reference point. If one of the nodes of the network is taken as the reference point the indefinite admittance matrix becomes a definite admittance matrix (because now the reference point is defined).

Suppose the r-th node of the network is taken as the reference point. Then the voltages of all other nodes must be expressed relative to this node, which is achieved by taking v_r as zero. This means that the r th column of the indefinite admittance matrix may be deleted.

Furthermore, the current flowing into the reference mode is usually not required. This implies that the rth row of the indefinite admittance matrix may also be deleted.

The matrix now obtained is referred to as the *definite admittance matrix* or simply the *admittance matrix* of the network relative to the common node r. Consider, by way of example, the indefinite admittance matrix of a network having four nodes with nodes 3 and 4 connected as given by Eq. (1.4.11) If the combined node 3 is regarded as the common reference node the definite admittance matrix becomes:

$$||y|| = \left\| \begin{matrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{matrix} \right\| . \tag{I.4.12}$$

If the definite admittance matrix of a network is known, the indefinite admittance matrix can be obtained by using the condition that all rows and columns add to zero. This means that a row and a column have to be added to the definite admittance matrix containing elements of such a value that these conditions are satisfied. For example, let the definite admittance matrix of a network be given by Eq. (I.4.12). The indefinite matrix then becomes:

$$||y|| \left\| \begin{matrix} y_{11} & y_{12} & -(y_{11}+y_{12}) \\ y_{21} & y_{22} & -(y_{21}+y_{22}) \\ -(y_{11}+y_{21}) & -(y_{12}+y_{22}) & y_{11}+y_{12}+y_{21}+y_{22} \end{matrix} \right\| . \tag{I.4.13}$$

I.4.3 SURVEY OF PROPERTIES OF THE INDEFINITE ADMITTANCE MATRIX

In sub-sections I.4.1 and I.4.2 various properties of the indefinite admittance matrix are derived. In this sub-section these properties are summarized, for ease of reference:

1) The indefinite admittance matrix is singular and the sum of the elements of any row or column is zero.

2) The definite admittance matrix of a network is obtained from the indefinite admittance matrix by deleting a row and a column corresponding to the reference node. The definite admittance matrix is non-singular.
3) The indefinite admittance matrix can be obtained from the definite admittance matrix by adding one row and one column with elements such that each row and column add to zero.
4) When two nodes of the network are connected the corresponding rows and columns are added to form one row and column.
5) Isolated nodes correspond to rows and columns of zero entries. Isolated nodes may be employed to increase the order of the indefinite admittance matrix.

I.4.4 THE INDEFINITE ADMITTANCE MATRIX OF NETWORKS IN PARALLEL

If two n-node networks have corresponding nodes at the same voltage level interconnection of these nodes is permissible (see Section I.3). If all permissible interconnections are made it is said that the two n-node networks are connected in parallel.

The indefinite admittance matrix of the paralleled networks can then be obtained by adding corresponding elements of the indefinite admittance matrices of the individual networks. To add these matrices it is required that they have the same number of rows and columns. This means that the networks must have the same number of nodes and this may be achieved by inserting a number of isolated nodes in one of the networks.

Furthermore, if interconnection of two nodes in the networks to be connected in parallel is not permissible an isolated node is inserted in each of the networks corresponding to the non-isolated node where interconnection was not permissible.

It will be evident from the above considerations that "paralleling" networks making use of the isolated node concept covers a very wide field of circuit applications. Using this technique it is possible to write down by inspection the indefinite admittance matrix of almost any circuit.

To elucidate the method of paralleling and obtaining the indefinite admittance matrix, the circuit of Fig. I.11 is analyzed step by step. This circuit has four nodes and therefore a 4×4 indefinite admittance matrix. In Fig. I.12 the indefinite admittance matrices for each of the separate elements of the circuit are derived, whereas in the final step (5) the complete indefinite admittance matrix is obtained. In practical circuit analysis it is, of course, not necessary to follow a step by step method of deriving the indefinite admittance matrix of the complete circuit. It can be written down by merely inspecting the circuit.

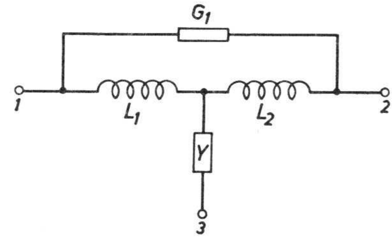


Fig. I.11. Four-node network (compensated trap circuit) used as an example for illustrating the method of employing the indefinite admittance matrix.

Taking node (terminal) 3 as common the definite admittance matrix of the circuit of Fig. I.11 becomes:

$$\left\| \begin{matrix} G_1 + \dfrac{1}{j\omega L_1} & -G_1 & -\dfrac{1}{j\omega L_1} \\ -G_1 & G_1 + \dfrac{1}{j\omega L_2} & -\dfrac{1}{j\omega L_2} \\ -\dfrac{1}{j\omega L_1} & -\dfrac{1}{j\omega L_2} & +\dfrac{1}{j\omega L_1} + \dfrac{1}{j\omega L_2} + Y \end{matrix} \right\| . \quad \text{(I.4.14)}$$

Obviously, the definite admittance matrix (I.4.14) of the circuit of Fig. I.11 can also be obtained without the intermediate step of the indefinite admittance matrix. For complicated circuits, however, the method employing the indefinite admittance matrix will prove to be more systematic.

I.5 Application of the General Admittance Matrix in Amplifier Analysis

The general admittance matrix of an n-node network as considered in the preceding section may readily be applied to the analysis of multi-stage amplifiers dealt with in Chapters 5, 7 and 8. Such amplifiers generally have only two terminal pairs of which one terminal is common.

To analyze the performance of the amplifier with respect to stability, gain and frequency response it is sufficient to calculate the transfer function from the input terminal pair to the output terminal pair. This calculation can be carried out either by the "determinant method" of solving a set of simultaneous equations using the definite admittance matrix or by reducing the order of the definite admittance matrix from (n—1) to 2 (assuming the amplifier has n-nodes).

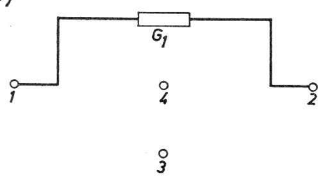
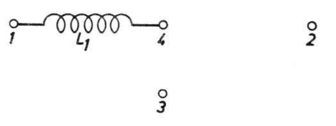
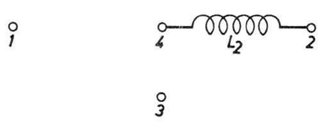
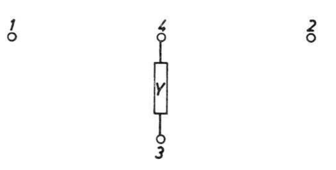
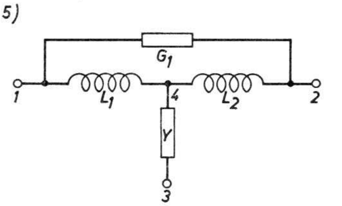
1) 	$\begin{Vmatrix} G_1 & -G_1 & 0 & 0 \\ -G_1 & G_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{Vmatrix}$
2) 	$\begin{Vmatrix} \frac{1}{j\omega L_1} & 0 & 0 & -\frac{1}{j\omega L_1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{j\omega L_1} & 0 & 0 & \frac{1}{j\omega L_1} \end{Vmatrix}$
3) 	$\begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{j\omega L_2} & 0 & -\frac{1}{j\omega L_2} \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{j\omega L_2} & 0 & \frac{1}{j\omega L_2} \end{Vmatrix}$
4) 	$\begin{Vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Y & -Y \\ 0 & 0 & -Y & Y \end{Vmatrix}$
5) 	$\begin{Vmatrix} G_1+\frac{1}{j\omega L_1} & -G_1 & 0 & -\frac{1}{j\omega L_1} \\ -G_1 & G_1+\frac{1}{j\omega L_2} & 0 & -\frac{1}{j\omega L_2} \\ 0 & 0 & Y & -Y \\ -\frac{1}{j\omega L_1} & -\frac{1}{j\omega L_2} & -Y & \frac{1}{j\omega L_1}+\frac{1}{j\omega L_2}+Y \end{Vmatrix}$

Fig. I.12. Tabular diagram showing the step by step method of obtaining the indefinite (or definite) admittance matrix of the network of Fig. I.11.

I.6 Reduction of the Order of an Admittance Matrix

The method of reducing the order of an admittance matrix as presented here is based on an article by Nichols (Bibliography (I.7)).

An admittance matrix of an n-node linear network may be split into columns and rows corresponding to nodes which have external connections and columns and rows which correspond to nodes that have no external connections and so have zero node currents. Preferably the admittance matrix should be reduced to such a degree that all rows and columns corresponding to the latter kind of nodes disappear. Then the remaining matrix relates currents and voltages at the terminals and this is sufficient to analyze its performance.

The reduction can be carried out as follows: Let r—1 nodes of the n-node network have external connections and let these nodes be numbered 1 to (r—1). Then n — (r—1) nodes have no external connections. Let these nodes be numbered r to n. The admittance matrix equation of this network then becomes as shown in Fig. I.13. The matrix:

$$\left\| \begin{array}{c:c} A & B \\ \hdashline C & D \end{array} \right\|, \tag{I.6.1}$$

is the definite or indefinite admittance matrix of the system partitioned into four matrices A, B, C and D. The matrices A, B, C and D are defined in Fig. I.13.

The matrix equation of Fig. I.13 can now be split into two matrix equations, namely:

$$\left\| \begin{matrix} i_1 \\ \cdot\,\cdot \\ \cdot\,\cdot \\ \cdot\,\cdot \\ i_{r-1} \end{matrix} \right\| = A \cdot \left\| \begin{matrix} v_1 \\ \cdot\,\cdot \\ \cdot\,\cdot \\ \cdot\,\cdot \\ v_{r-1} \end{matrix} \right\| + B \cdot \left\| \begin{matrix} v_r \\ \cdot\,\cdot \\ \cdot\,\cdot \\ \cdot\,\cdot \\ v_n \end{matrix} \right\| \tag{I.6.2}$$

and:

$$0 = C \left\| \begin{matrix} v_1 \\ \cdot\,\cdot \\ \cdot\,\cdot \\ \cdot\,\cdot \\ v_{r-1} \end{matrix} \right\| + D \left\| \begin{matrix} v_r \\ \cdot\,\cdot \\ \cdot\,\cdot \\ \cdot\,\cdot \\ v_n \end{matrix} \right\|. \tag{I.6.3}$$

The matrix D is a square matrix of order $n - r + 1$. Provided $|D| \neq 0$, the inverse $\|D\|^{-1}$ exists (see sub-section I.1.2) and Eq. (I.6.3) may be written:

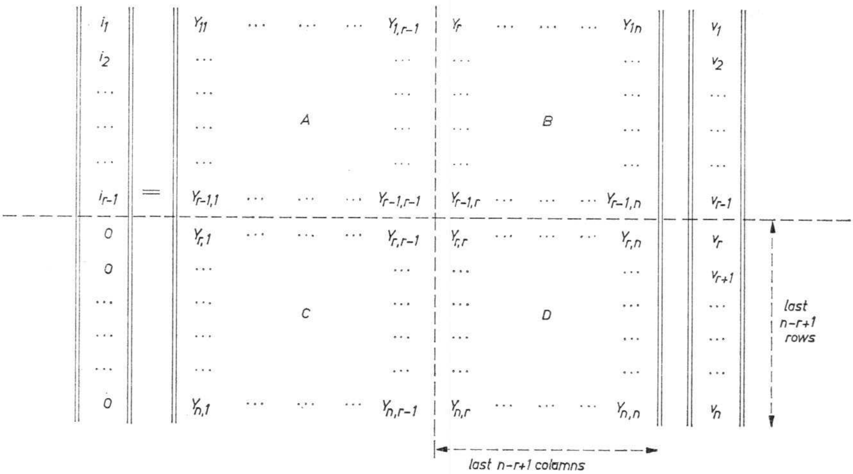


Fig. I.13. Partitioning of a general matrix equation for reduction of the order of the matrix.

$$\left\| \begin{matrix} v_r \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ v_n \end{matrix} \right\| = -D^{-1} \cdot C \cdot \left\| \begin{matrix} v_1 \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ v_{r-1} \end{matrix} \right\| . \tag{I.6.4}$$

Substituting Eq. (I.6.4) into (I.6.2) gives:

$$\left\| \begin{matrix} i_1 \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ i_{r-1} \end{matrix} \right\| = (A - B \cdot D^{-1} \cdot C) \cdot \left\| \begin{matrix} v_1 \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ v_{r-1} \end{matrix} \right\| . \tag{I.6.5}$$

The matrix $(A—B\cdot D^{-1}\cdot C)$ is an admittance matrix of order $(r—1)$, and Eq. (I.6.5) only contains voltages and currents appearing at the terminals of the network.

In multiplying the matrices $B\cdot D^{-1}$ and C care must be taken that these matrices are conformable in the order (see sub-section I.1.2.4). If not, this can be achieved by adding rows and columns of zeros at appropriate places in the complete matrix (as given in Fig. I.13).

BIBLIOGRAPHY

[I.1] R. BELLMAN, *Introduction to Matrix Analysis*, Mc Graw-Hill Book Comp., Inc., New-York, 1960.

[I.2] J. S. BELL and K. BREWSTER, *An Introduction to Matrices and their Use in Transistor Circuit Analysis*, Electronic Eng. **31**, p. 98-102. Febr, 1959.

[I.3] F. BOWMAN, *An Introduction to Determinants and Matrices*, The English University Press, Ltd., London, 1962.

[I.4] R. FELDTKELLER, *Einführung in die Vierpoltheorie der Elektrischen Nachrichtentechnik*, S. Hirzel Verlag, Stuttgart, 1959.

[I.5] C. R. LAUGLIN, *The Indefinite Y Matrix*, Electronic Ind., **21**, p. 121-122, Dec. 1962.

[I.6] M. M. NICOLSON, *Fundamentals and Techniques of Mathematics for Scientists*, Longmans, Green & Co. Ltd., London, 1961, Chapter 18.

[I.7] K. G. NICHOLS, *The Determination of the Admittance Matrix of a Linear Three Pole*, Electronic Eng. **34**, p, 476-482. July 1962.

[I.8] K. G. NICHOLS, *A Matrix Representation of Linear Amplifiers*, J. Brit. I.R.E. **23**, p. 517-533, June 1961.

[I.9] T. R. NISBET and W. W. HAPP, *Jacobians — a New Computational Tool*, Electronic Ind., **17**, p. 69-71, xxx 1958.

[I.10] L. A. PIPES, *The Matrix Theory of Four-Terminal Networks*, Phil. Mag. **30**, p. 370-395, 1940.

[I.11] R. F. SHEA, *Transistor Circuit Engineering*, John Wiley & Sons, Inc., New York.

[I.12] J. SHEKEL, *Matrix Representation of Transistor Circuits*, Proc. I.R.E. **40**, p. 1493-97, November 1952.

[I.13] M. N. STRIKANTASWANG and K. K. NAIR, *Analysis of Active Networks by Admittance Matrices*, J. Instn. Telecommun. Engrs. India **5**, p. 198-193, 1959, no. 4.

[I.14] C. R. WYLIE, JR., *Advanced Engineering Mathematics*, 2nd edition. McGraw-Hill Book Comp., Inc. New York, 1960, Chapter 1.

[I.15] A. VON WEISS, *Einführung in die Matrizenrechnung zur Anwendung in der Elektrotechnik*, R. Oldenbourg, München, 1961.

[I.16] G. ZERLINGER, *Matrix Analysis Applied to Transistor Amplifier Design*, J. Brit I.R.E., 25, p. 107-112, 1963.

[I.17] R. ZURMÜHL, *Matrizen*, Springer-Verlag, Berlin, 1958.

APPENDIX II

SINGLE-TUNED BANDPASS FILTERS

In this appendix single-tuned bandpass filters as used in the amplifiers analyzed in this book will be considered having regard to frequency-dependent properties as well as to power losses.

II.1 Frequency-Dependent Properties of a Single-Tuned Bandpass Filter

In Fig. II.1 a single-tuned bandpass filter, or single-tuned circuit as it is usually referred to, with elements L, C and G connected in parallel is shown.

For the admittance of this circuit we may write:

$$Y = G + j\omega C + \frac{1}{j\omega L}, \tag{II.1.1}$$

or:

$$Y = G\left\{1 + j\left(\frac{\omega C}{G} - \frac{1}{G\omega L}\right)\right\},$$

$$= G\left\{1 + j\left(\frac{\omega_0 C}{G}\cdot\frac{\omega}{\omega_0} - \frac{1}{\omega_0 LG}\cdot\frac{\omega_0}{\omega}\right)\right\}. \tag{II.1.2}$$

Now the quality factor Q of a tuned circuit equals:

$$Q = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 LG}. \tag{II.1.3}$$

Introducing moreover:

$$\beta = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}, \tag{II.1.4}$$

the admittance becomes:

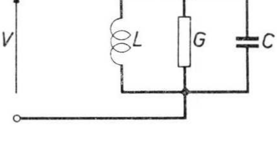


Fig. II.1. Representation of a single-tuned circuit with the elements L, C and G connected in parallel.

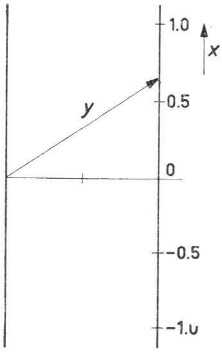


Fig. II.2. Polar plot of the normalized admittance of a single-tuned circuit.

$$Y = G\,(1 + \mathrm{j}\beta Q) \tag{II.1.5}$$

The quantity β given by Eq. (II.1.4) is a measure of the relative detuning of the circuit with respect to the reasonance angular frequency $\omega_0 = 2\pi f_0$. At the angular frequency ω_0, $\beta = 0$ and, according to Eq. (II.1.5) the admittance of the circuit is real.

By introducing:

$$x = \beta Q,$$

the admittance of the tuned circuit becomes:

$$Y = G\,(1 + \mathrm{j}x). \tag{II.1.6}$$

Denoting the admittance at the resonant frequency by $Y_0 = G$, the normalized admittance y equals:

$$y = 1 + \mathrm{j}x. \tag{II.1.7}$$

The quantity x, which forms the frequency-dependent part of the normalized admittance will be referred to as the normalized frequency.

In Fig. II.2 a polar plot of y as a function of x is shown. Fig. II.3 gives

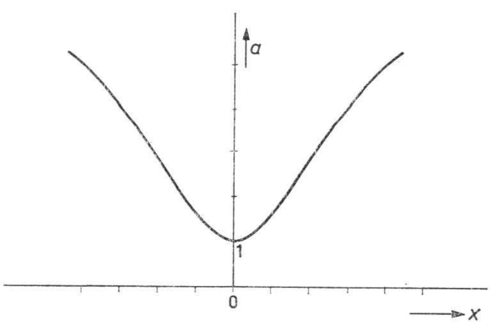


Fig. II.3. Normalized amplitude response curve of a single-tuned circuit; $a = |1 + \mathrm{j}x|$.

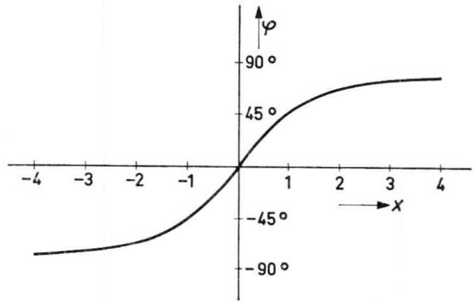


Fig. II.4. Phase angle $\phi = \tan^{-1} x$ of a single-tuned circuit.

the amplitude response curve $a = |y| = f(x)$ of the single-tuned circuit whereas Fig. II.4 represents the phase response curve $\varphi = f(x)$ in which $\varphi = \tan^{-1}x$. The envelope delay t_e of the circuit is shown in Fig. II.5. According to sub-section 2.5.3.5 the envelope delay as a function of x is given by:

$$t_e = \frac{Q}{\omega_0}\left\{1 + \left(\frac{\omega_0}{\omega}\right)^2\right\} \cdot \frac{1}{1+x^2} \simeq \frac{2Q}{\omega_0} \cdot \frac{1}{1+x^2}. \qquad \text{(II.1.8)}$$

For a tuned circuit with the elements L, C and R connected in series as represented in Fig. II.6 analogous expressions may be derived. For the impedance of the circuit we may write:

$$Z = R + \mathrm{j}\omega L + \frac{1}{\mathrm{j}\omega C}. \qquad \text{(II.1.9)}$$

The quality factor of the series-tuned circuit equals:

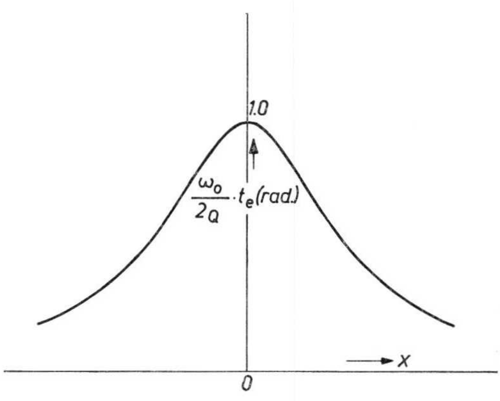


Fig. II.5. Envelope delay curve of a single-tuned circuit.

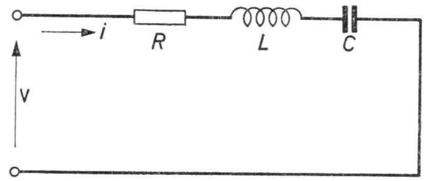


Fig. II.6. Representation of a single-tuned circuit with the elements L, C and R connected in series.

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}. \tag{II.1.10}$$

With Eqs. (II.1.4) and (II.1.6) the impedance becomes:

$$Z = R(1 + \mathrm{j}x). \tag{II.1.11}$$

The normalised impedance z equals:

$$z = 1 + \mathrm{j}x,$$

which is identical to the relation obtained for the normalised admittance of the parallel-tuned circuit. Hence Figs. II.2 to II.5 are also valid for the series-tuned circuit by changing y into z where necessary.

In the way described the frequency dependent properties of single-tuned circuits can easily be expressed provided the losses can be considered as a pure parallel damping or a pure series resistance. In practical circuits in most cases "mixed losses" will occur, but, except for low values of Q, these losses can be converted with sufficient accuracy in either of the two types considered.

II.2 Power losses in a Single-Tuned Bandpass Filter

If a single tuned circuit is inserted between a source and a load, power delivered by the source is lost in the parallel damping or the series resistance of the tuned circuit.

Fig. II.7 shows an equivalent circuit diagram for a parallel-tuned circuit with source and load. In this figure:

G_S denotes the conductance of the source,

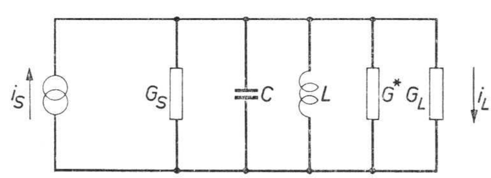


Fig. II.7. Equivalent diagram of a parallel-tuned circuit with source and load connected.

G_L is the load conductance, and

G^* is the damping of the tuned circuit itself.

The imaginary parts of source and load admittances are assumed to be contained in the tuning elements of the circuit.

At resonance the equivalent circuit of Fig. II.7 can be simplified to Fig. II.8. The power developed in the load is:

$$P_L = \frac{i_L^2}{G_L} = \left\{ i_S \cdot \frac{G_L}{G_S + G^* + G_L} \right\}^2 \cdot \frac{1}{G_L}, \tag{II.2.1}$$

whilst the power available from the source is:

$$P_{Sav} = \frac{i_S^2}{4\, G_S} \,. \tag{II.2.2}$$

The transducer gain is therefore:

$$\Phi_t = \frac{P_L}{P_{Sav}} = \frac{4\, G_S G_L}{(G_S + G^* + G_L)^2} \,. \tag{II.2.3}$$

Now, the quality factor Q of the loaded circuit is given by:

$$Q = \frac{\omega_0 C}{G_S + G^* + G_L}, \tag{II.2.4}$$

and the quality factor Q_0 of the unloaded circuit by:

$$Q_0 = \frac{\omega_0 C}{G^*} \,. \tag{II.2.5}$$

The ratio of these quality factors will be denoted by:

$$w = \frac{Q}{Q_0} = \frac{G^*}{G_S + G^* + G_L} \,. \tag{II.2.6}$$

Hence, with Eqs. (II.2.6) and (II.2.3):

$$\Phi_t = \frac{4\, G_S G_L}{(G_S + G_L)^2} \cdot (1 - w)^2 \,. \tag{II.2.7}$$

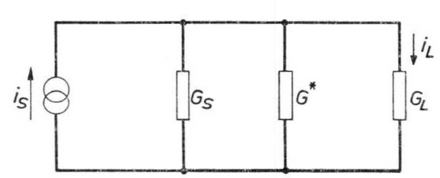


Fig. II.8. Equivalent circuit diagram of a parallel tuned circuit at resonance.

Eq. (II.2.7) represents the transducer gain, or better, transducer loss of a single-tuned circuit from a source having a damping G_S to a load with a damping G_L. The first factor represents the mismatch losses Φ_{mm}. When $G_L = G_S$ this term is unity.

The second term represents the insertion losses of the tuned circuit. These losses can be minimized by making the quality factor of the non-loaded circuit, Q_0, as large as possible, and that of the loaded circuit, Q, as small as possible. However, in practice the value of Q_0 will be limited by practical considerations or by stability requirements, whereas the value of Q will have to meet selectivity requirements. The limit imposed by stability requirements will become clear when it is realized that $(1 - w)^2$ represents the insertion losses. The losses may from necessity be such that they decrease the loop-gain of each stage to a value at which stability is ensured. Denoting these insertion losses by Φ_i gives:

$$\Phi_i = (1 - w)^2, \tag{II.2.8}$$

and

$$\Phi_t = \Phi_{mm} \cdot \Phi_i. \tag{II.2.9}$$

For a series connected single-tuned bandpass filter with a source with resistance R_S and a load with resistance R_L analogous expressions for Φ_{mm} and Φ_i can be derived:

$$\Phi_{mm} = \frac{4\, R_S R_L}{(R_S + R_L)^2}, \tag{II.2.10}$$

and

$$\Phi_i = (1 - w)^2 = \left(1 - \frac{R^*}{R_S + R^* + R_L}\right). \tag{II.2.11}$$

Here R^* denotes the resistance at resonance of the tuned circuit itself.

APPENDIX III

DOUBLE-TUNED BANDPASS FILTERS

In this appendix some of the properties of double-tuned bandpass filters are derived using the theory presented in Appendix I. The double-tuned bandpass filters are considered with respect to the frequency-dependent properties of input immittance, output immittance and transfer function. Also the power losses are calculated.

III.1 Four-Terminal Network Parameters of Double-Tuned Bandpass Filters

III.1.1 Y-PARAMETERS

In Fig. III.1 below the circuit diagram of a double-tuned bandpass filter with indirect inductive coupling is given. This figure which shows the various elements of the double-tuned bandpass filter is also used for defining the symbols related to these elements.

Furthermore, let the coefficient of coupling between the primary and secondary be defined by:

$$k = \frac{M}{\sqrt{L_p L_s}}. \tag{III.1.1}$$

The admittance parameters of the circuit then become:

$$\left.\begin{aligned} Y_{11} &= G_p + j\omega C_p + \frac{1}{j\omega L_p(1-k^2)}, \\ Y_{12} &= Y_{21} = j \cdot \frac{1}{\omega\sqrt{L_p L_s}} \cdot \frac{k}{1-k^2}, \\ Y_{22} &= G_s + j\omega C_s + \frac{1}{j\omega L_s(1-k^2)}. \end{aligned}\right\} \tag{III.1.2}$$

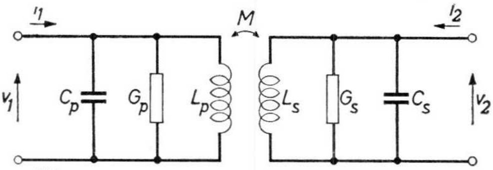


Fig. III.1. Double-tuned bandpass filter with indirect inductive coupling and parallel tuning of primary and secondary.

These expressions show that Y_{11} and Y_{22} denote admittances of single-tuned circuits with effective tuning inductances $L_p(1 - k^2)$ and $L_s(1 - k^2)$ respectively.

If primary and secondary are tuned to the same frequency it is obvious that

$$\omega_0{}^2 C_p L_p (1 - k^2) = \omega_0{}^2 C_s L_s (1 - k^2) = 1. \tag{III.1.3}$$

By introducing:

$$Q_p = \frac{\omega_0 C_p}{G_p}, \tag{III.1.4}$$

and:

$$Q_s = \frac{\omega_0 C_s}{G_s}, \tag{III.1.5}$$

and with the considerations of Appendix II. Eq. (III.1.2) may be written:

$$\left.\begin{aligned} Y_{11} &= G_p (1 + \mathrm{j}x_p), \\ Y_{12} &= Y_{21} = \mathrm{j} \cdot \frac{k}{\omega \sqrt{L_p (1 - k^2) L_s (1 - k^2)}}, \\ Y_{22} &= G_s (1 + \mathrm{j}x_s). \end{aligned}\right\} \tag{III.1.6}$$

Let furthermore:

$$G = \sqrt{G_p G_s}, \tag{III.1.7}$$

$$L = \sqrt{L_p (1 - k^2) L_s (1 - k^2)}, \tag{III.1.8}$$

and

$$Q = \sqrt{Q_p Q_s}, \tag{III.1.9}$$

which gives:

$$L = \frac{1}{\omega_0 G Q}. \tag{III.1.10}$$

With Eqs. (III.1.7) to (III.1.10) the admittance Y_{12} according to Eq. (III.1.6) can be written:

$$Y_{12} = \mathrm{j}kGQ \frac{\omega_0}{\omega}. \tag{III.1.11}$$

By putting:

$$k \cdot \frac{\omega_0}{\omega} = K, \tag{III.1.12}$$

and
$$q = KQ, \tag{III.1.13}$$
this expression is reduced to:
$$Y_{12} = \mathrm{j}qG. \tag{III.1.14}$$

Eq. (III.1.6) is thus simplified to:
$$\left.\begin{aligned} Y_{11} &= G_p\,(1 + \mathrm{j}x_p)\,, \\ Y_{12} &= Y_{21} = \mathrm{j}GQK\frac{\omega_0}{\omega} = \mathrm{j}Gq\,, \\ Y_{22} &= G_s\,(1 + \mathrm{j}x_s)\,. \end{aligned}\right\} \tag{III.1.15}$$

The parameters Y_{11} and Y_{22} are clearly frequency-dependent whereas Y_{12} may be considered frequency-independent provided $\omega_0/\omega = 1$, see Eq. (III.1.)

Identical expressions as derived above can be obtained for the admittance parameters of double-tuned bandpass filters with direct inductive or capacitive coupling. Because of the different arrangement of elements the effective tuning inductances or capacitances are different from those in the case of indirect inductive coupling considered above. In each case these effective inductances and capacitances are equal to the sum of the inductances and capacitances forming part of the admittances Y_{11} and Y_{22}.

Furthermore the frequency dependency of the Y_{12} parameter is different for the various types of coupling. This is of special importance when the frequency range of interest is such that the condition $\omega = \omega_0$ is not always fulfilled.

In Table III.1 relations giving the coefficient of coupling and the admittance parameter Y_{12} are compiled for double-tuned bandpass filters with the various types of coupling. The symbols L_m and C_m denote the coupling elements for direct inductive or capacitive coupling.

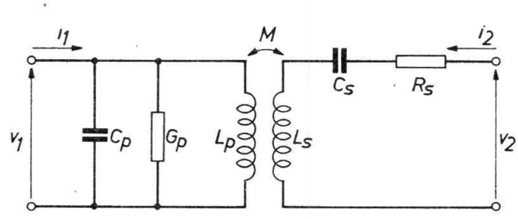


Fig. III.2. Double-tuned bandpass filter with indirect inductive coupling. The primary forms a parallel-tuned circuit whereas the secondary forms a series-tuned circuit.

TABLE III.1

Type of coupling of the bandpass filter	Transadmittance	Coefficient of coupling
indirect inductive coupling	$Y_{12} = Y_{21} = \mathrm{j}GQ\,k\,\frac{\omega_0}{\omega}$	$k = \frac{M}{\sqrt{L_p L_s}}$
capacitive π coupling	$Y_{12} = Y_{21} = -\mathrm{j}GQ\,k\,\frac{\omega}{\omega_0}$	$k = \frac{C_m}{\sqrt{(C_p+C_m)(C_s+C_m)}}$
inductive T coupling	$Y_{12} = Y_{21} = -\mathrm{j}GQ\,k\,\frac{\omega}{\omega_0}$	$k = \sqrt{\frac{C_p C_s}{(C_p+C_m)(C_s+C_m}}$
inductive π coupling	$Y_{12} = Y_{21} = -\mathrm{j}GQ\,k\,\frac{\omega_0}{\omega}$	$k = \sqrt{\frac{L_p L_s}{(L_p+L_m)(L_s+L_m)}}$
inductive T coupling	$Y_{12} = Y_{21} = -\mathrm{j}GQ\,k\,\frac{\omega_0}{\omega}$	$k = \frac{L_m}{\sqrt{(L_p+L_m)(L_s+L_m)}}$

III.1.2 *K*-PARAMETERS

For the indirect inductively coupled double-tuned bandpass filter with a parallel-tuned primary and a series-tuned secondary as represented in Fig.III.2 the *K*-parameters can be written:

$$\left.\begin{aligned} K_{11} &= G_p + \mathrm{j}\omega C_p + \frac{1}{\mathrm{j}\omega L_p}, \\ K_{12} &= K_{21} = -k\sqrt{\frac{L_s}{L_p}}, \\ K_{22} &= R_s + \mathrm{j}\omega L_s(1-k^2) + \frac{1}{\mathrm{j}\omega C_s}. \end{aligned}\right\} \qquad \text{(III.1.16)}$$

It follows from the expressions for K_{11} and K_{22} that the effective tuning inductances of primary and secondary now amount to L_p and $L_s(1-k^2)$.

Assuming that primary and secondary are tuned to the same angular frequency ω_0 such that:

$$\omega_0 C_p L_p = \omega_0\, C_s L_s(1-k^2) = 1, \qquad \text{(III.1.17)}$$

and introducing

$$Q_p = \frac{1}{\omega_0 L_p G_p}, \tag{III.1.18}$$

$$Q_s = \frac{\omega_0 L_s (1 - k^2)}{R_s}, \tag{III.1.19}$$

we obtain:

$$K_{12} = -K_{21} = -\frac{k}{\sqrt{1-k^2}} \cdot \sqrt{G_p \cdot R_s} \cdot \sqrt{Q_p Q_s}. \tag{III.1.20}$$

By putting:

$$q = \frac{k}{\sqrt{1-k^2}} \sqrt{Q_p Q_s}, \tag{III.1.21}$$

Eq. (III.1.16) becomes:

$$\left.\begin{aligned} K_{11} &= G_p (1 + \mathrm{j}x_p), \\ K_{12} &= -K_{21} = -q \sqrt{G_p R_s}, \\ K_{22} &= R_s (1 + \mathrm{j}x_s). \end{aligned}\right\} \tag{III.1.22}$$

Other practical versions of the parallel-series tuned double-tuned bandpass filter are those with capacitive — or inductive — T coupling for which analogous expressions may be derived. With respect to the effective tuning inductances or capacitances the remark made in the preceding sub-section applies.

III.2 Input and Output Immittance of a Double-Tuned Bandpass Filter

For a four-terminal network the input immittance can be expressed as:

$$Y_i = Y_{11} - \frac{Y_{12} Y_{21}}{Y_{22}},$$

or:

$$K_i = K_{11} - \frac{K_{12} K_{21}}{K_{22}}. \tag{III.2.1}$$

With Eqs. (III.1.15) and (III.1.22):

$$Y_i = K_i = G_p \left(1 + \mathrm{j}x_p + \frac{q^2}{1 + \mathrm{j}x_s}\right). \tag{III.2.2}$$

The output immittances of a fourpole network are given by:

$$\left.\begin{aligned} Y_0 &= Y_{22} - \frac{Y_{12}Y_{21}}{Y_{11}}, \\ K_0 &= K_{22} - \frac{K_{12}K_{21}}{K_{11}}. \end{aligned}\right\} \qquad \text{(III.2.3)}$$

With Eqs. (III.1.15) and (III.1.22) we obtain for the double-tuned bandpass filter:

$$Y_0 = G_s \left(1 + \mathrm{j}x_s + \frac{q^2}{1 + \mathrm{j}x_p}\right), \qquad \text{(III.2.4)}$$

and

$$K_0 = R_s \left(1 + \mathrm{j}x_s + \frac{q^2}{1 + \mathrm{j}x_p}\right). \qquad \text{(III.2.5)}$$

III.2.1 REDUCED INPUT- AND OUTPUT IMMITTANCES

It follows from Eqs. (III.2.2), (III.2.4) and (III.2.5) that the input and output immittances consist of a factor G or R and a frequency dependent factor. This frequency dependent factor is usually referred to as the *reduced input-* or *output immittance* respectively.

Now:

$$\left.\begin{aligned} Y_i &= G_p \cdot y_1, \\ K_i &= G_p y_i, \end{aligned}\right\} \qquad \text{(III.2.6)}$$

and

$$\left.\begin{aligned} Y_0 &= G_s \cdot y_0, \\ K_0 &= R_s \cdot y_0, \end{aligned}\right\} \qquad \text{(III.2.7)}$$

in which:

$$y_i = k_i = 1 + \mathrm{j}x_p + \frac{q^2}{1 + \mathrm{j}x_s}, \qquad \text{(III.2.8)}$$

$$y_0 = k_0 = 1 + \mathrm{j}x_s + \frac{q^2}{1 + \mathrm{j}x_p}. \qquad \text{(III.2.9)}$$

It appears from these equations that the reduced input and output immittances plotted in a polar diagram consist of the addition of the vectors $(1 + \mathrm{j}x)$ and $q^2/(1 + \mathrm{j}x)$. The first vector $(1 + \mathrm{j}x)$ represents a straight line in the complex plane parallel to the imaginary axis through the point $+ 1.0$ as shown in Fig. II.2. The second vector $q^2(/1 + \mathrm{j}x)$ represents a circle with diameter q^2 and centre at the point $(+0.5, 0)$, see Fig. III.3.

It follows from Eq. (III.2.8) that the polar diagram of the reduced input immittance can be constructed by adding to the vector $(1 + \mathrm{j}x_p)$ a vector

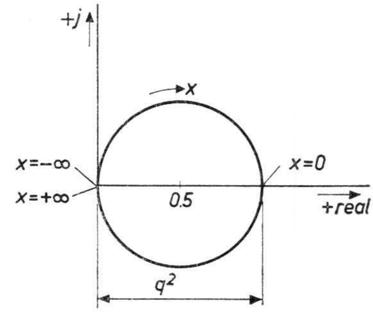


Fig. III.3. Polar plot of the vector $q^2/1 + jx$.

$q^2/(1 + jx_s)$ of which the extremity is situated on a circle moving along the $(1 + jx_p)$ line. This construction is given in Fig. III.4.

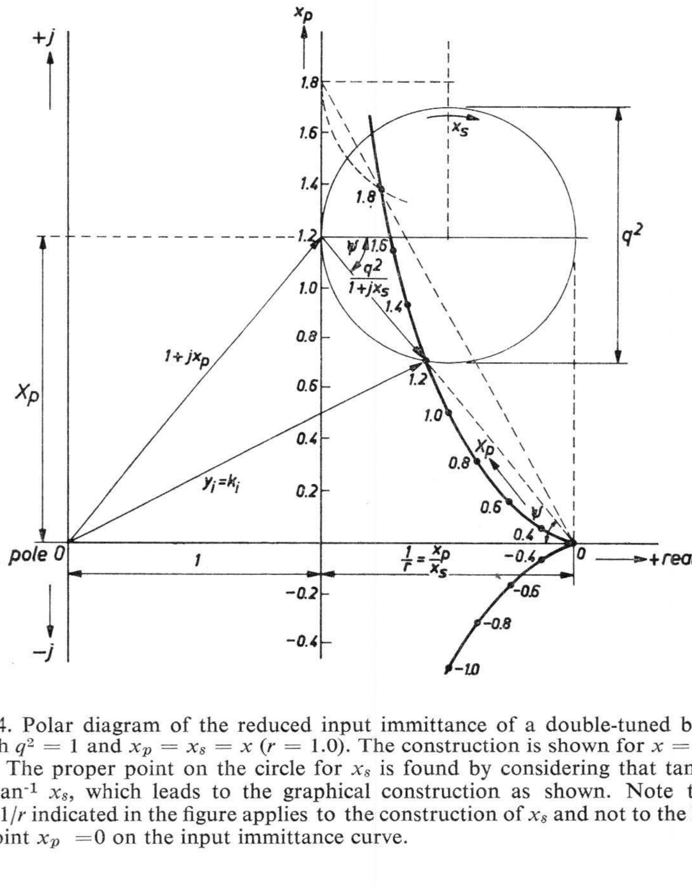


Fig. III.4. Polar diagram of the reduced input immittance of a double-tuned bandpass filter with $q^2 = 1$ and $x_p = x_s = x$ $(r = 1.0)$. The construction is shown for $x = 1.2$ and $x = 1.8$. The proper point on the circle for x_s is found by considering that $\tan^{-1} x_p = - r.\tan^{-1} x_s$, which leads to the graphical construction as shown. Note that the distance $1/r$ indicated in the figure applies to the construction of x_s and not to the location of the point $x_p = 0$ on the input immittance curve.

III.3 Transfer Function of a Double-Tuned Bandpass Filter

The transfer function of a double-tuned bandpass filter of which both primary and secondary consist of parallel-tuned circuits can best be expressed by means of its transimpedance Z_t. For the case the primary consists of a parallel-tuned circuit and the secondary of a series-tuned circuit the best method is found in the (forward) current transfer ratio H_t.

According to four-terminal network theory:

$$Z_t = -\frac{Y_{21}}{\Delta Y}, \tag{III.3.1}$$

and

$$H_t = -\frac{K_{21}}{\Delta K}. \tag{III.3.2}$$

For the sake of convenience we will use the reciprocal of the transfer functions Z_t and H_t in the calculations. With Eqs. (III.1.15) and (III.3.1):

$$\frac{1}{Z_t} = \mathrm{j}\sqrt{G_p G_s} \cdot \frac{(1 + \mathrm{j}x_p)(1 + \mathrm{j}x_s) + q^2}{q}; \tag{III.3.3}$$

and with Eqs. (III.1.22) and (III.3.2):

$$\frac{1}{H_t} = -\sqrt{G_p R_s} \cdot \frac{(1 + jx_p)(1 + \mathrm{j}x_s) + q^2}{q}. \tag{III.3.4}$$

III.3.1 NORMALIZED TRANSFER FUNCTION

It follows from Eqs. (III.3.3) and (III.3.4) that at $x = 0$:

$$\frac{1}{Z_{t0}} = \mathrm{j}\sqrt{G_p G_s} \cdot \frac{1 + q^2}{q}, \tag{III.3.5}$$

and

$$\frac{1}{H_{t0}} = -\sqrt{G_p R_s} \cdot \frac{1 + q^2}{q}. \tag{III.3.6}$$

The *normalized transfer function*, that is the transfer function of the bandpass filter relative to its value at $x = 0$ becomes therefore:

$$\frac{1}{z_t} = \frac{1}{h_t} = \frac{(1 + \mathrm{j}x_p)(1 + \mathrm{j}x_s) + q^2}{1 + q^2}. \tag{III.3.7}$$

By putting

$$\frac{x_s}{x_p} = r, \tag{III.3.8}$$

and

$$\sqrt{x_p x_s} = x\,, \tag{III.3.9}$$

Eq. (III.3.7) becomes:

$$\frac{1}{z_t} = \frac{1}{h_t} = \frac{1 + \mathrm{j}x\left(\sqrt{r} + \dfrac{1}{\sqrt{r}}\right) - x^2 + q^2}{1 + q^2}\,. \tag{III.3.10}$$

III.4 Amplitude Response Curve of a Double-Tuned Bandpass Filter

The amplitude response of the bandpass filter can be found by determining the modulus of the transfer function. The modulus of the reciprocal of the relative transfer function then represents the *normalized amplitude response curve*. By introducing an amplitude response curve shape factor:

$$\alpha = \frac{2q^2 - \left(r + \dfrac{1}{r}\right)}{1 + q^2}\,, \tag{III.4.1}$$

which is zero for an amplitude response curve of maximum flatness, it follows from Eq. (III.3.10) that:

$$a = \frac{1}{|z_t|} = \frac{1}{|h_t|} = \left\{1 - \alpha\frac{x^2}{1 + q^2} + \left(\frac{x^2}{1 + q^2}\right)^2\right\}^{\frac{1}{2}}. \tag{III.4.2}$$

In fig. III.5 amplitude response curves are plotted for $r = 1$ and $\alpha = -0.67$, 0 and 0.67 (that is for $q^2 = 0.5$, 1.0 and 2.0).

III.5 Envelope Delay Curve of the Double-Tuned Bandpass Filter

The normalized transfer function as given by eq. (III.3.7) can also be written as:

$$\frac{\{1 + \mathrm{j}\,(x_p + q)\}\,\{1 + \mathrm{j}\,(x_s - q)\}}{1 + q^2}\,. \tag{III.5.1}$$

Hence it follows for the phase angle φ of this transfer function:

$$\varphi = \tan^{-1}(x_p + q) + \tan^{-1}(x_s - q). \tag{III.5.2}$$

According to sub-section 2.5.3.5 the envelope delay t_e is given by

$$t_e = \frac{2Q}{\omega_0}\cdot\frac{\mathrm{d}\varphi}{\mathrm{d}x}\,,$$

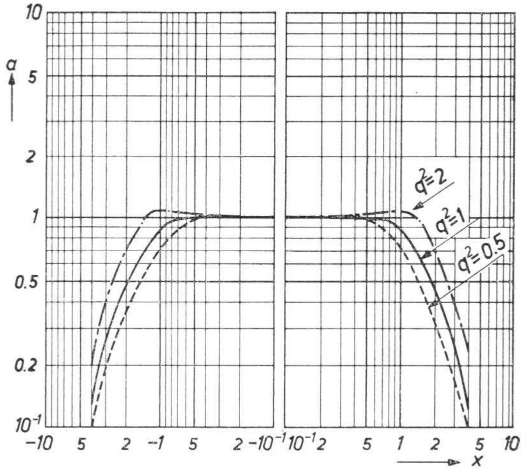


Fig. III.5. Amplitude response curves for a double-tuned bandpass filter with $r = 1$ and $q^2 = 0.5$, 1.0 and 2.0 respectively

in which $Q = \sqrt{Q_p Q_s}$, and $x = \sqrt{x_p x_s}$.

With Eq. (III.3.8) it follows from Eq. (III.5.2):

$$\frac{\mathrm{d}\varphi}{\mathrm{d}x} = \frac{1}{1 + \left(\frac{x}{\sqrt{r}} + q\right)^2} + \frac{1}{1 + (x\sqrt{r} - q)^2}. \tag{III.5.3}$$

In Fig. III.6 $\mathrm{d}\varphi/\mathrm{d}x$ is plotted as a function of x for $r = 1$ and $q^2 = 0.5$, 1.0 and 2.0.

III.6 The Transducer Gain of a Double-Tuned Bandpass Filter

Fig. III.7 shows an equivalent circuit of a double-tuned bandpass filter with parallel-tuned primary and secondary. In this circuit G_S denotes the source conductance and G_L the load conductance; the other symbols have the same meaning as in Fig. III.1, the asterisks accounting for the fact the corresponding dampings are those of the bandpass filter proper.

The transducer gain Φ_t at the tuning frequency is given by:

$$\Phi_t = 4\, G_S G_L\, |Z_{t0}|^2, \tag{III.6.1}$$

in which Z_{t0} is given by Eq. (III.3.5). Hence:

$$\Phi_t = 4\, G_S G_L \cdot \frac{1}{G_p G_s} \cdot \frac{q^2}{(1 + q^2)^2}. \tag{III.6.2}$$

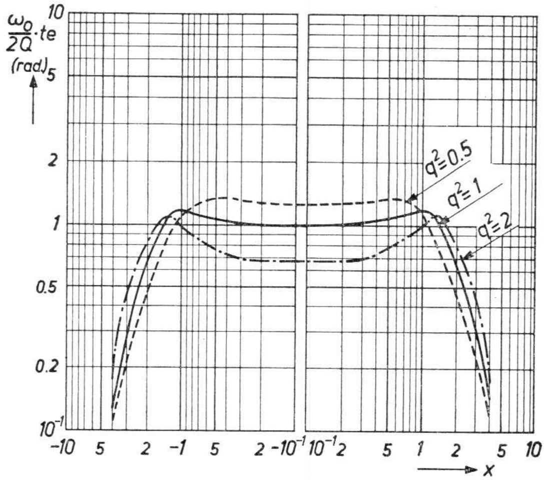


Fig. III.6. Envelope delay curves for a double-tuned bandpass filter with $r = 1$ and $q^2 = 0.5$, 1.0 and 2.0 respectively.

Denoting the ratio of the damping of the tuned circuit itself to the total damping by w, that is:

$$w = \frac{G^*}{G}, \tag{III.6.3}$$

it may be written:

$$G_p = G_S + G_p^*, \text{ or } G_S = (1 - w_p)G_p, \tag{III.6.4}$$

$$G_s = G_L + G_s^*, \text{ or } G_L = (1 - w_s)G_s, \tag{III.6.5}$$

whence:

$$\Phi_{tb} = (1 - w_p)(1 - w_s)\left\{\frac{2q}{1 + q^2}\right\}^2. \tag{III.6.6}$$

For the double-tuned bandpass filter with parallel-tuned primary and series-

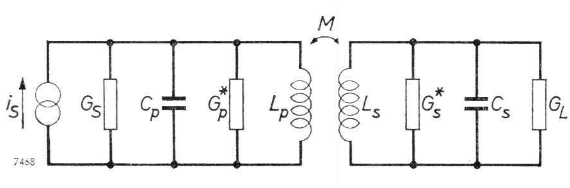


Fig. III.7. Equivalent circuit of a loaded double-tuned bandpass filter with parallel-tuned primary and secondary.

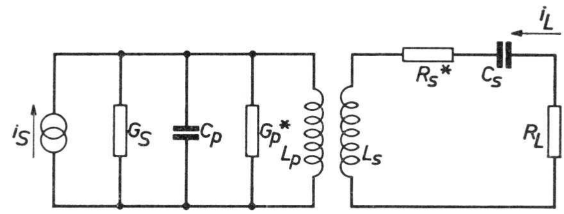


Fig. III.8. Equivalent circuit of a loaded double-tuned bandpass filter with parallel-tuned primary and series-tuned secondary.

tuned secondary as shown in Fig. III.8 analogous expressions can be derived. The transducer gain follows from

$$\Phi_t = 4\, G_S\, R_L\, |H_{t0}|^2, \tag{III.6.7}$$

which gives with Eq. (III.3.6):

$$\Phi_t = 4\, G_S R_L \cdot \frac{1}{G_p R_s} \cdot \frac{q^2}{(1+q^2)^2}. \tag{III.6.8}$$

With

$$w = \frac{R^*}{R},$$

$$R_s = R_L + R^*, \text{ or } R_L = (1 - w_s) R_s. \tag{III.6.9}$$

By substituting Eqs. (III.6.4) and (III.6.9) into Eq. (III.6.8), Eq. (III.6.6) is obtained.

Expressions (III.6.6) and (III.6.8) thus represent the transducer losses (gain) of a double-tuned bandpass filter. By putting

$$\Phi_p = 1 - w_p \tag{III.6.10}$$

$$\Phi_s = 1 - w_s \tag{III.6.11}$$

and

$$\Phi_q = \left(\frac{2q}{1+q^2}\right)^2 \tag{III.6.12}$$

the transducer loss becomes:

$$\Phi_{tb} = \Phi_p \cdot \Phi_s \cdot \Phi_p. \tag{III.6.13}$$

The quantities Φ_p and Φ_s thus represent the ratio of the source and load-damping to the total damping of the primary and secondary of the double-tuned bandpass filter, respectively.

BIBLIOGRAPHY

[III.1] H. H. VAN ABBE, *The Input Impedance and Transimpedance of I.F. Transformers*, Electronic Appl. Bull., **13**, p. 170-220, 1952 (No. 11/12).

[III.2] C. B. AIKEN, *Two Mesh-Tuned Coupled Circuits*, Proc. I.R.E., **25**, p. 230-272, 1937.

[III.3] B. G. DAMMERS, J. HAANTJES, J. OTTE and H. VAN SUCHTELEN, *Application of the Electron Tube in Radio Receivers and Amplifiers, I. Electronic Valves IV*, Philips Technical Library, Centrex Publishing Company, Eindhoven, 1948.

[III.4] W. F. DIL, *Principles of Measurements on Coupled Circuits*, Philips Research Rep., **5**, 91-115, 1950.

[III.5] R. FELDTKELLER, *Hochfrequenz Bandfilter*, Hirzel, Stuttgart, 1953.

[III.6] A. P. STERN, *Power Transfer in Double-Tuned Coupling Networks*, Proc. I.R.E., **44**, p. 1063-1064, 1956.

[III.7] B. D. H. TELLEGEN, *Coupled Circuits*, Philips Research Rep., **2**, p. 1-19, 1947

[III.8] A. G. W. UITJENS, *Television Receiver Design, Monograph I, I, F. Stages, Electronic Valves, Vol. VIII-A*, Philips Technical Library, Centrex Publishing Company, Eindhoven, 1953.

APPENDIX IV

DEFINITIONS OF GAIN IN POWER

The various definitions of gain in power used in this book are listed below. The definitions are in accordance with the "I.R.E. Standards on Electron Tubes: Definitions of Terms, 1957" (57.IRE 7.S2).

The *available power* of a source is defined as the maximum power which can be transferred from the source to a load.

Note: Maximum power transfer will take place when the immittance of the load is the conjugate of that of the source. The source immittance must have a positive real part.

The *power gain* of a four-terminal network is defined as the ratio of 1) the power that the network delivers to a specified load to 2) the power delivered to the input of the network. In Fig. IV.1:

$$\Phi = \frac{P_L}{P_i}. \qquad \text{(IV.1)}$$

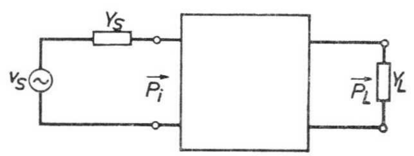


Fig. IV.1. The power gain of a four-terminal network equals $\Phi = P_L/P_i$.

Note: The power gain of a network is not defined unless its input immittance has a positive real part.

The *maximum power gain* of a four-terminal network is defined as the ratio of 1) the available power from the output of the network to 2) the power delivered to the input when the output is conjugately matched. In Fig. IV.2:

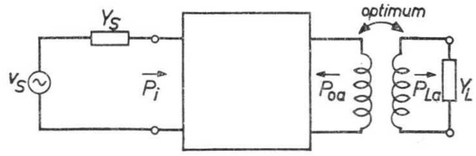


Fig. IV.2. The maximum power gain of a four-terminal network is obtained when the load immittance is conjugately matched to the output immittance of the amplifier. The conjugate matching is indicated symbolically by means of a transformer.

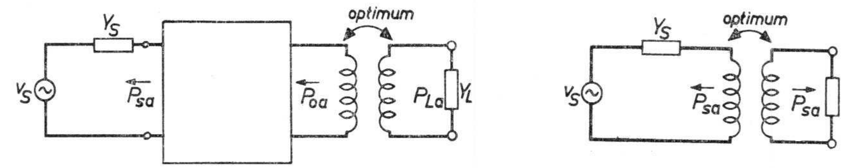


Fig. IV.3. The available power gain of a four-terminal network relates the power available from the output of the network to the power the source has available. Under conjugately matched conditions the power in the load equals the power available from the output of the amplifier, see Fig. 3a. The power delivered to the network by the source is, generally smaller than the power available from the source, compare Figs. 3a and 3b.

$$\Phi_{\mathrm{M}} = \frac{P_{La}}{P_i} = \frac{P_{oa}}{P_i}. \tag{IV.2}$$

The *maximum unilateralized power gain* of a four-terminal network is defined as the ratio of 1) the available power from the output of the network to 2) the power delivered to its input terminals, when the network is unilateralized.

The *available power gain* of a four-terminal network is defined as the ratio of 1) the available power from the output of the network to 2) the available power from the input source. In Fig. IV.3:

$$\Phi_a = \frac{P_{La}}{P_{Sa}} = \frac{P_{oa}}{P_{Sa}}. \tag{IV.3}$$

Note: The available power gain of a network is a function of the match between the source immittance and the immittance of the input of the network.

The *maximum available power gain* of a four-terminal network is defined as the available gain of the network when it is conjugately matched to source and load. In Fig. IV.4:

$$\Phi_{a\mathrm{M}} = \frac{P_{La}}{P_{Sa}} = \frac{P_{oa}}{P_{Sa}}. \tag{IV.4}$$

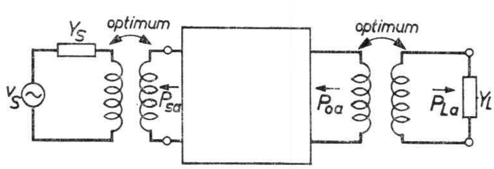


Fig. IV.4. The maximum available power gain of a four-terminal network is obtained when the input immittance of the amplifier is conjugately matched to the source immittance.

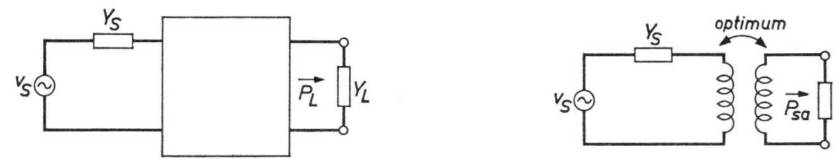


Fig. IV.5. The transducer gain of a four-terminal network equals $\Phi_t = P_L/P_S$av.

Note: The maximum available power gain of a network is not defined unless both input and output immittances of the network have positive real parts for arbitrary passive input and output terminations.

The *transducer gain* of a four-terminal network is defined as the ratio of 1) the actual power transferred from the output of the network to its load, to 2) the available power from the source driving the network. In Fig. IV.5:

$$\Phi_t = \frac{P_L}{P_{Sa}}. \qquad \text{(IV.5)}$$

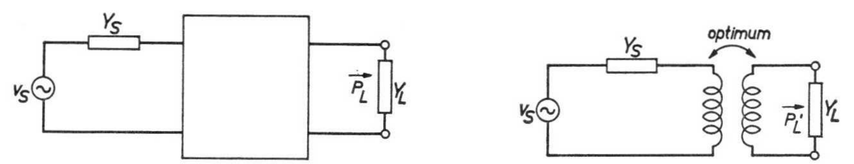


Fig. IV.6. The insertion gain of a four-terminal network is the ratio of the power the network, fed from a given source, delivers to the load to the power the source would deliver when the load was connected directly.

The *insertion gain* or *insertion loss* of a four-terminal network is defined as the ratio of 1) the actual power transferred from the output of the network to its load, to 2) the power that the same load would receive if driven directly by the source. In Fig. IV.6:

$$\Phi_i = \frac{P_L}{P_L'}. \qquad \text{(IV.6)}$$

APPENDIX V

MAXIMUM UNILATERALIZED POWER GAIN OF A TRANSISTOR

According to Appendix IV, the maximum power gain of a four-terminal network is obtained when it is conjugately matched at its output terminals. When a transistor, considered as a four-terminal network is matched accordingly it delivers the maximum unilateralized power gain when it is neutralized by a loss-less external network (unilateralized) so as to make the resultant feedback admittance equal to zero.

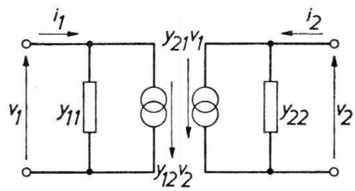


Fig. V.1. Equivalent admittance parameter fourpole network of a transistor.

In Fig. V.1 an admittance parameter equivalent circuit of a transistor is given. The input power P_i equals $v_1^2 g_{11}$ and the output power in the matched load G_L (which has a value equal to g_{22}) is:

$$P_o = \left(\frac{|y_{21}| \cdot v_1}{2}\right)^2 \cdot \frac{1}{G_L},$$

$$= \tfrac{1}{4}\,|y_{21}|^2\, v_1 \cdot \frac{1}{g_{11}}.$$

The maximum unilateralized power gain is therefore given by:

$$\Phi_{u\mathrm{M}} = \frac{|y_{21}|^2}{4\, g_{11} g_{22}}. \qquad \text{(V.1)}$$

It may be shown that $\Phi_{u\mathrm{M}}$ does not vary with the matrix environment so that:

$$\Phi_{u\mathrm{M}} = \frac{|h_{21}|^2}{4\, R_e(h_{11}) \cdot R_e(h_{22})}. \qquad \text{(V.2)}$$

APPENDIX VI

BOUNDARY OF STABILITY IN AN n-STAGE AMPLIFIER WITH $(n+1)$ SINGLE-TUNED BANDPASS FILTERS

Consider the reduced amplifier determinant given by Eq. (6.3.5), viz. [1]):

$$\delta_u = \left|\begin{array}{cccccccccc} 1 & u_n & - & - & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & - & - & 0 & 0 & 0 & 0 & 0 & 0 \\ - & - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - & - \\ 0 & 0 & - & - & 1 & u_5 & 0 & 0 & 0 & 0 \\ \cdots & & & & & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & - & - & 1 & 1 & u_4 & 0 & 0 & 0 \\ 0 & 0 & - & - & 0 & 1 & 1 & u_3 & 0 & 0 \\ 0 & 0 & - & - & 0 & 0 & 1 & 1 & u_2 & 0 \\ 0 & 0 & - & - & 0 & 0 & 0 & 1 & 1 & u_1 \\ 0 & 0 & - & - & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right| \begin{array}{l} \\ \\ \\ \\ \\ \cdots P_4 \\ \\ \\ \cdots P_1 \\ \cdots P_0 \\ \\ \end{array} \quad . \qquad \text{(VI.1)}$$

and let minor determinants P be defined as indicated. Then we may write:

$$\left.\begin{array}{lll} P_0 = 1, & & \\ P_1 = 1 - u_1 & \text{or} \; \dfrac{P_1}{P_0} & = 1 - u_1 , \\ P_2 = P_1 - u_2 P_0 & \text{or} \; \dfrac{P_2}{P_1} & = 1 - u_2 \dfrac{P_0}{P_1}, \\ P_3 = P_2 - u_3 P_1 & \text{or} \; \dfrac{P_3}{P_2} & = 1 - u_3 \dfrac{P_1}{P_2}, \\ P_n = P_{n-1} - u_n P_{n-2} & \text{or} \; \dfrac{P_n}{P_{n-1}} & = 1 - u_n \dfrac{P_{n-2}}{P_{n-1}}. \end{array}\right\} \qquad \text{(VI.2)}$$

---

[1]) The prefix n in the symbol ${}_nu$, denoting that ${}_nu$ applies to an n-stage amplifier, and the suffix g in the symbol u_g, denoting the value of u at the boundary of stability, are omitted in this appendix for simplicity in writing the various equations.

in which P_n equals δ which should be equated to zero to find the boundary of stability.

If all transistors and all single-tuned bandpass filters of the amplifier are assumed to be identical, all u's are equal (see Eq. (6.3.5)) and the quotient P_n/P_{n-1} may be written as a continued fraction. Consider, for example, the quotient P_5/P_4. According to Eq. (IV.2) this may be written as:

$$\begin{aligned}\frac{P_5}{P_4} &= 1 - u\frac{P_3}{P_4}, \\ &= 1 - \cfrac{u}{1 - u\cfrac{P_2}{P_3}}, \\ &= 1 - \cfrac{u}{1 - \cfrac{P_1}{P_2}}, \\ &= 1 - \cfrac{u}{1 - \cfrac{u}{1 - \cfrac{u}{1 - \cfrac{u}{1 - u}}}}.\end{aligned} \tag{VI.3}$$

Hence, for solving $P_n = \delta$, the theory of continued fractions may be applied [1])

For finding the boundary of stability of this n-stage amplifier the smallest positive root of u must thus be determined from

$$\frac{P_n}{P_{n-1}} = 1 - u\frac{P_{n-2}}{P_{n-1}}, \tag{VI.4}$$

for $P_n = 0$.

We try to solve this system by putting:

$$P_{n+1} = A\varepsilon^{an}, \tag{VI.5}$$

which, indeed, is possible provided:

$$\varepsilon^{2a} - \varepsilon^{a} + u = 0, \tag{VI.6}$$

---

1) See e.g. H. S. WALL, *Analytical Series of Continued Fractions*, Van Nostrand and Co. New-York, 1948.

$$\left.\begin{aligned} \varepsilon^{\alpha_1} &= \tfrac{1}{2}(1+\sqrt{1-4u}), \\ \varepsilon^{\alpha_2} &= \tfrac{1}{2}(1-\sqrt{1-4u}). \end{aligned}\right\} \qquad \text{(VI.7)}$$

Thus we have:

$$P_n = A\varepsilon^{\alpha_1 n} + B\varepsilon^{\alpha_2 n}, \qquad \text{(VI.8)}$$

in which A and B are functions of u not depending on n and satisfying:

$$\left.\begin{aligned} P_0 &= 1 = A + B \\ P_1 &= 1 - u = A\varepsilon^{\alpha_1} + B\varepsilon^{\alpha_2} \end{aligned}\right\} \qquad \text{(VI.9)}$$

Solving A and B from Eq. (VI.9) and substitution in Eq. (VI.8) yields:

$$P_n = \frac{\varepsilon^{\alpha_2} - 1 + u}{\varepsilon^{\alpha_2} - \varepsilon^{\alpha_1}}\,\varepsilon^{\alpha_1 n} + \frac{\varepsilon^{\alpha_1} - 1 + u}{\varepsilon^{\alpha_1} - \varepsilon^{\alpha_2}}\,\varepsilon^{\alpha_2 n}.$$

With Eq. (VI.7) the last expression can be written as:

$$P_n = \frac{1}{\sqrt{1-4u}}\left\{\varepsilon^{\alpha_1(n+2)} - \varepsilon^{\alpha_2(n+2)}\right\}. \qquad \text{(VI.10)}$$

Zeros of P_n are those values of u that make

$$\varepsilon^{\alpha_1(n+2)} = \varepsilon^{\alpha_2(n+2)} \qquad \text{(VI.11)}$$

with a possible exception of $u = \frac{1}{4}$.

Since

$$\varepsilon^{\alpha_1} \cdot \varepsilon^{\alpha_2} = u, \qquad \text{(VI.12)}$$

we have from (VI.11):

$$\varepsilon^{2\alpha_1(n+2)} = u^{n+2} \cdot \varepsilon^{j2\pi k}; \qquad \text{(VI.13)}$$

$$k = 0, 1, 2, 3, \ldots$$

Taking the 2 $(n+2)$th root from the last expression, we get with Eq. (VI.7):

$$\tfrac{1}{2}(1 + \sqrt{1-4u}) = \sqrt{u} \cdot \varepsilon^{j\frac{k\pi}{n+2}}. \qquad \text{(VI.14)}$$

Solving for u yields:

$$u = \frac{1}{4\cos^2 \dfrac{\pi k}{n+2}}. \qquad \text{(VI.15)}$$

Since we have derived an expression for every possible root for $\sqrt{u}$, we find every root for u twice. Moreover, instead of finding every root twice, we found it four times, since, by squaring repeatedly, we also solved:

$$\varepsilon^{2a_2(n+2)} = u^{\,n+2} \cdot \varepsilon^{\,j\,2\,\pi k}\,.$$

The value $u = \frac{1}{4}$ found from Eq. (VI.15) for $k = 0$, $k = n + 3$ and $n = \infty$ is a suspected value since it is also a zero of the denominator of Eq. (VI.10).

Indeed, further consideration shows:

$$\lim_{u \to \frac{1}{4}} P_{n+1} = 0$$

Because we are interested in the smallest root of u, we must take $k = 1$ which gives:

$$u = \frac{1}{4 \cos^2 \dfrac{\pi}{n+2}}\,. \qquad \text{(VI.16)}$$

APPENDIX VII

INFLUENCES OF IMPEDANCES IN SERIES WITH THE TRANSISTOR LEADS

When connecting a transistor to its external circuitry in an amplifier series-impedances for signals of the desired frequencies will be introduced between the (lumped) components of the circuitry and the transistor. These series-impedances may either be due to parasitic effects or may be provided for intentionally.

Series-impedances due to parasitic effects are, for example, the inductances of the wires connecting the transistors to the circuitry and the not completely decoupled d.c. biasing resistor in the lead common to input and output circuits of a transistor. Series-reactances due to the use of non-ideal transformers may also be considered to belong to this group. Series-impedances which are connected into the circuit intentionally are, for example, resistances in the collector lead of the transistor to prevent parasitic oscillations at lower frequencies and impedances which are connected in series with the transistor lead in order to increase the stability of the amplifier in a particular range of frequencies.

Because these series-impedances are, generally, constant over the pass band of the amplifier their influence can most easily be taken into account by calculating the parameters of a new four terminal network including the transistor as well as the series-impedances.

In the following sections we will confine ourselves to an admittance parameter representation of the transistors.

VII.1 Calculation of the Admittance Parameters of a Transistor with Impedances in its Leads

In Fig. VII.1 a schematic diagram is given of a transistor with impedances Z_1, Z_2 and Z_3 in leads 1, 2 and 3 respectively. The transistor four-terminal network will be denoted by the matrix:

$$||y|| = \begin{Vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{Vmatrix} . \qquad \text{(VII.1)}$$

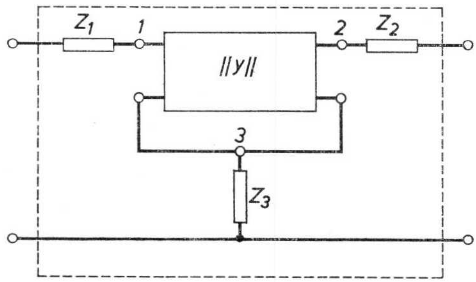


Fig. VII.1. Transistor four-terminal network with series-impedances in the transistor leads.

Using Table 1 of Appendix I this admittance matrix may be transformed into an impedance matrix $||z||$ as:

$$||z|| = \begin{Vmatrix} \dfrac{y_{22}}{\Delta y} & \dfrac{-y_{12}}{\Delta y} \\ \dfrac{-y_{21}}{\Delta y} & \dfrac{y_{11}}{\Delta y} \end{Vmatrix}, \tag{VII.2}$$

which may also be written as:

$$||z|| = \begin{Vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{Vmatrix}. \tag{VII.3}$$

The impedance matrix of the four-terminal network including the series-impedances is now obtained as:

$$||z||' = \begin{Vmatrix} z_{11} + Z_1 + Z_3 & z_{12} + Z_s \\ z_{21} + Z_3 & z_{22} + Z_2 + Z_3 \end{Vmatrix}, \tag{VII.4}$$

or:

$$||z'|| = \begin{Vmatrix} z_{11}' & z_{12}' \\ z_{21}' & z_{22}' \end{Vmatrix}. \tag{VII.5}$$

Converting the impedance matrix of Eq. (VII.1.5) into an admittance matrix we get:

$$||y'|| = \begin{Vmatrix} \dfrac{z_{22}'}{\Delta z'} & \dfrac{-z_{12}'}{\Delta'} \\ \dfrac{-z'_{21}}{\Delta z'} & \dfrac{z'_{11}}{\Delta z'} \end{Vmatrix}, \tag{VII.6}$$

which we put equal to:

$$||y'|| = \begin{Vmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{Vmatrix}. \tag{VII.1.7}$$

The dashes in Eqs. (VII.4) to (VII.7) refer thus to a new four terminal network including the transistor as well as the impedances Z_1, Z_2 and Z_3. After some calculations it follows that the admittance parameters of the new four-terminal network are given by:

$$y_{11}' = \frac{z_{22}'}{\Delta z'} = \frac{z_{22} + Z_2 + Z_3}{\Delta z'} = \frac{\dfrac{y_{11}}{\Delta y} + Z_2 + Z_3}{\Delta z'},$$

$$= \frac{y_{11} + \Delta y \cdot (Z_2 + Z_3)}{1 + \Sigma y \cdot Z_3 + y_{11}Z_1 + y_{22}Z_2 + \Delta y \cdot \Sigma Z^2}, \tag{VII.8}$$

$$y_{12}' = -\frac{z_{12}'}{\Delta z'} = -\frac{z_{12} + Z_3}{\Delta z'} = -\frac{-\dfrac{y_{12}}{\Delta y} + Z_3}{\Delta z'},$$

$$= \frac{y_{12} - \Delta y \cdot Z_3}{1 + \Sigma y \cdot Z_3 + y_{11}Z_1 + y_{22}Z_2 + \Delta y \cdot \Sigma Z^2}, \tag{VII.9}$$

$$y_{21}' = -\frac{z_{21}'}{\Delta z'} = -\frac{z_{21} + Z_3}{\Delta z'} = -\frac{-\dfrac{y_{21}}{\Delta y} + Z_3}{\Delta z'},$$

$$= \frac{y_{21} - \Delta y \cdot Z_3}{1 + \Sigma y \cdot Z_3 + y_{11}Z_1 + y_{22}Z_2 + \Delta y. \cdot \Sigma Z^2}, \tag{VII.10}$$

$$y_{22}' = \frac{z_{11}'}{\Delta z'} = \frac{z_{11} + Z_1 + Z_3}{\Delta z'} = \frac{\dfrac{y_{22}}{\Delta y} + Z_1 + Z_3}{\Delta z'},$$

$$= \frac{y_{22} + \Delta y\,(Z_1 + Z_3)}{1 + \Sigma y \cdot Z_3 + y_{11}Z_1 + y_{22}Z_2 + \Delta y \cdot \Sigma Z^2}. \tag{VII.11}$$

In these expressions:

$$\Delta y = y_{11}y_{22} - y_{12}y_{21}, \tag{VII.12}$$

$$\Sigma y = y_{11} + y_{12} + y_{21} + y_{22}, \tag{VII.13}$$

and

$$\Sigma Z^2 = Z_1Z_2 + Z_1Z_3 + Z_2Z_3. \tag{VII.14}$$

INDEX